

WIGGLER FOR ILC COOLER

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Abstract

We represented the development of concept of wiggler with linear piecewise field dependence. This eliminates nonlinearities in wiggler. This type of wiggler can be recommended for usage in ILC cooler.

INTRODUCTION

It is known, that minimal emittances in a wiggler dominated damping ring have the form [1]

$$(\gamma\mathcal{E}_x) \cong \frac{1}{2} \cdot \tilde{\lambda}_c \bar{\beta}_x (1 + K^2/2) K / \tilde{\lambda} \quad (1)$$

$$(\gamma\mathcal{E}_y) \cong \frac{1}{2} \cdot \tilde{\lambda}_c \bar{\beta}_y K / \tilde{\lambda}, \quad (2)$$

where $\bar{\beta}_{x,y}$ – are averaged values of envelope functions in the wiggler, $\lambda = 2\pi \cdot \tilde{\lambda}$, is the wiggler period, $K = eH_{\perp} \tilde{\lambda} / mc^2$. Together with the cooling time

$$\tau_{cool} \cong \frac{3}{2} \cdot (\tilde{\lambda}^2 / r_0 c \bar{K}^2 \gamma), \quad (3)$$

where \bar{K}^2 represents K^2 averaged over the ring, these formulas define the cooling dynamics under SR. One can see that equilibrium invariant emittances *do not depend on energy*. In addition, quantum equilibrium *vertical emittance and the cooling time* do not depend on the wiggler period. Vertical emittance in (2) defined by opening angle of radiation in vertical plane $\cong 1/\gamma$. According to (1), radial emittance asks for smaller period, meanwhile nonlinearities in vertical dependence of field generated by pure sinusoidal dependence

$$B_y(y, s) = B_0 \text{Sin}\left(\frac{s}{\tilde{\lambda}} \left(1 + \frac{3y^2}{8\tilde{\lambda}^2} + \frac{5y^4}{192\tilde{\lambda}^4} + \dots\right)\right) \quad (4)$$

ask for the longer period.

While for NLC/JLC time pattern it was possible to work without wigglers at all, just rising operational energy to ~3-5 GeV and get parameters better, than in official variant [4], for ILC pattern this is not possible. This is due to the extremely big circumference and low field value in bending magnet required. For 5GeV and $R=1$ km, $H = pc/300R \cong 167G$ only. So wiggler becomes inevitable here. We identified some improvements of field quality, which can be easily done practically for any wiggler [3], [4], [5]. There for the first time we suggested and realized a *wide pole* wiggler.

Other things proposed were active pole correction at each poles and extended law of tapering for zero average trajectory displacement. Meanwhile one thing remained unsolved until [2] identified how to build the wiggler practically without nonlinearities.

Our look at dipole wiggler is that it can be represented as series of partial *transversely oriented quadrupoles*, so the particle passes them *across* longitudinal coordinate s through central region and wiggling along x , [6].

One can see that nonlinear terms associated with the higher derivatives. So as particle is moving along x ,

towards and backwards from viewer in Fig.1, one can easily see, why the wiggler always focusing particle in vertical direction: with reverse of longitudinal field direction in neighbouring quads, the angle is reversed too.

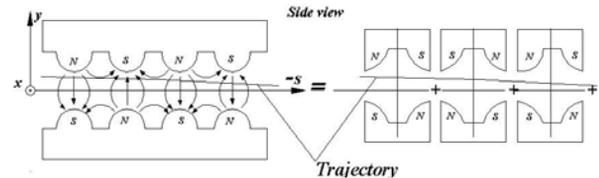


Figure 1: Representation of wiggler as series of quadrupoles with *transverse* orientation.

So the basic concept emerging is if the field is represented as series of quadrupoles, one needs to have the field in these quadrupoles as linear as possible.

FIELDS IN A WIGGLER

Let us consider a wiggler with finite pole width across transverse coordinate which directed along Cartesian coordinate x , vertical axes y and longitudinal one $-s$. Particles supposed to move along s -axis. If zero x lies in the medial plane $y=0$ and transversely coincides with the middle of the pole, then analytical representation for the wiggler fields is the following [3], [5]

$$B_x(x, y, s) = -\frac{xy}{4} B''(s) + 2S(s)xy + \frac{x^3y + xy^3}{48} B^{(IV)} + \dots \quad (5)$$

$$B_y = B(s) + S(s)(x^2 - y^2) - \frac{x^2 + 3y^2}{8} B''(s) + D(s) \cdot (x^4 + y^4 - 6x^2y^2) + \dots$$

$$B_s = y \cdot B'(s) - \frac{x^2y + y^3}{8} B'''(s) + \frac{3x^2 - y^3}{3} S'(s) + D'(s) \frac{5x^4 - 10x^2y^2 + y^5}{5} + \dots$$

where $B(s)$, $S(s)$, $D(s)$ stand for generation functions for dipole, sextupole, decapole, ... field along the wiggler respectively. Here derivatives are taken along s . So these expansions are valid for particular choice of coordinates. We also suggested that plane sy is a plane of symmetry for the wiggler. In this case function $B(s)$ can be defined simply by measurements along s -axis or by calculations, as in this case $x=y=0$. After that, all necessary derivatives appeared in (1), can be taken numerically with necessary accuracy. So now the difference between calculated (or measured) transverse field variations and found from (1) can be treated as higher harmonics, with sextupole $S(s)$ as a lowest one. Or with other words, if one has transverse field roll-off, say at field maximum, calculated or measured, sextupole component $S(s)$, having dependence $\sim x^2$ can be identified *after subtraction* the terms, associated with derivatives. Other harmonics, such as $D(s)$ can be found in the same manner, taking in consideration dependence $\sim x^4$ and so on. This gives a *simple and powerful recipe* for representation of wiggler

field for the purposes of calculation of nonlinearities and dynamical aperture associated with this.

In case, when the poles are much wider, than aperture opening, the field can be described by single generating function $B(s)$, which stands for the vertical dipole field at the axis[3].

$$B_y(y,s) = B(s) - \frac{y^2}{2!} B''(s) + \frac{y^4}{4!} B^{(IV)}(s) - \dots$$

$$B_s(y,s) = y \cdot B'(s) - \frac{y^3}{3!} B'''(s) + \frac{y^5}{5!} B^{(V)}(s) - \dots, \quad (6)$$

One can easily conclude from (2), that if longitudinal field distribution is a (piecewise) linear one, then all derivatives are going to be zero. For the wiggler, of cause linear dependence everywhere is not possible, just piecewise smooth linear function, so these derivatives emerge at points where the field reaches its extremes, see Fig 2. Let us check if these points can give an input to the vertical kick however. Formally as the second derivative is proportional to the local curvature, it can be very big at these points. Fortunately there might be two factors helping in kick reduction. First one is that the vertical forces acting to the particle, is proportional to the transverse angle, $\alpha \cong \alpha'$, which is zero at points of extreme. Other is that the second derivative is limited by physical means, going to be $B''(s) \cong B_{\max} / a^2$, where a stands for characteristic dimension, which might be an aperture opening at this place.

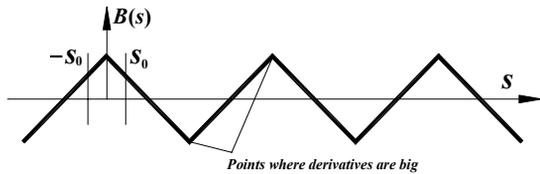


Figure 2: Longitudinal profile of magnetic field with linear dependence between extremes.

Vertical field B_y defines local amplitude of horizontal wiggling angle,

$$\alpha(s) = \frac{1}{(HR)} \int_0^s \frac{B_y(\sigma) d\sigma}{\cos \alpha(\sigma)} \cong \frac{1}{(HR)} \int B_y(\sigma) d\sigma, \quad (7)$$

Cosine in denominator is due to the fact that integration is going along straight line. Meanwhile B_s defines vertical kick $y' \cong \alpha \cdot B_s$. The regions around maximal or minimal values of field, the particle passing with zero angle with respect to longitudinal axis. So we can suggest, that around maximal field value the angle depends like odd function of longitudinal coordinate s , $\alpha \cong k_1 \cdot s + k_3 \cdot s^3 + \dots$, where coefficients k_1, k_3, \dots defined from (2) and (4). While particle passes from $-s_0$ to $+s_0$, where s_0 is arbitrary coordinate, not far from maximum, so resulting nonlinear vertical kick is proportional to the following

$$y' \cong -\frac{y^3}{3!(HR)} \int_{-s_0}^{s_0} (k_1 s + k_3 s^3 + \dots) B'''(s) ds \quad (8)$$

Calculating this integral one can obtain

$$y' \cong \frac{s_0 y^3}{3(HR)} (k_1 + \frac{1}{4} k_3 s_0^2 + \dots) \cdot B''_s(s_0) \quad (9)$$

So one can see if second derivative of field distribution at this point $s = s_0$ is zero, then the integral (6) goes to be a zero. Although assumption was about symmetry of field distribution around maximum/minimum of magnetic field this result still valid under broader conditions, as soon as second derivation is zero at the ends of integration interval.

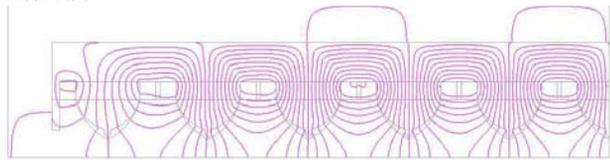


Figure 3: Magnetic lines for $\sim 1/4, 1/2, 1$ tapering.

Again, now one can see that requirements of nonlinearity for linear field dependence all derivatives are disappeared except in points around extremes. But namely here the angle is zero, so there is no nonlinear action at all. Of cause this idealized jump of curve slope, represented in Fig.1 is not possible in practice, but it was interesting to investigate on how close to this distribution it is possible to approach.

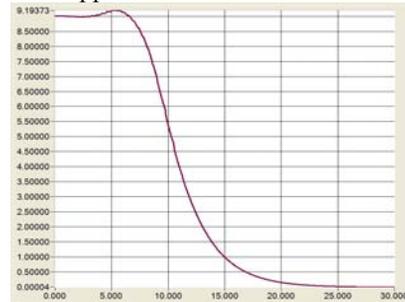


Figure 4: Vertical field across the regular pole in median plane.

Eleven pole wiggler in Fig. 3 has period 25 cm and the pole width 30 cm, shimmed by slight increase of profile at the ends. Poles approximated by cylinders with radius 5.5 cm with ribs having height 0.83 cm, width 0.8 cm with fillet radius 0.4 cm. So the pole in lowest point runs 2.7 cm above medial plane. All iron in this model is Steel 1010. Current running in central pole coil is 60kA. In the end coils the current is ~ 14 and ~ 43 kA respectively. Coil itself has dimensions 2.3×2 cm². Supposedly this is a coil with SC wires. Details of cryostat are not shown, just cold mass only. Tracking in this field done with help of numerical code [7]. Result is represented in Figs. 5 and 6. Vertical focusing could not be eliminated, of course. Once again, the purpose of our investigation is to make this focusing linear.

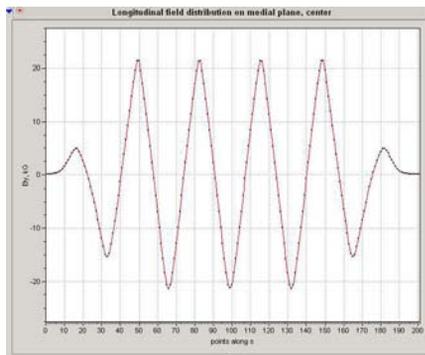


Figure 5: Longitudinal field, kG, dependence (as function of s) at $x=0$ calculated for the wiggler in Fig. 3 with tapering $\sim 1/4, 3/4, 1$. Right point on this picture corresponds to $s=150$ cm.

WIGGLER HARDWARE

Vacuum chamber occupied by the beam has room temperature. Vacuum chamber interlaced by a sleeve, cooled to liquid nitrogen temperature, Fig. 6.

To see how our considerations can be applied to the real case, we have modeled real wiggler. 3D model was erected and the field calculations with MERMAID have been carried for the hardware represented in Figs. 6, 7.

One can see from Fig. 8 that at least for ± 1 -cm vertical coordinate there is no nonlinear action to the particle.

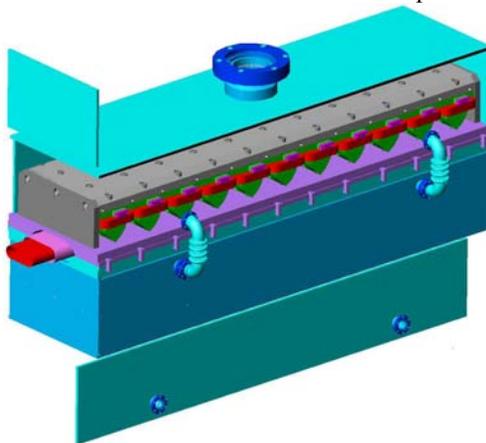


Figure 6: Cold mass view. Two upper side walls opened.

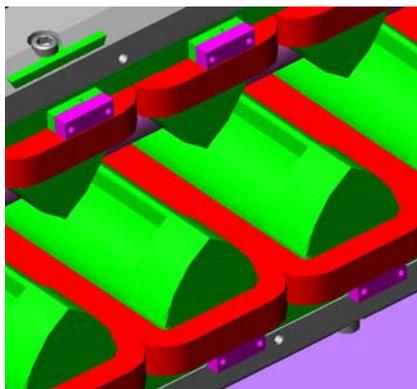


Figure 7: Wiggler cold mass fragment magnified.

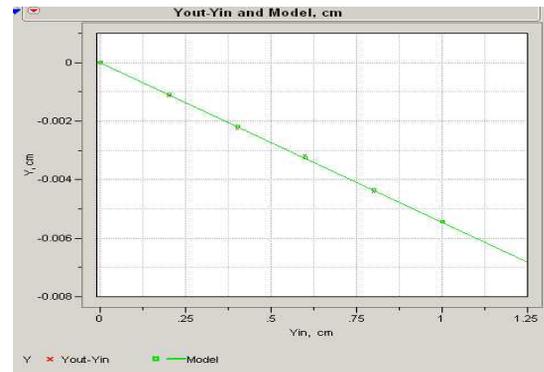


Figure 8: Difference between exit and entrance y – coordinates and the linear model fit.

CONCLUSIONS

On the basis of new approach to the wiggler design [2] we developed engineering of wiggler having piecewise linear field dependence along particle trajectory. It is based on presentation of wiggler as series of partial quadrupoles installed transversely to the direction of particle motion. The field can be generated by poles, having a profile of hyperbolas. Other recommendation can be given is that the pole must be as wide as possible in a sense that transverse slope of vertical field must be small.

Minimal period required for minimization of emittance now allowed as the nonlinearities in vertical motion do not manifest due to zigzag dependence of vertical field.

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