

Review of Beam-Beam Interaction in Electron-Positron Circular Colliders

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1 Introduction

For the last few years theory of beam-beam interaction in electron-positron storage rings has shown a remarkable progress due to the demand of high luminosity B-factory. In particular, problems related to asymmetric colliders have been studied intensively as a new feature of this field. Recent studies have been summarized in review talks by Keil in the preceeding conferences[1] and in summaries of workshops by Gareyte[2] and Keil[3].

This paper summarizes the investigations mostly last and this years. Those which were described in the preceeding reviews in detail will appear only briefly in this report. The notation is not fully defined but I believe it to be standard.

No general conclusion was attempted in this review.

2 Beam-Beam Limit in General

During the last few years analytic models of strong-strong interaction to reveal the physics of the beam-beam limit have been developed, which was initiated by Hirata's work on the mapping of the second moment $\langle x^2 \rangle$, $\langle xx' \rangle$ etc.[4] The theory has explained various phenomena such as flip-flop qualitatively. Hirata[5,6] has extended his theory to higher moments using Stratonovich (generalized Hermite) expansion. In the strong weak case he got a good agreement with computer simulation, taking as high as 18-th moment. For strong-strong case, only 4-th moments were included, due to a numerical problem, but the agreement with simulation was still improved.

Tennyson[7] has developed a new method of computing the beam-beam limit, which may be called self-consistent strong-weak model. He assumes that (a) the final state is time-independent and that (b) the time variation of the r. m. s. bunch length $\sigma_i (i = 1, 2)$ of each beam is a function of σ_1 and σ_2 (and other constant parameters) only, i.e., $d\sigma_i/dt = F_i(\sigma_1, \sigma_2)$. Then, by weak-strong simulation, he finds the weak bunch size $S_1(\sigma_2)$ as a function of the strong bunch size σ_2 . The same procedure gives $S_2(\sigma_1)$ by changing the role of strong and weak. Intersections of the two curves on (σ_1, σ_2) plane give the equilibrium bunch sizes (Fig. 1). He proved that $dS_1/d\sigma_2 \times dS_2/d\sigma_1 < 1$ is the necessary and sufficient condition for the stability of the equilibrium point. He claims that the beam-beam physics can be reduced to strong-weak problem by this method and that strong-strong simulations are not very powerful because of numerical noises.

He tried round beams only. In principle non-round beams can be treated by introducing four-dimensional space $(\sigma_{x1}, \sigma_{y1}, \sigma_{x2}, \sigma_{y2})$ but this will require even more computing time than strong-strong simulation. Extremely flat beams are nearly one-dimensional and may be treated by this method if we can ignore the energy flow from the horizontal degree of freedom.

It is not clear in his formalism whether or not the 'stable' solutions found in this way are stable against coherent motions such as dipole. It seems the assumption (b), based on which the stability condition was proved, excludes such possibility. This

assumption is much stronger than the Gaussian approximation employed in most strong-strong simulation codes because it does not contain terms involving x' .

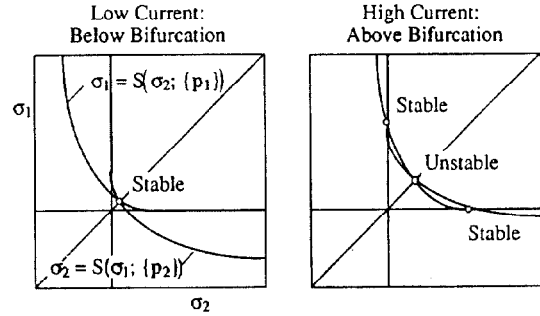


Fig.1. Beamsize function and equilibrium points[7].

3 Round and Flat Beams

In the case of equal rings, the luminosity is given by

$$L = \frac{(I/\epsilon)\gamma\xi}{2r_e\beta_y^*}(1+r) \quad r = \sigma_y^*/\sigma_x^* \quad (1)$$

Apparently round beams have an advantage over flat by factor of 2 because of $(1+r)$. In addition Krishnagopal and Siemann[8] exaggerated that in the case of round beams the beam-beam parameter

$$\xi_y = \frac{Nr_e\beta_y}{2\pi\gamma\sigma_y(\sigma_x + \sigma_y)} \quad (2)$$

does not depend on the longitudinal position s in spite of the variation of β and σ 's with s . (Normally, round beams mean not only $\sigma_x = \sigma_y$ but also $\beta_x = \beta_y$). When the bunch length σ_s is considerably shorter than the beta function, the synchrotron-betatron coupling is excited. Round beams can avoid (some part of) this coupling. Krishnagopal and Siemann[8] showed by computer simulation that ξ_{max} will be larger in the case of round beams.

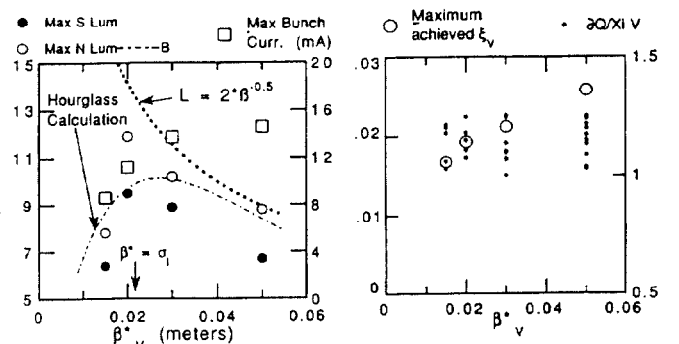


Fig.2. Luminosity and ξ_{max} vs. β^* at CESR[9].

Experiments of changing the betafunctor were carried out at CESR[9] (with flat beams). The bunch length was fixed constant.

It turned out, contrary to the first expectation, that $\beta_y^* \sim \sigma_s$ was the optimum for the luminosity as seen in Fig.2. As β^* increases, ξ_{max} becomes larger but the increase is less than linear. This result was explained by Krishnagopal and Siemann[10] by the fact that the bunch length causes the modulation of its own betatron oscillation as well as relaxes the effect on the opposing beam due to phase averaging. In computer simulations, therefore, it overestimates the modulation effects to include the shift of the collision point due to synchrotron oscillation with one thin-lens kick. Taking into account these facts, Krishnagopal and Siemann[11] again compared round and flat beams by strong-weak simulations. After optimization of each case, they got $\xi_{max}(R) \sim 0.10$ and $\xi_{max}(F) \sim 0.05$. (In Fig.3 ξ_{max} is plotted against σ_s/β^* for two different cases.)

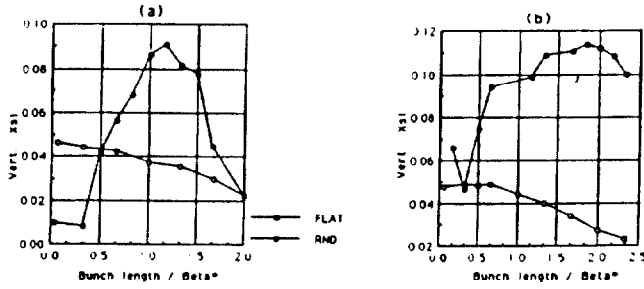


Fig.3. ξ_{max} for round and flat beams[11].

Chin[12] has also compared round and flat beams for APIARY rings by strong-strong simulation but obtained different result $\xi_{max}(R) \sim 0.035$ and $\xi_{max}(F) \sim 0.04$ with an interesting observation that the round beam has an equilibrium distribution sharply cut off whereas the flat beam has a long vertical tail.

Hirata[13] made analytic comparison using a Gaussian model. Under the assumption $\beta^*(R) = \beta_y^*(F)$, which is quite in favor of the round beam, he found the maximum luminosity $L(R) \sim 1.4L(F)$ when the luminosity is limited by flip-flop bifurcation and $L(R) \sim L(F)$ when ξ saturates without bifurcation. In both cases he found $\xi_{max}(R) \sim \xi_{max}(F)/2$. Actually, since the above assumption is very difficult to achieve for round beams, this result says the round beams cannot give higher luminosity than the flat beams.

These results strongly contradict with each other. It should be mentioned that the bunch-length effect, which is the key of Krishnagopal and Siemann's simulation, is not taken into account in Hirata's simple model. (The detail of Chin's simulation is not known to me.) As is seen in Fig.3, when the bunch length is short, the round beam does not have an advantage. Also, to be fair, the luminosity limitation in Hirata's model comes from the strong-strong nature of the interaction which is not taken into account in Krishnagopal and Siemann's.

It is impossible to give a definite conclusion at this moment. In recent designs of asymmetric colliders, however, people tend to adopt flat beams, admitting that the round beam cannot give overwhelmingly higher luminosity which can pay for the considerable efforts to make the beam round. (An exception is Siemann's suggestion[14] that the round beam allows a long bunch which can reduce the required rf voltage.) I am afraid that the round beam issue might come to an end without conclusions of beam-beam theory because of the practical need. We have not only to wait for the round beam experiments at CESR but also to try more simulations and analytic works.

4 Energy Transparency

Since there is no e^+e^- collider with asymmetric energies upto now, it would be nice if we can resort to our experience in symmetric ones by imposing some conditions, called 'energy transparency conditions' by somebody, such that an asymmetric collider looks like a symmetric one.

It has been usual in designing asymmetric colliders to make (1) the beam sizes and (2) beam-beam parameter ξ same in the two rings. Y. Chin[15] added two more items to the list, namely (3) δ (relative energy loss between collisions) and (4) $\sigma_s Q_s/\beta^*$ (betatron phase modulation due to finite bunch length) through try and errors of computer simulation for the APIARY rings.

Krishnagopal and Siemann[16] replaced (4) with (5) fractional part of tunes, (6) Q_s , (7) β^* and (8) σ_s . They got these conditions by perturbation treatments of Hamiltonian so as to make equal the resonance structures and strengths of the two rings. As a result, Chin's condition (4) was split into three conditions. If we demand all these conditions to be satisfied, the only parameter which can compensate the energy inequality is the number of particles per bunch; $N_1 \gamma_1 = N_2 \gamma_2$.

Tennyson[7] argues, by his self-consistent strong-weak method, that symmetric machines have no obvious advantage over asymmetric ones and the latter may have their own optimum. He claims to have an example for which $\delta_1 \neq \delta_2$ is an optimum.

The issue of the energy transparency seems to have become obscure. Chin started with an optimization of the luminosity and obtained the equality conditions for the particular set of the machine parameters. Krishnagopal and Siemann's work is more general but it is not clear whether the conditions are needed for the luminosity optimization. Computer simulations of wider range of parameters are desired in order to see if the equalities make the optimum. Among the equalities, Chin's condition (3) has to be examined because it is expensive to get the same damping rates in rings with same circumference but different energies and also because this problem seems to be the simplest. It is not clear whether Chin's work insists the damping rates must be the same or be just as large as possible.

5 Tail Distribution, Life Time

When a machine is well tuned, the luminosity limitation usually comes from the life time due to the particle loss from the tail of the (quasi-)equilibrium distribution. Since the particles in the tail do not take part in the strong-strong dynamics, we can employ strong-weak picture to find the tail distribution after finding the core (or by simply assuming a Gaussian core).

Chin[17] has developed a technique, which he calls renormalization, to compute the distribution function. A simple application of perturbation theory often causes infinity due to the resonance denominator $1/(Q(I) - Q_{res})$ when integrated over the action I . He solved this problem by treating the average and the oscillating parts separately and gave finite results.

Comprehensive studies have been done for the life time estimation by Gerasimov and Dikansky[18,19,20]. They start with the Fokker-Planck equation and show the (quasi-)equilibrium distribution can be written as $Z \exp(-\phi/\eta + O(\eta))$ in the limit of $\eta \rightarrow 0$, where η is proportional to the radiation excitation ('weak noise asymptotics'). They studied non-linear isolated resonances $mQ_x(I_x, I_y) + nQ_y(I_x, I_y) = \text{integer}$ and pointed out the essential difference between 1-dimension and ≥ 2 -dimension. In the case

of 1-dim $\phi - \phi_0 = O(\Delta I)$ (ϕ_0 is the Gaussian distribution and ΔI is the resonance width) but, in the case of ≥ 2 -dim, $\phi - \phi_0$ can be the same order as ϕ_0 due to the mechanism which they call 'phase convection'. A particle initially at $I_x = I_y = 0$ may come to the point marked with a circle (see Fig.4) slowly by radiation diffusion and, then, it can go along the resonance tube much more rapidly. As a result, ϕ is almost constant in the region II which shows a long plateau if projected onto the I_y axis. This explains the energy flow from the horizontal to vertical degrees of freedom in the case of flat beams.

They compared the theory with experimental results in VEPP-4 and got qualitative agreements. The method is very promising and we hope comparisons in more detail with computer simulations and with scraper experiments.

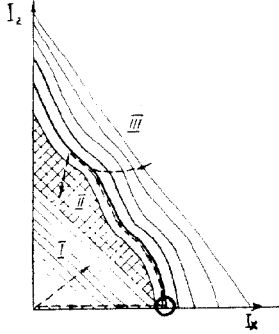


Fig.4. Contour of $\phi(I_x, I_y)$ [20].

n_2	n_1						
	1	2	3	4	5	6	7
1	32.5	34.9	41.9	43.2	44.0	48.6	51.9
2	-	-	48.4	-	55.3	-	61.6
3	-	-	-	57.1	61.7	-	67.9
4	-	-	-	-	66.6	-	73.6
5	-	-	-	-	-	74.8	78.2
6	-	-	-	-	-	-	82.6

Tab.1. Fraction of unstable area (%) [21].

6 Coherent Beam-Beam Interaction

The latest review (or rather a 'textbook') was given by Hirata[21].

Dikansky and Pestrikov[22] and Simonov[23] have studied coherent phenomena (single bunch per beam). Various formulas are summarized in these papers, including growth rates, stop-band widths, multipole ($>$ dipole) modes, synchro-beta resonances, etc.

The work that gave the strongest impact in the field of coherent interaction during the last year was presumably the one by Hirata and Keil[24,25]. They studied the coherent interaction between beams with arbitrary numbers of bunches, which has become important because of asymmetric colliders, using rigid Gaussian model with linear approximations and numerical solutions of matrix eigenvalue problem. Their main results can be summarized as follows. (The ratio of the numbers of bunches is n_1/n_2 with n_1 and n_2 being relatively prime.)

- The sum resonances $n_2 Q_1 + n_1 Q_2 \sim \text{integer}$ are the most dangerous.
- The area of the unstable region in (Q_1, Q_2) plane is approximately proportional to $\sqrt{n_1^2 + n_2^2}$. Tab.1 shows the area of the unstable region for some combinations of n_1 and n_2 for the coherent beam-beam parameter $\Xi_1 = \Xi_2 = 0.03$. (Alexandrov and Pestrikov[26] have shown, in the case $n_1/n_2 = 1/n$, that the resonances overlap when $n \gtrsim 1/16\Xi_1\Xi_2$, by using a dispersion relation and a resonance approximation. Note that the relation between Ξ and the incoherent beam-beam parameter ξ is not clear so long as one employs the rigid Gaussian approximation. Hirata and Keil assume $\Xi = \xi/2$ and Alexandrov and Pestrikov $\Xi = \xi$.) In order that the unstable region is less than 50%, the bunch number ratio must be less than ~ 6 .

- When the operating point is in the stopband of a sum resonance, the beams separate and fall into a limiting cycle. See Fig.5.
- It seems very hard to damp the instability by a feedback system because a large gain and a large band width are required.

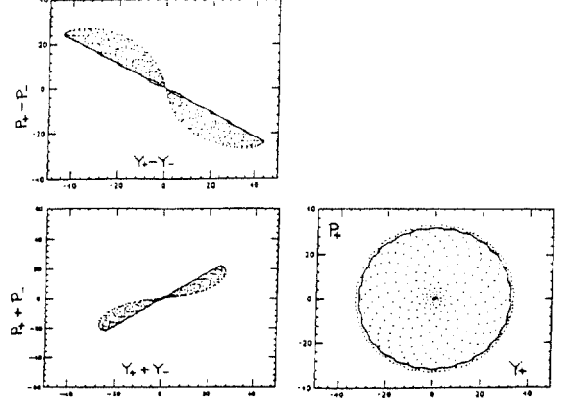


Fig.5. Limiting cycle at a sum resonance[21].

It seems that this theory is almost eliminating the possibility of inexpensive B-factories which add a very small ring to a large existing ring, if not many other advantages of equal-size rings[27]. The maximum possible ratio is hard to say (the above value need not be considered seriously). In principle one may be able to find a good operating point between numbers of stopbands. But in order to convince oneself with the existence of such a point in a large ratio collider, one has also to study carefully non-Gaussian interaction, higher multipoles, etc.

Upto now the coherent beam-beam instability has not been harmful for real operations of colliders except an example found at DORIS II[28]. The stable coherent oscillation with small amplitudes has been used as a diagnostics tool of the beam: by measuring the tune splitting ΔQ_π between the π - and σ - modes, one can estimate the incoherent beam-beam parameter ξ . When ξ is small, the relation between them were theoretically found to be $\Delta Q_{\pi,x} = 1.33\xi_x$, $\Delta Q_{\pi,y} = 1.24\xi_y$ (flat beams) and $\Delta Q_\pi = 1.21\xi$ (round beams)[29]. An experiment was done by Koiso *et al*[30] and the agreement was excellent especially for $\Delta Q_{\pi,x}$. The comparison of $\Delta Q_{\pi,y}$ is limited by the accuracy of the luminosity monitors but the results at CESR[31] also seem to confirm the above relation.

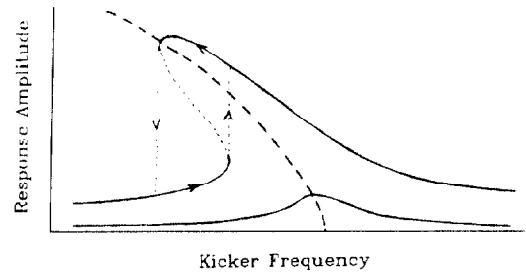


Fig.6. Hysteresis of the σ -mode response.

An interesting phenomena of coherent oscillation with large amplitudes were observed by Ieiri and Hirata[32]. They excited the oscillation by applying rf kicker with a slowly changing frequency. The response amplitude showed a hysteresis behavior: it took different courses for increasing and decreasing frequencies (Fig.6).

This is a well-known characteristic of non-linear damped oscillators. The skelton curve (dashed) gives the tune-amplitude relation $Q(I)$ if the system consists of two rigid particles. Our system is ensembles of particles, however, and we only know that the intersection of the skelton and the horizontal axis, ΔQ_π , gives ξ . What we desire is a theory. With it, we should be able to extract at least one more parameter which will tell us about the particle distribution function in the core, *e.g.*, how it is close to Gaussian.

7 Simulation Techniques

Recent status of the computer simulations is summarized by Siemann[33] who says that our simulations can reproduce many of the experimental features of beam-beam interaction (indeed the CESR luminosity has been explained to $\sim 10\%$ accuracy[34]) but the predictive power is still poor. Here, we only mention some works after his review.

To find the beam life time due to the particle loss from the tail as long as a few minutes to hours, we need a tremendous computing time (typically 10^{10} to 10^{11} particle-turns). Irwin[35] proposed a fine technique of tracking particles emphasising the tail. The whole process after finding the core distribution consists of several steps each of which has an amplitude boundary increasing from step to step. In each step he starts with the initial distribution determined by the data accumulated in the previous step and track the particles for about one damping time, recording the coordinates of particles going across the boundary outwards for the next step. By this way he is able to track more particles in the tail.

It is very interesting to see whether this algorithm can reproduce the 'phase convection' studied by Gerasimov and Dikansky. This is not trivial because Irwin had to assume that the particles must forget their initial conditions during each step so that the correlation is lost. But the phase convection is a rapid and correlated phenomena.

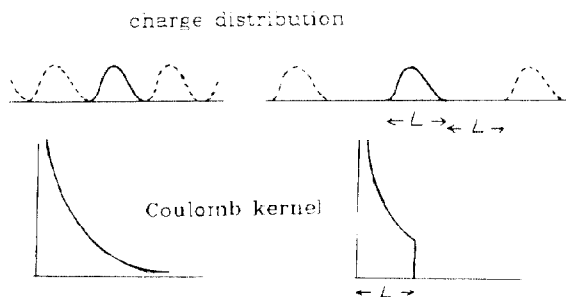


Fig.7. Solving Poisson equation using FFT.

Kikutani[36] has developed a code in which the beam-beam force is computed using Fourier transformation without assuming the Gaussian distribution. The method is shown in Fig.7 (1-dim for illustration). First, Fourier transform the distribution function (solid line at top-left), multiply Fourier kernel of the Coulomb potential (bottom-left) and transform back to the real space. If one naively applies FFT, the ghost charges (dotted line), which inevitably come from the periodicity of FT, exert forces onto the real charge. In Kikutani's code the range of FFT is twice as large as the real charge area (top-right) with some price of the computing time, and a truncated Coulomb kernel (bottom right) is used so that the ghost charges do not contribute to the field in the real charge region.

He was able to confirm the relation $\Delta Q_{\pi,x} = 1.33\xi_x$ which can only be explained only with the non-Gaussian nature. The computing time is still tolerable but the noise problem has not yet been studied fully.

Hirata, Moshhammer and Ruggiero[37] claimed that, when one considers the synchrotron oscillation effects on the beam-beam force, he must also take into account the corresponding energy change in order to make the force symplectic in the six-dimensional phase space. Although the effect does not seem to be large because the longitudinal emittance is usually much larger than the transverse, one can include these effects for safety as the proposed algorithm to compute the counterpart does not require much computing time.

There has been an argument as to how to take into account the emittance change due to the optics change by the linear part of the beam-beam interaction[33,38]. The last paper on this issue is by Hirata and Ruggiero[39] who state that the standard technique of generating radiation can automatically reproduce the correct emittance if the betatron phase advances between the IP and the bending magnets are almost uniformly distributed, which is normally satisfied in large rings.

(Channel[40] has reported a new method of tracking the moments of the distribution function but, unfortunately, it has not been available to me.)

8 Crab Crossing

The so-called crab crossing[41] is a new topic of the beam-beam interaction. It is proposed in order to make the crossing angle large without losing the luminosity and without exciting the synchro-betatron resonances. If everything is perfect, the interaction with a finite crossing angle plus the crab tilt is the same as that of head-on. The problem is, therefore, the tolerances of the various parameters. An estimation for a τ -charm factory is presented in[42].

Piwinski[43] made a strong-weak simulation for round beams and considered the cavity power and phase errors (static), the error in the betatron phase advance and the effect of the bunch length with respect to the crab-cavity wave-length. He found the tolerances for the emittance blow-up to be small are not tight. The tolerances for tail particles are a little tighter but still easily manageable.

Koiso and Oide[44] considered the tolerances for the errors fluctuating from turn to turn using strong-strong simulations. They found, if the four cavities have the same errors, the tolerances are not a problem at all. When the four cavities are independent, the required tolerances look tight but do not make problems if the four cavities are fed by the same klystron.

We can say through these studies that the crab crossing is not a problem for the beam-beam interaction.

Acknowledgement

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Following abbreviations are used in the list above.

Novosibirsk Workshop: Third Advanced ICFA Beam Dynamics Workshop on Beam-Beam Effects in Circular Colliders, 29 May-3 Jun, 1989, Novosibirsk, USSR.

Chicago Conf: Proc. 1989 IEEE Part. Acc. Conf., 20-23, Mar. 1989, Chicago, Ill, USA.

Tsukuba Conf: Proc 1989 XIV International Conference on High Energy Accelerators, 22-26 Aug. 1989, Tsukuba, Japan.

LBL Workshop: Proc. Workshop on Beam Dynamics Issues on High-Luminosity Asymmetric Collider Rings, 12-16 Feb. 1990, LBL, Berkeley, CA, USA.