THE TRANSVERSE FORCES IN WAKEFIELD ACCELERATORS*

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The purpose of this paper is an attempt to compare beam breakup problems in dielectric lined waveguide to plasmas as they pertain to wakefield accelerators. This is difficult for various reasons. In the waveguide position is measured relative to the physical center of the guide but in a plasma position is relative to the centroid of the bunch creating the wakes. Dielectrics are very linear making their behavior well suited for analytical study. Plasmas are very nonlinear so one makes a great many approximations to put them into an analytical regime.

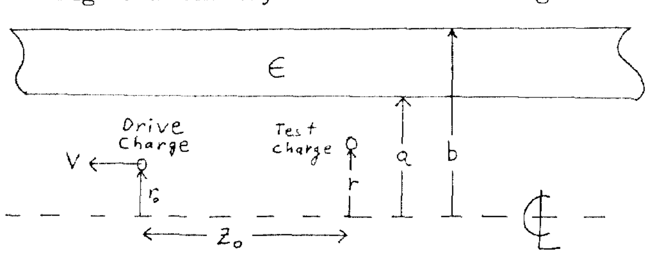
The electromagnetic fields radiated by a relativistic charge moving inside a dielectric lined cylindrical waveguide have been given in detail elsewhere [1]. Doing a Fourier analysis one finds that m=0 modes give rise to only longitudinal forces in the ultrarelativistic limit. All higher order modes give rise to transverse forces.

For an azimuthally symmetric disk traveling thru a plasma only m=0 modes exist [2]. This is because the electrostatic waves induced in the plasma are centered about the charge distribution. The transverse forces act about this centroid.

Dielectrics

The geometry of the dielectric filled waveguide is shown in figure 1. The drive charge is located at $(r_0, 0)$ and the test charge position is (r, z_0). In ref. [1] the m=0 longitudinal electric field is found to be independent of r in the ultrarelativistic limit. There are no transverse forces associated with this mode (or more accurately, they fall off as γ^{-2}).

Fig. 1 Geometry of Dielectric Waveguide



From [1] the $m \neq 0$ ultrarelativistic electric field is

$$E_z^m = -\frac{8em}{s\frac{dC}{ds}}\left(\frac{rr_0}{a^2}\right)^m \cos m\theta \cos k_0 z_0 \qquad (1)$$

with

$$C(s) = -\frac{m(\epsilon+1)}{s^2a^2} + \frac{1}{m+1} + \frac{1}{sa}\left(\frac{S'_m(sa)}{S_m(sa)} + \epsilon\frac{R'_m(sa)}{R_m(sa)}\right) \qquad (2)$$

where $C(s) = 0$ defines the frequencies s, a is the inner radius of the waveguide, ϵ is the dielectric constant of the liner, r_0 is the offset of the driving bunch, r is the offset of the test bunch, z_0 is the distance behind the driving bunch and $k_0 = s/(\epsilon - 1)^{1/2}$.

The functions R_m, R'_m, S_m, S'_m are derived from the geometry of the problem and are defined as

$$\begin{aligned} R_m(sa) &= N_m(sb)J_m(sa) - J_m(sb)N_m(sa) \\ R'_m(sa) &= N_m(sb)J'_m(sa) - J_m(sb)N'_m(sa) \\ S_m(sa) &= N'_m(sb)J_m(sa) - J'_m(sb)N_m(sa) \\ S'_m(sa) &= N'_m(sb)J'_m(sa) - J'_m(sb)N'_m(sa) \end{aligned}$$

and J_m, J'_m, N_m, N'_m are the mth order Bessel functions of the first and second kind.

The transverse force is found from the Panofsky-Wenzel theorem shown in [1]:

$$F_\perp = \frac{-8e^2m^2}{sak_0\frac{dC(s)}{ds}}\left(\frac{r_0}{a}\right)^m\left(\frac{r}{a}\right)^{m-1}(\cos m\theta\,\mathbf{r} - \sin m\theta\,\boldsymbol{\theta})\sin k_0 z_0 \qquad (3)$$

For $m = 1$ this is a pure dipole moment. It does not matter where the trailing bunch is in the waveguide for this case. The amplitude of this force is proportional to the driving beam offset only and this can be reduced by getting the centroid of the charge distribution close to the axis of the waveguide.

Plasmas

The geometry of the plasma problem is shown in figure 2. The center line is assumed for general expansion of nonuniform distributions. The linear model is purely electrostatic and assumes that the ions are a continuous uniform background with an electron fluid superposed.

In [3] an example is given for the wakefield response of a cold plasma in 3-d. There is an a priori assumption of azimuthal symmetry. Working without that assumption ref. [4] finds that the same formalism used in the dielectric analysis leads to a delta function response with a longitudinal electric field:

$$E_z = -8ek_p^2 \sum_{m=-\infty}^{\infty} e^{im(\theta-\theta_0)} I_m(k_p r_<) K_m(k_p r_>) \cos(k_p z_0) \qquad (4)$$

where $k_p = \frac{\omega_p}{c}$ and $\omega_p^2 = \frac{4\pi e^2 n_0}{m_e}$, n_0 is the plasma density, and v is the beam velocity. The symbols $r_<$ and $r_>$ are chosen as r or r_0 depending on which is smaller and larger. For the case similar to [3] we take $r_< = 0$ and $\theta = \theta_0 = 0$ to find the identical (within a factor of 4) result:

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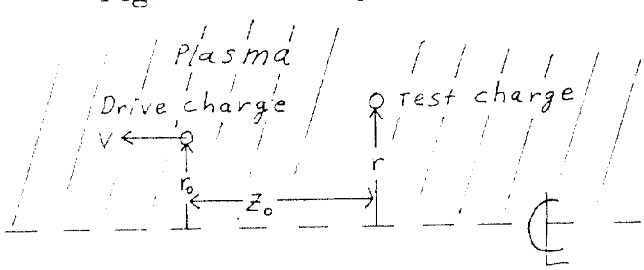
$$E_z = -8ek_p^2 K_0(k_p r)\cos(k_p z_0) \quad (5)$$

The infinite response of (5) as r goes to zero is due to the linear fluid model of the electrons. As shown in [3] the fluid response is a delta function along the line of the drive charge. In a more accurate model one would include temperature effects which will spread out the fluid response by a Debye length. In the dielectric case this artificial problem does not exist.

Fig. 2 Geometry of Plasma



To compare (4) to the example mentioned in [2] we integrate equation (4) over a charge distribution suggested in ref [2] as

$$\begin{aligned}\sigma(r) &= \frac{2N}{\pi a^2}(1 - r^2/a^2) \quad r < a \\ &= 0 \qquad r > a \qquad (6)\end{aligned}$$

The result of this calculation is:

$$\begin{aligned}E_z &= -\frac{32eN}{a^2}\cos(k_p z_0)[K_0(k_p r)[I_1(k_p r)(1 - \frac{r^2}{a^2}) \\ &+ \frac{2r^2}{a^2} I_2(k_p r) + I_0(k_p r)[k_p r K_1(k_p r) - \frac{2r^2}{a^2} K_2(k_p r) \\ &+ 2K_2(k_p a)]] \qquad r < a \qquad (7) \\ E_z &= -\frac{64eN}{a^2}\cos(k_p z_0) K_0(k_p r) I_2(k_p a) \qquad r > a \qquad (8)\end{aligned}$$

Ruth et al's[2] solutions have the form

$$E_z = -\frac{16eN}{a^2}\cos(k_p z_0)[K_2(k_p a) I_0(k_p r) + \frac{1}{2} - \frac{2}{(k_p a)^2} + \frac{r^2}{2a^2}] \quad (9)$$

for $r < a$ but is identical to (8) for $r > a$ (again a factor of 4 different). Plotting the terms within brackets of (7) and (9) gives different amplitudes but the curves are the parabolic shape of the source (see figs. 3 and 4).

Equations (5), (7), and (9) all show a strong radial dependance. The Panofsky-Wenzel theorem on (5) gives

$$F_\perp = -8e^2 k_p^2 K_1(k_p r) sin(k_p z_0) \quad (10)$$

which indicates a strong focusing force. As Ruth et. al. showed [2] there is a region where one can accelerate and focus. This is a major difference between the dielectric waveguide and the plasma.

If one does not have an azimuthally symmetric drive beam, (4) shows that $m \neq 0$ modes will exist. Integration of (4) over some arbitrary nonuniform distribution will also give rise to dipole terms similar to (1). It may be possible to transform to a new coordinate system where the $m = 1$ mode is eliminated, such as the centroid of the charge distribution. In a nonuniform plasma, there would be an $m = 1$ mode physically excited.

Fig. 3 Equation (7)

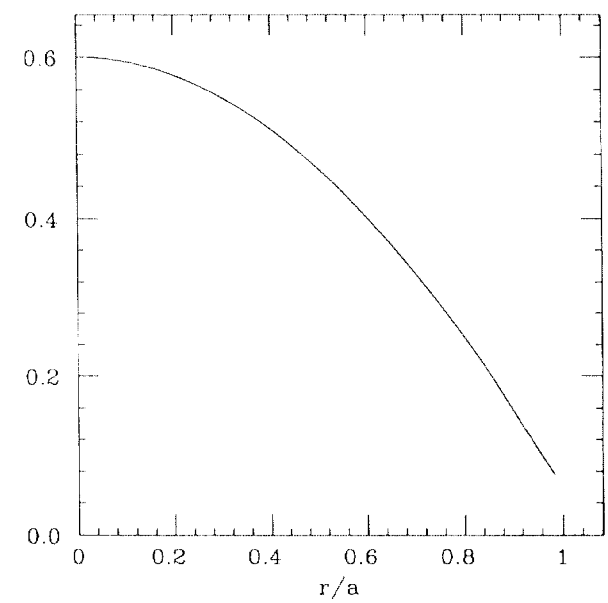
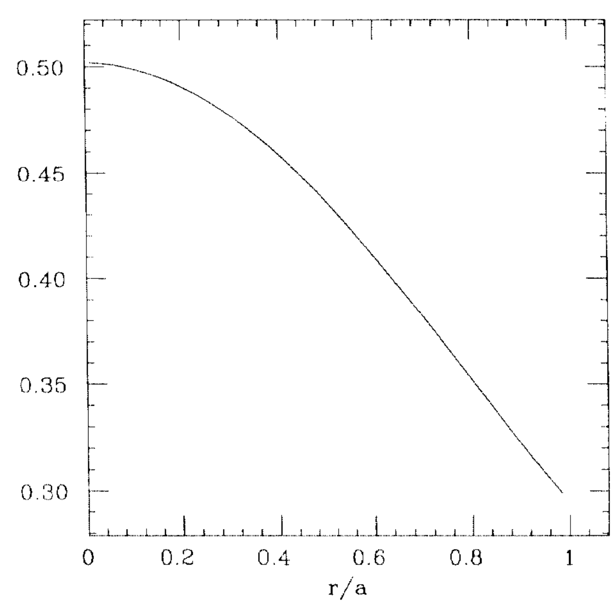


Fig. 4 Equation (9)



Both the plasma and dielectric pipe exhibit similar problems. In the plasma even a uniform distribution in the drive bunch gives rise to transverse forces seen by the trailing bunch. In the dielectric an offset beam from the center of the pipe is necessary to excite the transverse forces.

The model used on the dielectric is well understood and reasonably accurate. The model used for the plasma is somewhat questionable since only linearized approximations to the fluid equations were used. Past experiments show that prediction of the dielectric response is as expected [1]. The work done with plasmas [5] is not quite as successful but close enough for the linear approximation.

At this point in time, Argonne is attempting to build a 100 nC, 10 psec, 30 MeV accelerator to further test these ideas [6]. Transverse forces are difficult to measure in our present apparatus and higher currents with shorter bunches will alleviate some of these problems. In the future we hope to compare the absolute magnitudes of transverse forces in both dielectric waveguide and plasmas.

Because dielectric waveguides are easier to build and alignment is less critical than for plasmas we expect these devices to be more immediately useful for the accelerator community. External focusing can be added to dielectric waveguides which also reduces the problems associated with beam alignment. Since plasmas have similar problems (although for different reasons) it will be some time before their higher gradients will be used to build an accelerator.

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