Disk Loaded Waveguides Versus Dielectrically Loaded Waveguides

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Abstract

The wake fields of a bunch of charged particles moving with the velocity of light through a dielectric loaded circular waveguide are calculated. For the computation the method of mode analysis is used. The wake potential of monopole and dipole modes as well as the quality factor is compared with a typical disk loaded S-Band structure (SLAC). In an equivalent dielectric structure the peak wake potential is about 1.5 times smaller than in a cavity structure, but the quality factor with an assumed loss tangent of $\tan\delta = 10^{-4}$ is much smaller too. Because of these results and break down problems in dielectric structures, there seems to be no advantage compared to standard copper structures.

Introduction

Dielectric loaded waveguides have often been considered as an alternative to disk loaded waveguides for particle acceleration. Due to dielectric losses the metallic structure seems to be advantageous. Recently, dielectric tubes have again been discussed with respect to transverse wake field effects [1].

In this paper we calculate both the longitudinal and transverse wake fields and the quality factor in a dielectric structure with the method of mode analysis. Very good agreement is found with wake fields calculated in time domain [2].

Finally the solutions are compared to a typical disk loaded S-Band structure as used by SLAC.

Mode Analysis

First the modes of a circular waveguide coated with a dielectric material as shown in Figure 1 are calculated. For the solution of the modes it is assumed that the contributions of the free charges vanish. The time harmonic modes must satisfy the Maxwell equations which can be solved in cylindrical coordinates by a vector potential $\vec{A} = A_\varrho \vec{e}_\varrho + A_\varphi \vec{e}_\varphi$ and

$$\vec{H} = \mathrm{rot}\vec{A}. \tag{1}$$

$\vec{A}$ satisfies the wave-equation

$$\nabla^2 \vec{A} + k^2 \vec{A} = 0 \tag{2}$$

with $k = \omega/c = \omega\sqrt{\mu\varepsilon}$. The Helmholtz equation (2) can be solved by separation of variables,

$$A_\varrho = \left(C_1 Z_{(m+1)}(K\varrho) + C_2 Z_{(m-1)}(K\varrho)\right)\cos m\varphi \, e^{-jk_z z},$$

$$A_\varphi = \left(C_1 Z_{(m+1)}(K\varrho) - C_2 Z_{(m-1)}(K\varrho)\right)\sin m\varphi \, e^{-jk_z z}, \tag{3}$$

with the separation condition $k^2 = K^2 + k_z^2$. Here Z is a linear combination of Bessel functions. For $k_z = k$ the vector potential can be written as

$$A_\varrho = \left(C_1\varrho^{(m+1)} + C_2\varrho^{(m-1)} + C_3\varrho^{-(m+1)} + C_4\varrho^{-(m-1)}\right) * \cos m\varphi \, e^{-jk_z z},$$

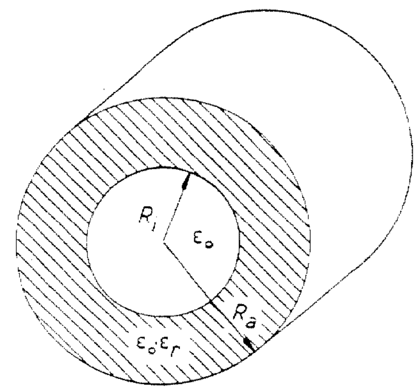

Fig. 1: Dielectric loaded circular waveguide

$$A_\varphi = \left(C_1\varrho^{(m+1)} - C_2\varrho^{(m-1)} + C_3\varrho^{-(m+1)} - C_4\varrho^{-(m-1)}\right) * \sin m\varphi \, e^{-jk_z z}. \tag{4}$$

In Eqs. (3) and (4) we assume that the wave propagates in positive z-direction and that the φ-dependence of the A_ϱ-component is $\cos m\varphi$. For electron and positron acceleration only waves with phase velocity $v_p = c_0$, the velocity of light, and the phase constant $\beta = \omega/c_0 = k_0$ are of interest.

In the inner region the vector potential as shown in Eq. (4) is used and the field components can be formulated with the condition that the electric and magnetic field is finite for $\varrho = 0$. Furthermore C_2 must be zero for $m = 0$.

$$\begin{aligned}\vec{H} = &+\left[C_1\varrho^{(m+1)} - C_2\varrho^{(m-1)}\right]\sin m\varphi \, e^{-jk_0 z}\vec{e}_\varrho \\ &-\left[C_1\varrho^{(m+1)} + C_2\varrho^{(m-1)}\right]\cos m\varphi \, e^{-jk_0 z}\vec{e}_\varphi \\ &-\frac{j}{k_0}C_1 2(m+1)\varrho^m \sin m\varphi \, e^{-jk_0 z}\vec{e}_z\end{aligned} \tag{5}$$

The electric field can be calculated from the magnetic field

$$\begin{aligned}\vec{E} = &-Z_0\left[C_1\varrho^{(m+1)} + \left(C_1\frac{2m(m+1)}{k^2} + C_2\right)\varrho^{(m-1)}\right] * \cos m\varphi \, e^{-jk_0 z}\vec{e}_\varrho \\ &-Z_0\left[C_1\varrho^{(m+1)} - \left(C_1\frac{2m(m+1)}{k^2} + C_2\right)\varrho^{(m-1)}\right] * \sin m\varphi \, e^{-jk_0 z}\vec{e}_\varphi \\ &+\frac{j}{k_0}Z_0 C_1 2(m+1)\varrho^m \cos m\varphi \, e^{-jk_0 z}\vec{e}_z.\end{aligned} \tag{6}$$

with $Z_0 = \sqrt{\mu_0/\varepsilon_0}$.

In the outer region $K \neq 0$, and Eq. (3) is used for the field representation. Using the boundary condition at $\varrho = R_a$ and the recurrence relations for the cylindrical functions [3], the field in

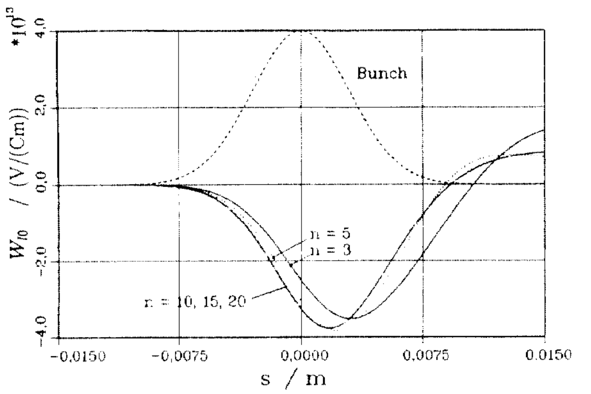

Fig. 2: Longitudinal wake potential of a Gaussian bunch with $\sigma = 3\,\mathrm{mm}$ for $m = 0$. $R_i = 1.15\,\mathrm{cm}$, $R_a = 4.08\,\mathrm{cm}$ and $\varepsilon_r = 2$. The parameter n is the number of used modes.

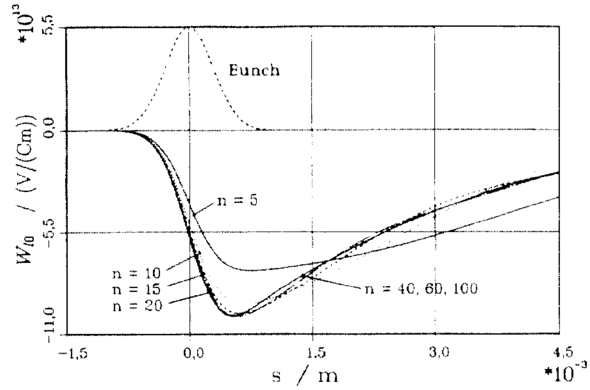

Fig. 3: Longitudinal wake potential of a Gaussian bunch with $\sigma = 0.3\,\mathrm{mm}$ for $m = 0$, $R_i = 1.15\,\mathrm{cm}$, $R_a = 4.08\,\mathrm{cm}$ and $\varepsilon_r = 2$. The parameter n is the number of used modes.

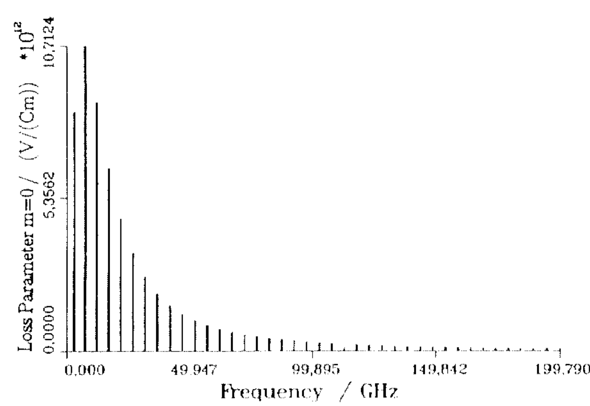

Fig. 4: Loss parameter k_μ as a function of the discrete modes for $m = 0$, $R_i = 1.15\,\mathrm{cm}$, $R_a = 4.08\,\mathrm{cm}$ and $\varepsilon_r = 2$.

the outer region is

$$\begin{aligned}\vec{H} &= \Big[-Y^H K_2 C'^H_m + \frac{m}{\varrho} C^E_m\Big] \sin m\varphi \, e^{-jk_0 z} \vec{e}_\varrho \\ &+ \Big[-Y^H \frac{m}{\varrho} C^H_m + K_2 C'^E_m\Big] \cos m\varphi \, e^{-jk_0 z} \vec{e}_\varphi \\ &- \frac{j}{\omega\mu_0} K_2^2 C^H_m \sin m\varphi \, e^{-jk_0 z} \vec{e}_z \,, \end{aligned} \tag{7}$$

$$\begin{aligned}\vec{E} &= \Big[-\frac{m}{\varrho} C^H_m + Z^E K_2 C'^E_m\Big] \cos m\varphi \, e^{-jk_0 z} \vec{e}_\varrho \\ &+ \Big[K_2 C'^H_m - Z^E \frac{m}{\varrho} C^E_m\Big] \sin m\varphi \, e^{-jk_0 z} \vec{e}_\varphi \\ &+ \frac{j}{\omega\varepsilon_2} K_2^2 C^E_m \cos m\varphi \, e^{-jk_0 z} \vec{e}_z \,, \end{aligned} \tag{8}$$

with

$$\begin{aligned} C^H_m &= C^H\Big[\mathrm{N}'_m(K_2R_a)\,\mathrm{J}_m(K_2\varrho) - \mathrm{J}'_m(K_2R_a)\,\mathrm{N}_m(K_2\varrho)\Big], \\ C'^H_m &= C^H\Big[\mathrm{N}'_m(K_2R_a)\,\mathrm{J}'_m(K_2\varrho) - \mathrm{J}'_m(K_2R_a)\,\mathrm{N}'_m(K_2\varrho)\Big], \\ C^E_m &= C^E\Big[\mathrm{N}_m(K_2R_a)\,\mathrm{J}_m(K_2\varrho) - \mathrm{J}_m(K_2R_a)\,\mathrm{N}_m(K_2\varrho)\Big], \\ C'^E_m &= C^E\Big[\mathrm{N}_m(K_2R_a)\,\mathrm{J}'_m(K_2\varrho) - \mathrm{J}_m(K_2R_a)\,\mathrm{N}'_m(K_2\varrho)\Big]. \end{aligned} \tag{9}$$

J and N are Bessel functions of the first and second kind and order m. The prime (') indicates the differentiation of the Bessel functions with respect to their arguments. Z^E and $Z^H = 1/Y^H$ are the wave impedances.

The unknown frequency and constants can be found by matching the tangential fields at $\varrho = R_i$. Thus the eigenvalue equation is formulated in matrix form.

$$\begin{vmatrix} Z_0\Big(R_i^2 - \dfrac{2m(m+1)}{k_0^2}\Big) & -Z_0 & K_2C'^H_m & -Z^E\dfrac{m}{R_i}C^E_m \\ -2(m+1)R_i & 0 & 0 & \dfrac{K_2^2}{\varepsilon_r}C^E_m \\ -R_i^2 & -1 & Y^H\dfrac{m}{R_i}C^H_m & -K_2C'^E_m \\ -Z_0 2(m+1)R_i & 0 & K_2^2C^H_m & 0 \end{vmatrix} = 0 \tag{10}$$

The factor $R_i^{(m-1)}$ is taken into the constants C_1, C_2. The eigenvalue equation is solved numerically.

Wake Potential

When the eigenvalue and the constants of the the μ-th mode are known, the stored energy W_μ and the induced voltage V_μ at $\varrho = R_i$ can simply be computed and the loss parameter

$$k_{\mu m} = \frac{V_\mu V_\mu^*}{4W_\mu R_i^{2m}} \tag{11}$$

is determined. The factor m is the azimuthal dependence of the mode. $k_{\mu m}$ descibes the coupling of the charged particle to the μth-mode.

A test particle following at a distance s behind a point charge q sees a longitudinal wake potential, which is written as an infinite sum [4].

$$W_{lm}(s) = -2q\varrho^{2m}\sum_{\mu=1}^{\infty} k_{\mu m}\cos\frac{\omega_\mu}{c}s \tag{12}$$

We assume that both the point charge and the particle travel at the speed of light and at the radius ϱ along the axis. For an arbitrary charge distribution Eq. (12) can be used as Green's function to calculate the longitudinal wake potential

$$W^\lambda_{lm}(s) = \int_{-\infty}^{s} \lambda(x)\frac{W_{lm}(x-s)}{q}\,dx. \tag{13}$$

For a Gaussian bunch with the total charge q and the standard deviation σ, $W^\lambda_{lm}(s)$ is given by

$$W^\lambda_{lm}(s) = -\frac{2q}{\sqrt{2\pi}\sigma}\sum_{\mu=1}^{\infty} k_{\mu m}\int_{-\infty}^{s}\exp\Big(-\frac{1}{2}\Big(\frac{x}{\sigma}\Big)^2\Big)\cos\frac{\omega_\mu}{c}(x-s)\,dx. \tag{14}$$

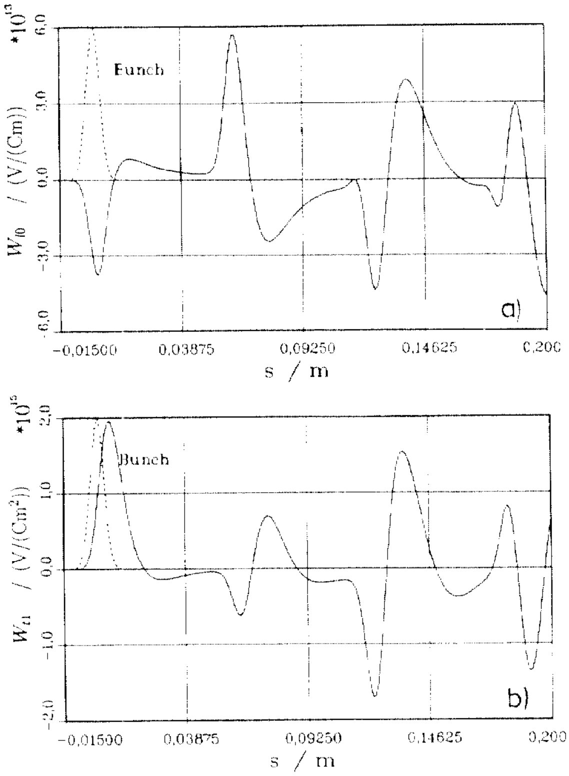

Fig. 5: Longitudinal (a) and transverse (b) wake potential of a Gaussian bunch with $\sigma = 3\,\mathrm{mm}$ for $m = 1$, $R_i = 1.15\,\mathrm{cm}$, $R_a = 4.08\,\mathrm{cm}$ and $\varepsilon_r = 2$.

With the longitudinal wake potential the transverse wake potential can be found.

$$W_{tm}^{\lambda}(s) = -\int_{-\infty}^{s} \frac{\partial W_{lm}^{\lambda}(x)}{\partial \varrho}\, dx \qquad (15)$$

Another interesting parameter of an accelerating structure is the quality factor Q

$$Q = \frac{\omega W}{P} = \frac{Q_d + Q_w}{Q_d Q_w}, \qquad (16)$$

where W is the stored energy and P the RF power loss per unit length. Q_d and Q_w represent the power loss in the dielectric material and in the metallic wall respectively.

Results

For numerical results the infinite sum in Eq. (12) must be truncated. Therefore the convergence of the azimuthal symmetric wake potential of a Gaussian bunch with $\sigma = 3\,\mathrm{mm}$ (Fig. 2) and $\sigma = 0.3\,\mathrm{mm}$ (Fig. 3) is proven. A comparison between the two figures shows that for a smaller bunch more modes have to be regarded. In Fig. 4 the loss parameters of these modes are shown as a function of the frequency. The values of the loss parameters decrease quite fast, which is the reason for the good convergence of the calculation of the wake potential. For a bunch with $\sigma = 3\,\mathrm{mm}$ only 15 modes need be used.

The results were compared with [2] and with one exception very good agreement was found.

ε_r	R_a [cm]	k_0 $\left[\frac{V}{C}/m\right]$	Monopole $W_{\parallel}$ $\left[\frac{V}{C}/m\right]$	Dipole $W_{\parallel}$ $\left[\frac{V}{Cm^2}/m\right]$	Dipole W_t $\left[\frac{V}{Cm}/m\right]$
SLAC		$9.0\ 10^{12}$	$57.0\ 10^{12}$	$5.45\ 10^{17}$	$4.14\ 10^{15}$
2	4.08	$8.3\ 10^{12}$	$37.7\ 10^{12}$	$3.21\ 10^{17}$	$1.96\ 10^{15}$
6	2.35	$14.\ 10^{12}$	$51.0\ 10^{12}$	$5.88\ 10^{17}$	$4.05\ 10^{15}$
30	1.63	$9.3\ 10^{12}$	$33.8\ 10^{12}$	$4.50\ 10^{17}$	$3.17\ 10^{15}$

ε_r	R_a [cm]	Q_d	Q_w	Q
SLAC			$1.5\ 10^4$	$1.5\ 10^4$
2	4.08	$1.07\ 10^4$	$3.00\ 10^4$	$7.89\ 10^3$
6	2.35	$1.10\ 10^4$	$1.20\ 10^4$	$5.80\ 10^3$
30	1.63	$1.08\ 10^4$	$4.41\ 10^3$	$3.13\ 10^3$

Fig. 6: Comparison of the peak wake potential within the Gaussian bunch and the quality factor between the dielectric and the SLAC structure for $R_i = 1.15\,\mathrm{cm}$, $\tan\delta = 10^{-4}$, $\kappa = 5.8\,10^7\ 1/\Omega\mathrm{m}$.

For comparison with the disk loaded structure the inner radius ($R_i = 1.15\,\mathrm{cm}$) and the frequency of the accelerating mode ($f = 3\,\mathrm{GHz}$) were chosen equal to the disk hole and the frequency of the SLAC cavity structure. Furthermore the relative permittivity was selected in order to have similar loss parameters for the accelerating mode. For the calculation of the quality factor a realistic loss tangent of $\tan\delta = 1 \times 10^{-4}$ and a conductivity of $\kappa = 5.8 \times 10^7\ 1/\Omega\mathrm{m}$ were assumed. The reference wake potential and quality factor of the SLAC structure were calculated with the program TBCI [5]. In Fig. 6, a table of the values of the peak wake potential within the Gaussian bunch and the quality factor is shown. The wake potential of the dielectric structure is about 1.5 times smaller than in a disk loaded structure. But the quality factor is about 2 times smaller too. Therefore the advantage of the smaller wake potential does n't seem to compensate the increased losses.

References

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