A SLED Type Pulse Compressor with Rectangular Pulse Shape

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ABSTRACT: Pulse compressors based on the SLED principle are used for electron linacs to reduce RF power[1], [2]. *In their original form, their output signal is characterized by a steep transient, followed by an exponential decay. This shape is not very well suited for constant gradient accelerating structures, which are designed for rectangular pulses. To achieve rectangular output pulses, a short – circuited transmission line has been proposed as a storage device*[3]. *In the present contribution an alternative way to produce rectangular pulses will be suggested, which does not need a different storage device. The complete high power part can be kept, the only modification is the replacement of the phase switch by a continuous phase modulator. The gradient achieved in a "constant gradient" structure does not differ very much from that with the original circuit*[2], *but the voltage peak at the beginning of the pulse is avoided.*

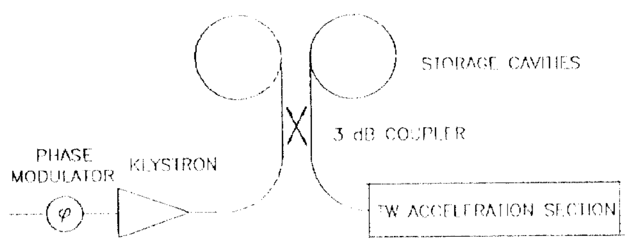

Fig. 1: *SLED scheme*

1. DESCRIPTION

Fig. 1 shows the layout for the pulse compressor. A 3 dB coupler, connected to two equal storage cavities, separates the forward and backward running wave. When switching on the klystron, the empty cavities form nearly a short circuit, so that the voltage is reflected with opposite sign. While the cavities are charged, the reflection coefficient increases, and finally the reflected voltage has the same phase as the incident wave. In the original device, now the phase of this wave is inverted, yielding a voltage step of two times the incident amplitude. From this maximum the output voltage decays exponentially, until the klystron is switched off (fig. 2).

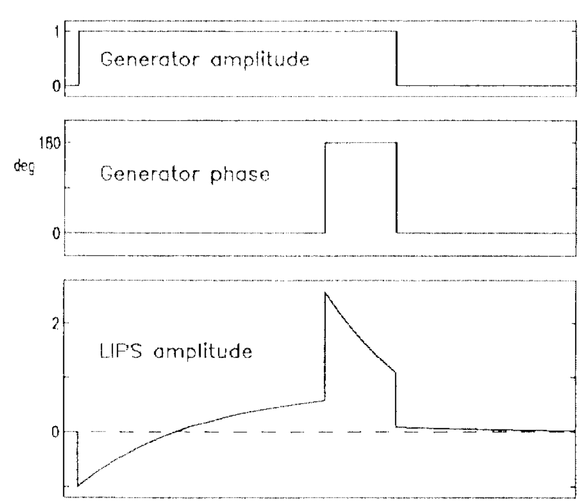

Fig. 2: *Waveforms of the conventional SLED*

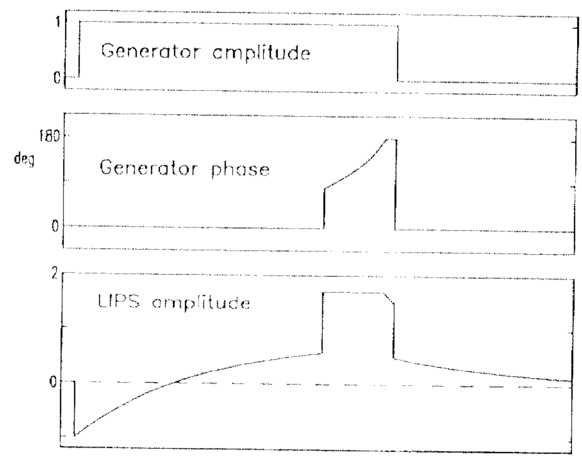

Fig. 3: *Waveforms of the continuous – phase SLED*

If we replace the phase switch by a continuous phase modulator, we can try to control the phase of the klystron voltage such that the magnitude of the reflected voltage remains constant. In fact, this can be done by starting with a phase step smaller than 180° which yields a smaller reflected voltage, and then raising the phase continuously until 180° are reached (fig. 3). After this, of course, the voltage will decay as in the former case. The length of this flat top depends on its voltage and of course the coupling and Q – value of the storage cavities. If it is made as long as the filling time, a fully usable rectangular shape is achieved.

From the energetic viewpoint, it is preferable to admit a certain decay time, until the reflected voltage equals the incident voltage. Then the reflected voltage becomes exactly zero after switching off the klystron, and no energy is left in the system. Because of the quadratic dependance of the energy versus the voltage, however, this zero voltage need not be very exactly adjusted.

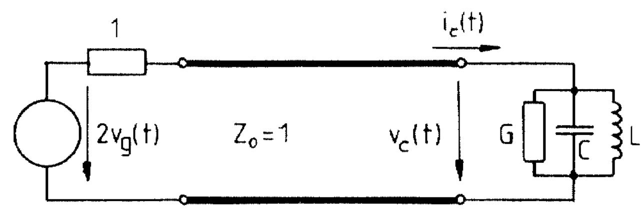

Fig. 4: *Equivalent circuit of the arrangement*

2. THEORY

The wave amplitudes on a transmission line of characteristic impedance Z_0 are defined as

$$\text{forward wave:} \quad a = \frac{1}{2}\left(\frac{v}{\sqrt{Z_0}} + i\sqrt{Z_0}\right)$$
$$\text{backward wave:} \quad a = \frac{1}{2}\left(\frac{v}{\sqrt{Z_0}} - i\sqrt{Z_0}\right) \qquad (1)$$

with v and i voltage and current on the line. Fig. 4 shows the transmission line terminated by the storage cavity and fed by a generator (klystron). The characteristic impedance Z_0 is normalized to 1, and the generator voltage is chosen to yield an incident wave amplitude of v_g. The elements representing the cavity are calculated from the coupling coefficient β and the Q-value Q_0. We have now to calculate voltage v_c and current i_c at the cavity. The reflected wave amplitude, which is fed via the coupler to the accelerating structure, is then $v_c - i_c$ (eq.(1)). Without losing generality, the length of the transmission line can be set zero. Then the voltage v_c can be shown to follow the differential equation (dots mean derivatives with respect to time):

$$2\dot{v}_g = \frac{1}{L} v_c + (1+G)\dot{v}_c + C\ddot{v}_c \qquad (2)$$

or

$$\alpha \dot{v}_g = \frac{\omega_0^2 \tau}{2} v_c + \dot{v}_c + \frac{\tau}{2}\ddot{v}_c$$

$$\text{where } \alpha = \frac{2\beta}{1+\beta} \text{ and } \tau = \frac{2Q_0}{(1+\beta)\omega_0} . \qquad (3)$$

The current is $i_c = 2v_g - v_c$, and hence the reflected amplitude

$$v_r = \frac{1}{2}(v_c - i_c) = v_c - v_g . \qquad (4)$$

Eq.(3) shall now be solved for a modulated generator voltage. Designating the (complex) amplitude values with uppercase letters, we get

$$v_c = \mathrm{Re}\{V_c e^{j\omega_0 t}\}$$

$$\dot{v}_c = \mathrm{Re}\{(\dot{V}_c + j\omega_0 V_c) e^{j\omega_0 t}\} \qquad (5)$$

$$\ddot{v}_c = \mathrm{Re}\left\{\left(\ddot{V}_c + j2\omega_0 \dot{V}_c - \omega_0^2 V_c\right) e^{j\omega_0 t}\right\} .$$

v_g and its derivatives have the same form. Now we insert the expressions of eq.(5) into the differential equation (3). To make this equation valid for any value of $j\omega_0 t$, it must also hold for the complex amplitudes. The result is

$$\frac{\alpha}{j\omega_0}\dot{V}_g + \alpha V_g = V_c + \left(\frac{1}{j\omega_0} + \tau\right)\dot{V}_c + \frac{\tau}{j2\omega_0}\ddot{V}_c . \qquad (6)$$

For high Q-values of the cavity, $1/\omega_0$ can be neglected against τ. The further interpretation of eq.(6) depends on the variation of the voltages. If V_g is constant or varies only smoothly (i.e. $\dot{V}_g << \omega_0$), also the resonator voltage will vary smoothly, so that all terms in eq.(6) with ω_0 in the denominator can be neglected. This leads to

$$\alpha V_g = V_c + \tau\dot{V}_c , \qquad (7)$$

which is formally equivalent to eq.(1) in [4], but generalized for complex amplitudes. For the reflected wave $V_r = V_c - V_g$ we get the differential equation

$$V_r + \tau\dot{V}_r = \Gamma V_g - \tau\dot{V}_g$$

$$\text{where } \Gamma = \frac{\beta - 1}{\beta + 1} \text{ the reflection coefficient.} \qquad (8)$$

The other limiting case is a step function of V_g. The resonator voltage cannot change discontinuously, and only its slope varies by an amount which can be found by integrating eq.(6). The change of the reflected voltage is

$$\Delta V_r = -\Delta V_g . \qquad (9)$$

Now we are in a position to calculate the reflected waveform completely. The generator signal is shown in fig. 3. The process can be divided into the following phases:

$$\begin{array}{llll} \text{phase 1:} & -t_a < t < 0 & V_g(t) = 1\,e^{j0} & \\ \text{phase 2:} & 0 < t < t_b & V_g(t) = 1\,e^{j\phi_g(t)}, & \phi_g(t) = \phi_0 \ldots \pi \\ \text{phase 3:} & t_b < t < t_c & V_g(t) = 1\,e^{j\pi} & \\ \text{phase 4:} & t > t_c & V_g(t) = 0 & \end{array} \qquad (10)$$

Phase 1 is known from the original SLED circuit. The reflected voltage is the solution of eq.(8) with $\dot{V}_g = 0$, and rises in an exponential function

$$V_r^{(1)}(t) = \Gamma - (1+\Gamma)e^{-(t+t_a)/\tau} \qquad (11)$$

until it reaches

$$V_r^{(1)}(-0) = \Gamma - (1+\Gamma)e^{-t_a/\tau} \equiv V_{-0} . \qquad (12)$$

The step in V_g to phase 2 is reflected (eq.(9)), so that

$$V_r^{(2)}(+0) = V_{-0} - \left(e^{j\phi_0} - 1\right)$$

$$= V_0 \exp\left[j\arctan\left(\frac{-\sin\phi_0}{V_{-0} + 1 - \cos\phi_0}\right)\right] \qquad (13)$$

$$\text{where } V_0 = \sqrt{V_{-0}^2 + 2(V_{-0} + 1)(1 + \cos\phi_0)} .$$

During phase 2, the ansatz

$$V_g^{(2)}(t) = e^{j\phi_g(t)} \qquad \dot{V}_g^{(2)}(t) = j\dot{\phi}_g(t)\,e^{j\phi_g(t)}$$

$$V_r^{(2)}(t) = V_0\,e^{j\phi_r(t)} \qquad \dot{V}_r^{(2)}(t) = j\dot{\phi}_r(t)\,V_0\,e^{j\phi_r(t)} , \qquad (14)$$

which takes into account that the magnitude V_0 of the reflected wave shall be constant, is put into eq.(8), and we get the following system of differential equations:

$$\Delta\dot{\phi} = \frac{V_0 - \frac{\Gamma}{V_0} + (1-\Gamma)\cos\Delta\phi}{\tau\sin\Delta\phi}$$

$$\dot{\phi}_g = \frac{-V_0 + \Gamma\cos\Delta\phi}{\tau\sin\Delta\phi} \qquad (15)$$

$$\text{where } \Delta\phi = \phi_r - \phi_g .$$

This equation is integrated numerically from $\phi_g = \phi_0$ and $\Delta\phi = \arg\{V_r^{(2)}(+0)\} - \phi_0$ (eq.(13)) until $\phi_g = \pi$ is reached at the time $t = t_b$. This is the transition to phase 3. During this phase the reflected voltage decays exponentially. As in phase 1, it is given as a solution of eq.(8) with $\dot{V}_g = 0$:

$$V_r^{(3)}(t) = -\Gamma + (\Gamma + V_0 e^{j\phi_r(t_b)}) e^{-(t-t_b)/\tau} \qquad (16)$$

Switching off the generator at $t = t_c$ means a voltage step of $\Delta V_g = 1$, which is subtracted from V_r. Later on the voltage continues to decay:

$$V_r^{(4)}(t) = \left(V_r^{(2)}(t_c) - 1\right) e^{-(t-t_c)/\tau} . \qquad (17)$$

In contrary to the original SLED, the phase of the voltage runs through values which deviate from 0° or 180°, so that the integration over the section to get the available accelerating voltage must be done vectorially. For a constant gradient structure, the group velocity decreases linearly with the length coordinate z (normalized to section length):

$$v_g(z) = v_{g0}(1 - gz) = \frac{1}{gT_a}\ln\left(\frac{1}{1-g}\right)(1 - gz) \qquad (18)$$

T_a is the filling time of the section. The slope g is chosen such that the decrease of group velocity exactly compensates for the loss, so that the amplitude of the travelling wave is constant over the section, and there is only a retardation between the field at different points on the section:

$$E(z,t) = E(0, t - t_{trans}) = E\left(0, t - \int_0^z \frac{d\zeta}{v_g(\zeta)}\right)$$

$$= E\left(0, t - T_a \frac{\ln(1 - gz)}{\ln(1 - g)}\right) . \qquad (19)$$

The accelerating voltage as a function of time is then

$$V_{acc}(t) = \int_0^1 E(\zeta, t)d\zeta = \int_0^1 V_r\left(t - T_a \frac{\ln(1 - g\zeta)}{\ln(1 - g)}\right) d\zeta . \qquad (20)$$

The magnitude of $|V_{acc}(t)|$ from eq.(20) is the available accelerating voltage, divided by its value if the section would be fed by a constant wave of unit amplitude.

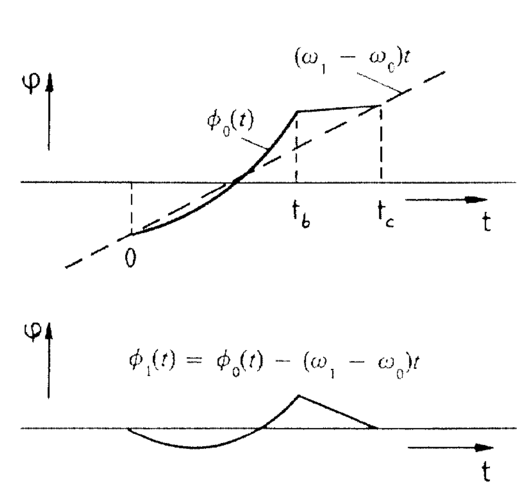

Fig. 5: Relationship between phase and frequency modulation

There is a little trick to further improve the efficiency. As stated above, there is a residual phase modulation in the signal, e. g. as shown in fig. 5. This phase modulated voltage with the angular frequency ω_0 can also be regarded as a voltage with a different frequency ω_1 and a different phase modulation $\phi_1(t)$:

$$Ve^{j\phi_0(t)}e^{j\omega_0 t} = Ve^{j\phi_1(t)}e^{j\omega_1 t} \quad \text{where } \phi_1(t) = \phi_0(t) - (\omega_1 - \omega_0)\,t\,. \qquad (21)$$

Thus we can operate the klystron with a lower frequency than the accelerating structure, and let a part of the residual phase modulation care for the frequency shift. As shown in fig. 3, the phase deviations from 0° can be made smaller, and the voltages add more efficiently. To calculate V_{acc}, the integrand in eq (20) must be replaced by $V_r(t')\exp(-j(\omega_1-\omega_0)t')$ where $t' = t - T_a \ln(1-g\,\zeta)/\ln(1-g)$.

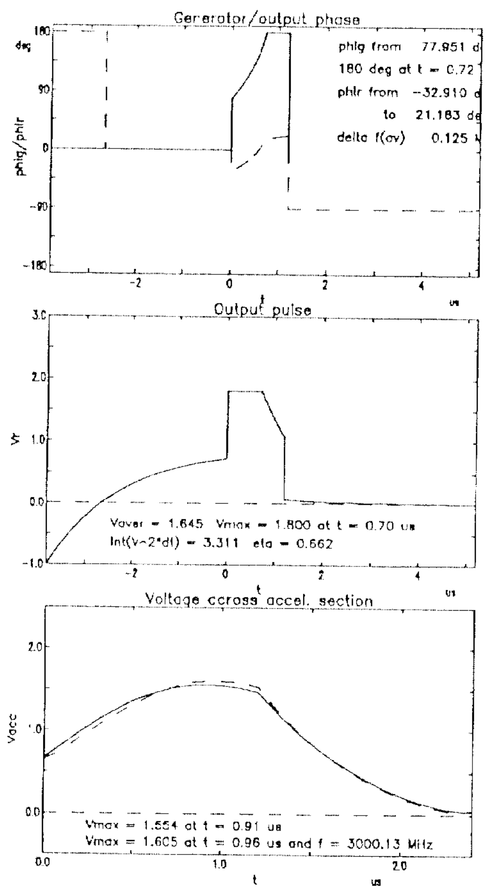

Fig. 6: *Calculated waveforms of the continuous−phase SLED, V_{max} = 1.8*

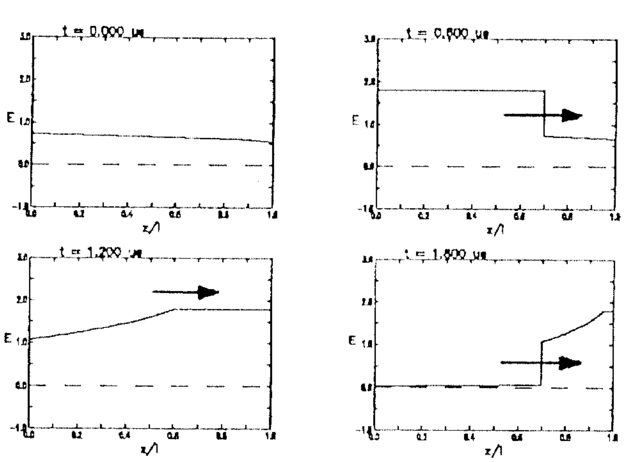

Fig. 7: *E−field across accelerating section, continuous−phase SLED, V_{max} = 1.8*

3. PERFORMANCE

Some numerical results based on the parameters of the LIPS cavities of the CERN LIL injector shall be given. The Q of the storage cavities is 200000, coupling β = 13.4, the filling time of the accelerating structure is 1.2 μs, and g = 0.8126. The amplitude of the incident wave from the klystron is taken as one. It is switched on at $t = -t_e$ with zero phase. From $t = 0$ to $t = t_b$ its phase is changed according to eq.(15), so that the reflected wave is constant in amplitude. From $t = t_b$ until switch−off at $t = t_c$ the phase is 180°. This is shown as the solid line in the top diagram of fig. 6. The charging time has been chosen as $t_e = 3.8\mu$s, and the decharging time as $t_c = 1.2\mu$s, equal to the accelerating section filling time. The reflected amplitude from the cavity is shown in the middle diagram. Up to $t = t_b$ the voltage can be held constant, afterwards there is an exponential decay. Obviously the flat top can be made the longer the lower this voltage level is chosen. The broken line in the upper diagram shows the phase of the pulse. During decharging, it deviates several 10° from zero. The lower diagram shows the accelerating voltage across the structure, i.e. the momentary voltage multiplication factor. There is a flat maximum at about 1.55. The broken curve shows the available voltage if a higher frequency is chosen for acceleration, as described in Chapter 3. A maximum voltage of 1.6 is reached. With the phase switched, the maximum voltage is 1.6 as well, but it is reached earlier in time. Fig. 7 shows the field distribution in the accelerating structure at different times.

In fig. 8, finally, a lower flat top voltage is chosen (1.6), shaping the output pulse nearly rectangular. The accelerating voltage is by 0.1 lower than for V_{max}, but with the higher operating frequency the value of 1.6 is reached again.

It can be stated that the continuous phase modulation scheme does not enhance the available acceleration energy. It causes, however, a more continuous distribution of the energy over the accelerating structure, which resembles more a true "constant gradient".

Further investigations have shown, that neither the value of the coupling coefficient nor the exact control of the generator phase are very critical for the circuit behaviour.

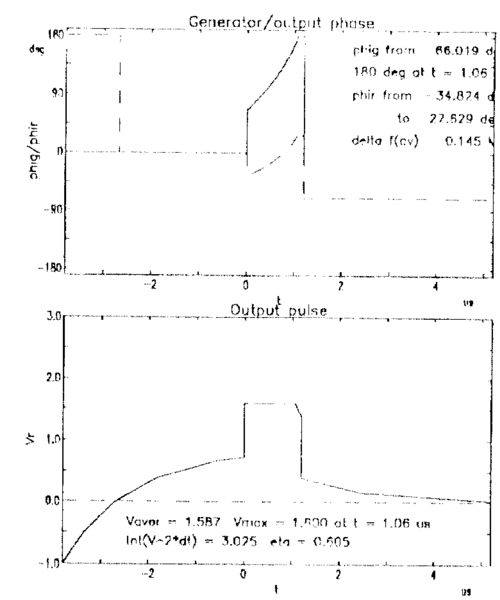

Fig. 8: *Calculated waveforms of the continuous−phase SLED, V_{max} = 1.6*

REFERENCES

1. Z. D. Farkas, H. A. Hogg, G. A. Loew, P. B. Wilson, *IX. Conference on High Energy Accelerators* 1974, pp. 576−582
2. A. Fiebig, R. Hohbach, P. Marchand, J. O. Pearce, *Particle Accelerator Conference*, Washington 1987
3. A. Fiebig, Ch. Schieblich, *European Particle Accelerator Conference*, Rome 1988, pp. 1075−1077
4. A. Fiebig, CERN PS−RF Note 83−3, 1983