COMPUTATIONS OF SPIRAL ACCELERATING CAVITIES FOR CYCLOTRONS

A. Susini
PS Division, CERN, 1211 Geneva 23, Switzerland

Introduction

The main parameters of the Eulima Cyclotron have already been presented in this conference [1]. We recall only that there are two accelerating cavities in the machine, which have a quite unconventional shape as they have to be accommodated in the space available between two highly spiralized hills. Fig. 1 shows an artist's conception.

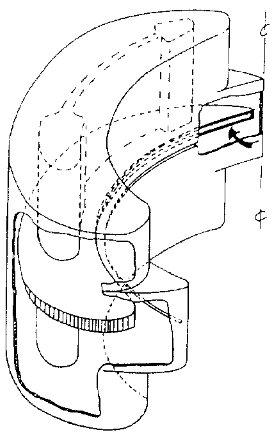

Fig. 1. The Eulima cavity; artist's conception.

A high-accelerating voltage, order of 1000 kV/turn, is required at the periphery for the beam extraction, whereas at injection half of that, or less, is sufficient. In between, the accelerating voltage can follow an arbitrary decreasing law, possibly a linear one. The accelerating frequency is 69.6 MHz, and the harmonic number h=4.

Basic principles

As the angular width of the magnetic hills is 35°, there is enough room, at the lowest mode frequency, to accommodate an accelerating structure delivering two kicks to the circulating particle at each cavity traversal. The latter, whose 3/4 height section is shown Fig. 2, cannot, evidently consist of a standard delta + stem arrangement.

The conceptually corresponding coaxial cable version.

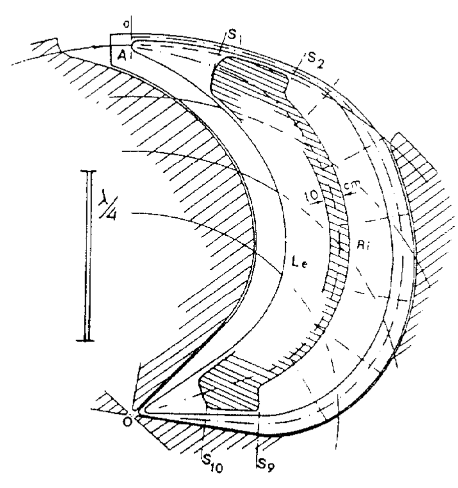

Fig. 2. The Eulima cavity. Horizontal section at 3/4 h.

consists, at the lowest mode frequency, of two 60° long cables connected at both ends. One of these ends represents the highest voltage point A at the periphery; the other one, earthed through an inductance $L\omega \cong Z_0$ (tan 30°)/2 represents the central point 0 where $V_0 \cong V_A/2$.

Using TE 01 waveguide instead, the phase velocity v_g is higher than c and can be made such that $\lambda_g/4 >> 2.1$ meters, (the machine radius). To decrease the rf losses, still with a reasonable gap width, the cross-section of the right waveguide Ri has a re-entrant shape, Fig. 3, and it is smoothly variable along the curvilinear coordinate s, defined as the mid-point of the accelerating gap. The

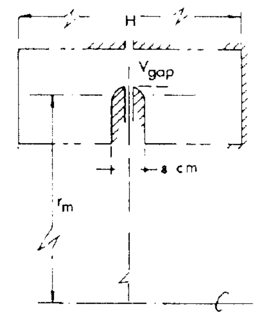

Fig. 3. Cross-section of the conducting waveguide.

local average curvature radius is $r_m(s)$. The left waveguide Le has a similar form, but the position of the accelerating lips is reversed with respect to the curvature center. Between Ri and Le waveguides a spacing of 10 cm is made available for mechanical supports, plumbing etc. Lack of space towards the two extremities A and center 0, prevents the waveguide configuration being kept throughout and the accelerating structure is changed locally into a horizontally flat electrode between two vertical walls, which builds up a delta-like accelerating structure, see Fig. 1. The phase constant being ω/c in this case, the impedances, which have to be infinite at points A and 0, decrease rapidly when s increases (at periphery) and s decreases (at center). For the Ri side, the calculations show that the delta structure can be maintained up to the coordinate $s_1 = 50$ cm. The impedance is -j120 Ω.

To be able to continue with the conducting waveguide Fig. 3, a 40 cm long segment of TE 01 cut-off waveguide is needed to raise the impedance up to -j780 Ω, Fig. 4.. The conducting waveguide begins at $s_2 = 90$ cm. The section is smoothly variable depending on the valley geometry. The cut-off frequency f_c, always $< f$, is chosen such that the phase constant

$$\beta = \frac{\omega}{c\sqrt{1-(f_c/f)^2}} \qquad (1)$$

causes the impedance to be still capacitive at $s_2 = 324$ cm : Z = -j381Ω, with a voltage of 109 kV, close to that required for the injection. To be resonant at $f = 69.6$ MHz both Ri and Le sides must be matched to an inductive impedance, which, for Ri must be equal to j381 Ω.

At the center there is again a flat electrode (delta) which carries the injection geometry. Proceeding as for the periphery, at 30 cm from the center this electrode shows an impedance of -j92.5 Ω, which is equally shared among Le and Ri sides.

Therefore, the Ri sides are loaded with -j185 Ω at $s = s_{10}$. The need exists to transform -j185 Ω into j381 Ω at $s = s_9$ to fulfill the resonance conditions. This is obtained with a 29 cm long cut-off waveguide. The resulting configuration of the Ri side is shown in Fig. 4 together with the main parameters resulting from the calculations.

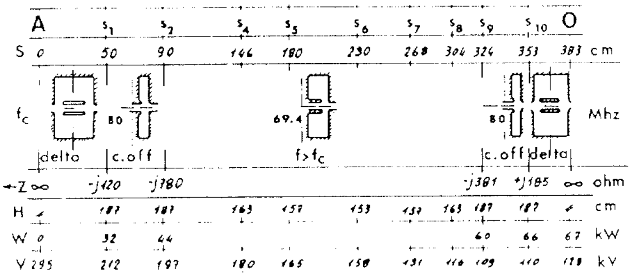

Fig. 4. Summarizing the results.

Organisation of the calculations

Starting from point A, it is arbitrarily supposed that a lumped capacitance C_A = 2 pF exists there. The initial voltage is set equal to 295 kV to ensure 500 kV energy gain for the outmost particle path.

For the first delta-like sector A - S_1 several cross-sections are traced and the line constants are computed by means of URMEL program (2) or graphical tables (3). Applying the non-uniform line program, (called MIRC92 in our case) voltages, currents, energies etc. are computed, which have to be shared among Le and Ri. Increasing the impedance at $s = s_1$ is obtained by means of a cut-off waveguide section following :

$$Z_2 = Z_1 \frac{1 + j(Z_0 / Z_1) tgh\, \gamma L}{1 - j(Z_1 / Z_0) tgh\, \gamma L} \quad (2)$$

with

$$\begin{cases} \gamma = \dfrac{\omega}{c}\sqrt{\left(f / f_{c_1}\right)^2 - 1} \\ Z_0 = \dfrac{Z_{00}}{\sqrt{\left(f / f_{c_1}\right)^2 - 1}} \end{cases} \quad (3)$$

When Z_2 and Z_0 are given, this leads to a second degree equation for Z_{00} with f_{c_1} and L as parameters. Z_{00} = 25.6 Ω, f_{c_1} = 80 MHz, L = 40 cm. The point S_2 is reached.

From S_2 on, the conducting waveguide is met. The aim is to reach the point 0 with the required voltage V = 109 kV. Several cross-sections are considered at different distances s from A in order to be able to apply the program MIRC92 as before, with the only difference that the phase constant and the characteristic impedance are given by (1 and :

$$Z = \frac{Z_{00}}{\sqrt{1 - (f_c / f)^2}} \quad (4)$$

where Z_{00} has to be found for every section. Whilst the lateral sides depend on the geometry of the given section, the height H has to be guessed to obtain the proper cut-off frequency f_c. For each cross-section a table has to be prepared for the waveguide constants as functions of H. Rotating the cross-section along its local curvature axis an axisymmetrical cavity is generated, following Fig. 3. Assuming zero axial periodicity, the program URMEL computes f_c, establishes an arbitrary value V_{gap} between the accelerating lips and gives the Q and the total energy content (not assuming half-cell symmetry). Then the value E' is calculated .

$$E' = \frac{\textit{total energy content}}{2\pi r_m V_{gap}^2 / 2} \quad (5)$$

thus leading to the energy based definition :

$$Z_0 = \frac{V_{gap}^2}{2E' c\sqrt{1 - (f_c / f)^2}} \quad (6)$$

which is also entered into the tables. A candidate f_c is tried at first.

From all tables the waveguide constants are interpolated first which correspond to the given f_c. Now MIRC92 can proceed from section S_3 until S_9 with steps of 2 cm. At each step, new constants are interpolated from the past S_i and the next S_{i+1} section. The calculations are repeated with different f_c until a satisfactory final voltage is reached. At this moment, one starts from the central point 0, where again an arbitrary capacitance C_0 = 2 pF is supposed to exist. At 30 cm from the center, MIRC92 indicates an impedance of -j185 Ω. Finally, a cut-off waveguide has to be found which transforms -j185 Ω input impedance into j381 Ω to resonate with the remaining part of the structure. One obtains : Z_{00} = 16.8 Ω, f_{c2} = 80 MHz, L = 29 cm.

The Ri side requires 67 kW and the Le one, though quite shorter, still 59 because most of the power is lost in the peripheral delta and subsequent cut-off waveguide. This occurs because no effort has been done up to now to optimize this part of the resonator. Combined engineering and electrical studies have to be undertaken to improve this point. For the Le side, there is one constraint more, i.e. to arrive at point S_{10} with the same voltage and impedance as the Ri side. There are enough free parameters and the solution can be found.

Sensitivities

Once the mathematical model is set up, some parameters are slightly changed. Tentatively, a new value of the working resonant frequency f has to be found which gives zero admittance at point 0. The results are shown in Table 1. For the Le section, the computations have not been carried out; similar figures are expected.

Table 1 - Sensitivities etc.

$\frac{df}{dC_A} = -250\,\text{kHz/pf}$	$V_A = \text{const.}$
$\frac{df}{df_{c_2}} = +112\,\text{kHz/MHz}$	$\frac{df}{df_{c_1}} = -330\,\text{kHz/MHz}$
$\frac{df}{df_{c_1}} = +396\,\text{kHz/MHz}$	$\frac{df_{c_1}}{dH} = -370\,\text{kHz/cm}$
$\frac{dV_0/V_0}{dC_A} = -9{,}5\%/\text{pf}$	$\frac{dV_0/V_0}{dC_0} = +12.5\%/\text{pf}$
$\frac{dV_0/V_0}{df_{c_2}} = -10.7\%/\text{MHz}$	$\frac{dV_0/V_0}{df_{c_1}} = +12\%/\text{MHz}$
$\frac{dV_0/V_0}{df_c} = +2.6\%/\text{MHz}$	Q = 29500
$\frac{df_{c_1,c_2}}{dH} = -429\,\text{kHz/cm}$	

Conclusion

Much further work either on paper or on models has to be done since this mathematical model is quite crude. First, the influence of the delta-to-waveguide transition has not been evaluated as well as the transition from cut-off-to-conducting waveguide and vice versa. Many assumptions have been made without previous engineering studies which, in turn, would have required previous knowledge of the structure losses and sensitivities. Nevertheless, it is felt that, at the present state, this mathematical model gives an idea how the cavity works and provides information for the construction of the unavoidable models.

References

[1] P. Mandrillon et al, "Eulima Status Report", this conference.

[2] C. Pal et al, "Urmel and Urmel User Guide", DESY M-85-11 December 1985.

[3] W.S. Metcalf, "Why not use rectangular coax?" Microwaves, April 1968, p. 52 and 56.