A NEW BENCH METHOD TO SIMULATE ELECTROMAGNETIC FIELDS OF SLOW BEAMS

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Abstract

For transverse impedance and pick-up measurements an image plane with an array of interleaved small probes and loops fed by delay lines was built. This set-up has been used to simulate the field of a slow particle beam with a beta of 0.065. By varying the delay line length and the probe-and loop-power dividers, the beta value and the ratio of the transverse E and H-fields can be adjusted independently. We discuss the simulated and actual beam fields and present first results of bench measurements.

Introduction

The well known coaxial wire method [1] permits the simulation of the electromagnetic field of beams with beta close to unity. One may slow down the phase velocity of the simulator line by dielectric loading, but in this case the ratio of E and H does in general not correspond to that of the slow beam. In this paper, we discuss a set-up of small loop and probe antennas which has been developed for transverse pick-up measurements.

The loops and probes are lined up on an aluminium plate with a close spacing between adjacent elements (Fig. 1). The installation in the PU tank is sketched in Fig. 2.

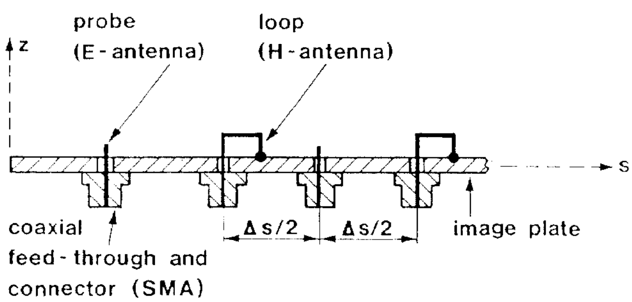

Fig. 1 : Cross-section of the beam-simulator.

The elements are fed via delay lines (Fig. 3) of properly adjusted length $\ell \propto s/\beta$ to create the $e^{i\omega(s/\beta c - t)}$ dependence. The loop and probe power levels are separately adjusted. Attenuators at the entrance to the elements avoid reflections. The signal is coupled out at the normal exit port of the pick-up and displayed using the network analyzer. A simple loop coupler type of pick-up with known response is installed downstream in the same tank for calibration.

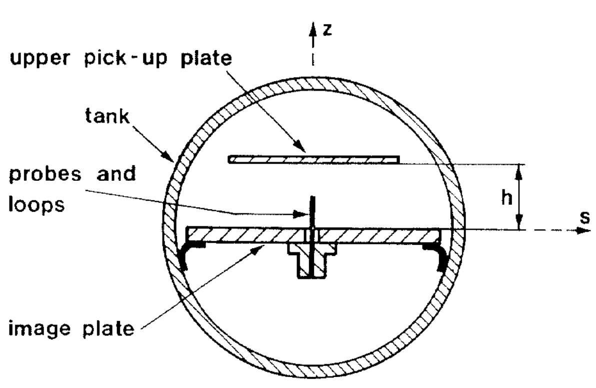

Fig. 2 : Beam-simulator installed in pick-up tank.

Electromagnetic Fields of Beam and Simulator

We wish now to compare the fields of a coasting beam in a storage ring with the fields created by the simulator. We assume that the beam oscillates vertically, such that each particle at position s and time t is displaced by

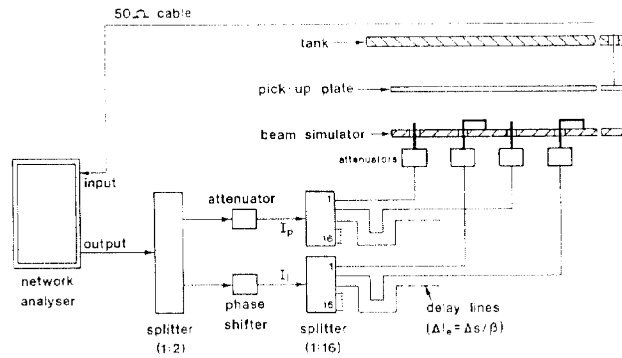

Fig. 3 : Measurement set-up (schematic).

$$\Delta z = \xi\, e^{i(ks-\omega t)} \tag{1}$$

The longitudinal particle velocity is $v_s = \beta ct$ and the linear charge density $\lambda = Ne/2\pi R$. For small beam size ($a \to 0$, $b \to 0$) the corresponding current and charge-density can be modelled as [2]:

$$j_x = 0, \quad j_s = j_{s0} + j_{s1}, \quad j_z = j_{z1}, \quad \rho = \rho_0 + \rho_1$$

with

$$\begin{aligned} j_{s0} &= \beta c \rho_o = \beta c \lambda\, \delta(x)\delta(z) \\ j_{s1} &= \beta c \lambda\, \delta(x)\left[\frac{\delta_{(z-b)} - \delta_{(z+b)}}{2b}\right]\xi e^{i(ks-\omega t)} \\ j_{z1} &= i\beta c\lambda\left[k - (\omega/\beta c)\right]\delta(x)\delta(z)\xi e^{i(ks-\omega t)} \\ \rho_1 &= j_{s1}/\beta c \end{aligned} \tag{2}$$

Here $\delta(x)$ is Dirac's function.

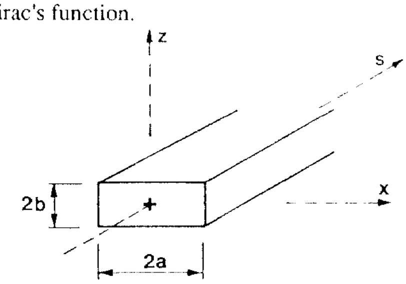

Fig. 4 : Beam and coordinate system.

Replacing $\lambda \to e/2\pi R$, the currents (2) can also be interpreted as the Fourier components of a single charge circulating in a storage ring and oscillating transversely with

$$\Delta z = \xi e^{-i\omega_\beta t} = \xi e^{-iQ(\beta c/R)t}$$

Wave number and frequency in (2) are given by [2]:

$$k = n/R, \quad \omega = \frac{n \pm Q}{R}\beta c \tag{3}$$

and hence

$$k = \frac{\omega}{\beta c}\frac{n}{n \pm Q} \qquad \left(\approx \frac{\omega}{\beta c} \quad for\ n >> Q\right) \tag{4}$$

Here $2\pi R$ is the circumference and Q the vertical betatron tune of the storage ring; n is the mode number of the beam oscillation.

We are mainly interested in the AC components (j_1) of (2). The corresponding electromagnetic fields can be obtained from the vector potential. In free space, and for the monochromatic time dependence one has [3]:

$$\vec{B} = rot\,\vec{A}, \quad \vec{E} = -\dot{\vec{A}} - grad U \tag{5}$$

where

$$\vec{A} = \frac{\mu_0}{4\pi}\int j(\vec{x}\,')\frac{e^{i(\omega/c)|\vec{x}-\vec{x}\,'|}}{|\vec{x}-\vec{x}\,'|}d^3x' \tag{6}$$

is a solution of the inhomogeneous wave equation

$$\Delta\vec{A} + (\omega/c)^2\vec{A} = -\mu_0\vec{j}$$

The scalar potential U is obtained from the charge density by an equation similar to (6) with $\mu j \to \rho/\varepsilon_0$. To satisfy the boundary conditions, solutions of the homogeneous wave equation have to be added. However, as the beam and the simulator are in an identical environment, these additional fields are the same in both situations provided that the free space solutions at the boundaries -including the pick-up plates- are the same.

We now turn to the currents and charge densities as given by the probes and loops (Figs. 5 and 6). We write

$$\begin{aligned}
j_x &= 0\\
j_s &= I_\ell 2bd\,\delta(x)\left[\frac{\delta_{(z-b)} - \delta_{(z+b)}}{2b}\right]e^{-i\omega t}\sum_m \delta(s - s_m)e^{iks_m}\\
j_z &= I_p 2\ell\,\delta(x)\delta(z)e^{-i\omega t}\sum \delta(s - s_m - (\Delta s/2))e^{ik(s_m - (\Delta s/2))}\\
&\quad + I_\ell 2b\,\delta(x)\delta(z)e^{-i\omega t}\sum\left[\delta(s - s_m) - \delta(s - s_m - d)\right]e^{iks_m} \qquad (7)\\
-i\omega\rho &= I_p 2\ell\,\delta(x)\left[\frac{\delta(z-\ell) - \delta(z+\ell)}{2\ell}\right]e^{-i\omega t}\times\\
&\quad \times\sum \delta(s - s_m - \Delta s/2)e^{ik(s_m - \Delta s/2)}
\end{aligned}$$

where the position of the m^{th} loop and probe is $s_m = m\Delta s$ and $s_m + \Delta s/2$, respectively. The expression for j_s contains the horizontal loop current at z = b and its image at z = -b (Figs. 5 and 6). They are approximated by Delta functions of all three coordinates. The equation for j_z has contributions from the probes and from the vertical loop currents at s_m and s_m + d. The charge density can be obtained from the continuity equation div j + $\dot{\rho}$ = 0; only the probes contribute to the charge density.

For small spacing Δs the sums in Eq. (7) can be approximated by integrals to obtain

$$\begin{aligned}
j_s &\cong \frac{I_e 2bd}{\Delta s}\delta(x)\left[\frac{\delta_{(z-b)} - \delta_{(z+b)}}{2b}\right]e^{i(ks-\omega t)}\\
j_z &\cong \left(\frac{I_p 2\ell}{\Delta s} - ik\frac{I_e 2db}{\Delta s}\right)\delta(x)\delta(z)e^{i(ks-\omega t)} \qquad (8)\\
-i\omega\rho &\cong \frac{I_p 2\ell\,\delta(x)}{\Delta s}\left[\frac{\delta(z-\ell) - \delta(z+\ell)}{2\ell}\right]e^{i(ks-\omega t)}
\end{aligned}$$

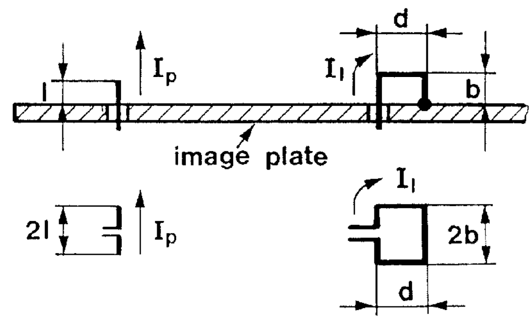

Fig. 5 : Probe and loop on image plate (a) and equivalent sources in free space (b).

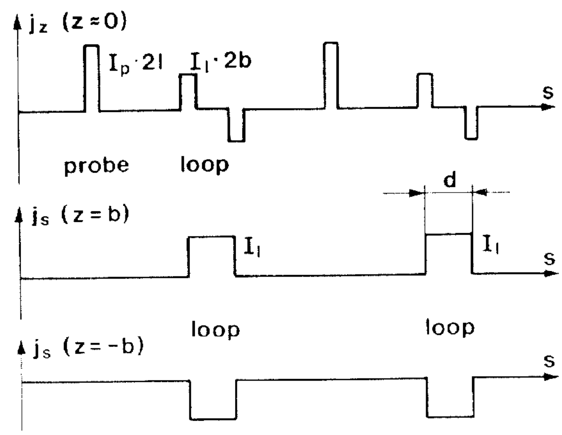

Fig. 6.: Vertical and longitudinal current sources produced by the beam simulator.

The use of these simplified sources can be justified in a more rigorous manner by inserting the Fourier expansion of the s dependence of the current (7) (not including the exponential dependence on iks) into (6). At a distance h from the axis, where $|\vec{x} - \vec{x}\,'| = \sqrt{h^2 + (s - s_m)^2}$,the contribution of the harmonics exp(ip2πs/Δs) to the potential is negligible if the variation of the remaining integrand is small over the period length Δs. This requires

$$k << 2\pi/\Delta s \quad and \quad h >> \Delta s/2\pi \tag{9}$$

Then only the harmonic sero has to be retained, which leads to the sources (8). Eq. (9) determines the required spacing of the loops and probes for given pick-up to beam distance h and given frequency $\omega/\beta c \approx k$ (Eq. 4).

To establish equivalence between simulator and beam we wish to approximate the perturbed beam sources, Eq. (2) by those of Eq. (8). One notes that full equivalence is only possible for two of the three quantities j_s, j_z, ρ. We choose to equalise j_s and ρ such that in the long wavelength limit ($\omega h/c << 1$) the transverse magnetic and electric field is correct. We then require for $\ell \to 0$, $b \to 0$:

$$\frac{I_e 2bd}{\Delta s} = \beta c\lambda\xi \quad and \quad \frac{I_p 2\ell}{\Delta s} = -i\omega\lambda\xi \tag{10}$$

It is clear that for this choice j_z and hence for example,the longitudinal magnetic field is not exactly that of the beam. Similar results can be obtained by adding the fields rather than current and charge densities, requiring that the near fields of the probes and loops approximate the transverse electric and magnetic beam fields.

The conditions (10), together with Eq. (4), can be satisfied by proper choice of the current levels and delays. In fact the required difference in electrical length $\Delta\ell = c\Delta t$ of the delay lines (Fig. 3) is given by $k\Delta s = \omega\Delta t$. Using Eq. (4) we then obtain $\Delta\ell = \Delta\ell\ s/\beta$, valid for all, except for very low frequencies where n is not yet large compared to Q.

Measurements

A simplification is possible if the interaction is mainly via the electric beam field. In this case only probes are necessary. This approach has been used in preliminary measurements. The increase of current with frequency as required by the second Eq. (10) is achieved feeding the probes from a constant voltage source. To the extent that the probes present a capacitive load, the current is $I_p = i\omega C U_p$ and thus proportional to frequency.

A beam simulator with probe antennas spaced by Δs = 17 mm was used to measure the sensitivity of a pick-up for stochastic cooling at $\beta = 0.065$ (2 MeV kinetic energy) in LEAR. The device [4] tested is a slow wave structure formed by a meander shaped strip line coated on dielectric plates at h = ± 30 mm above and below the beam (Fig. 7).The phase velocity in beam direction is matched to 0.065 c.In the measurement the required electrical delay length from one probe to the next $\Delta\ell = \Delta s / \beta$ is therefore about 260 mm.

In Fig. 8 the response of the 600 mm long meander coupler is compared to that of a 200 mm long solid electrode installed as loop coupler downstream in the P.U. tank at the same distance above the beam. One concludes that the longer meander structure has a 13 db (factor 20) higher average sensitivity. At equal length; i.e. assuming several short loop couplers in series with proper delay, this reduces to 4 db (factor 2.5). One also notes the expected $\sin^2\left[\omega(\ell/2c)\left(\beta^{-1}-1\right)\right]$ response of the loop with the first maximum at f ≈ 50 MHz compared to f ≈ 80 MHz for the longer meander structure.

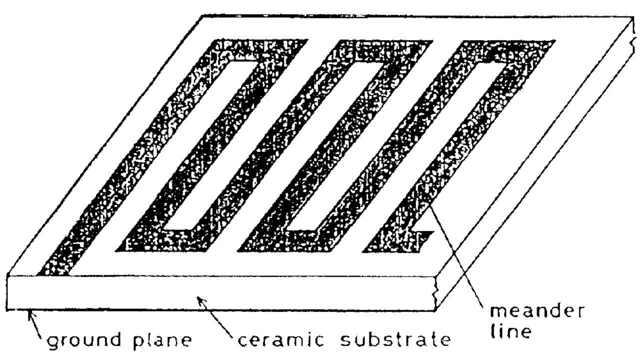

Fig. 7 : Sketch of the meander pick-up (adapted from [4]). Only the lower electrode is shown.

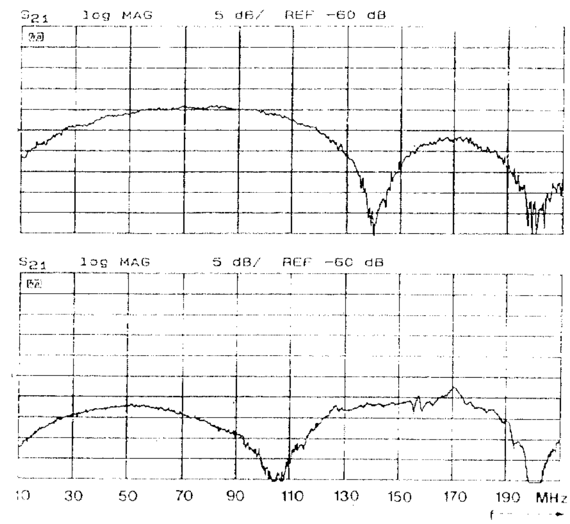

Fig. 8 Sensitivity of meander coupler (upper curve) and loop coupler v.s. frequency. Vertical scale : 5 db/division

Conclusions

The array of loop and probe antennas looks useful for synthesizing the transverse field of slow beams. Results obtained at β = 0.065 for a transverse pick-up agree in a satisfactory manner with expectation [4]. Earlier measurements using a wire with dielectric coating failed to give useful results.

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