MAGNET WAVEFORMS FOR THE TRIUMF KAON FACTORY SYNCHROTONS

M.K. Craddock* and R. Baartman
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C. V6T 2A3

Abstract

This note considers various possible waveforms for the magnets in the 50 Hz 3 GeV Booster and 10 Hz 30 GeV Driver synchrotrons. A dual-frequency piece-wise harmonic waveform is shown to offer a significant advantage for the Driver but probably not for the Booster. Adding a second harmonic to the fundamental is found to be less effective than dual frequency. Finally the question of an ideal waveform is considered from the point of view of minimizing the rf voltage; such a waveform is found to offer other advantages as well.

Introduction

This note will explore various waveforms for the magnets in the 50 Hz 3 GeV Booster and 10 Hz 30 GeV Driver synchrotrons from the point of view of minimizing the radiofrequency voltage gain per turn and assuming a resonant power supply circuit. The waveforms to be considered are:

- dual-frequency harmonic
- added second harmonic
- "ideal" waveform, requiring minimum rf voltage.

The programs for rf voltage $V(t)$ and synchronous phase $\phi_s(t)$ for the synchrotrons are set by a number of factors:

(i) The rate of rise of dipole magnetic field $\dot{B}$, which fixes the energy gain per turn:

$$V \sin \phi_s = C\rho\dot{B} \tag{1}$$

Here C is the orbit circumference and ρ the radius of curvature within each dipole magnet. For a resonant magnetic circuit, $\dot{B}$ and therefore the energy gain are normally zero at the beginning and end of each acceleration cycle, rising to some maximum in between. In practice it is the control settings for B and V which determine the effective value of ϕ_s; here we are asking how $\dot{B}$ and V are themselves determined.

(ii) Sufficient bucket area A in the energy-time plane to enclose the bunch emittance ϵ, which is invariant during acceleration

$$A = \frac{4\tau_c}{\pi h}\sqrt{\frac{2E_0}{\pi h}\frac{\gamma}{\mid\eta\mid}\epsilon V}\ \alpha(\phi_s) \tag{2}$$

where e and E_0 are the charge and rest energy of a proton, $\tau_c = C/c$ is the limiting orbital period, h is the harmonic number, γ is the relativistic energy factor and $\eta = \gamma^{-2} - \gamma_t^{-2}$. For energies well below the transition energy γ_t we can write $\eta \simeq 1/\gamma^2$ and therefore the bucket area can be written

$$A \sim \sqrt{V\gamma^3}\,\alpha\,(\phi_s)\,. \tag{3}$$

The dimensionless factor $\alpha(\phi_s)$ cannot be written as an exact function of ϕ_s, but can be approximated[1] by the equation

$$\alpha\,(\phi_s) = \left(1 - \overline{\sin\phi_s}^{0.79}\right)^{\frac{1}{0.79}} \tag{4}$$

with an error $<1\%$ for $\phi_s \leq 71°$. In the first part of the cycle V is set so that the bucket height is kept about 25% higher than the bunch height, ensuring that $A \simeq 1.7\epsilon$.

(iii) Two intensity-related effects:

- the longitudinal focusing forces provided by the rf must be kept larger than the forces arising from collective effects in order to avoid microwave instabilities;
- higher cavity voltages make beam loading easier to handle.

Both of these effects argue for $V\cos\phi_s$ to be large compared with a quantity proportional to beam current and to the appropriate beam coupling impedance.

(iv) Special requirements near injection:

- The transverse space charge tune shift $\Delta\nu_z$ must not be allowed to grow too much through narrowing of the bunch as ϕ_s increases from 0.
- The synchrotron tune ν_s must not be allowed to rise through too rapid an increase in V.

(v) At extraction the bunch must be matched to the requirements of the next machine.

Figure 1 illustrates the voltage program specified for the Booster, and the associated bunch parameters, for a harmonic magnet waveform. For the first 10% of the cycle the injection considerations (iv) dictate rather a slow rise in V. Then for the next part of the cycle the fill fraction (i.e. ratio of bunch height to bucket height) is kept constant so that factors (i) and (ii) together determine V and ϕ_s. The maximum voltage required in the cycle occurs in this section.

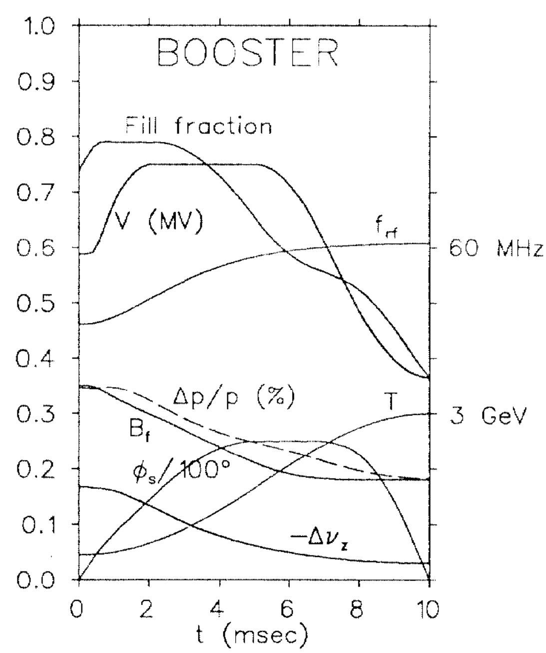

Fig. 1. RF voltage program and associated bunch parameters for the Booster with a single-frequency magnet waveform.

*On leave from University of British Columba.

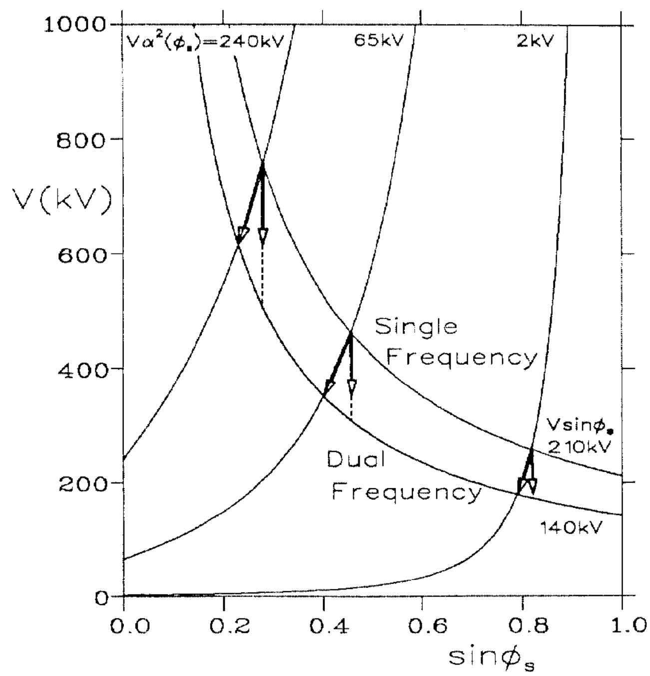

Fig. 2. Curves of constant energy gain per turn and constant bucket area for the Booster. The arrows show the reduction in rf voltage achieved by using a dual-frequency magnet waveform.

Over the remainder of the cycle V is gradually reduced, bringing the bunching factor B_f and the momentum spread $\Delta p/p$ down to values matching those in the next ring. The rf voltage at extraction is chosen as low as possible consistent with the constraints (iii) set by beam stability and beam loading, in order to minimize the rf costs in the next machine.

Figure 2 shows curves of constant energy gain and bucket area in the space of the two parameters, V and $\sin\phi_s$, which we are trying to determine, for the case of the Booster. The intersections of the curves for the required values of bucket area and energy gain give the solutions for V and ϕ_s. These curves are of universal form although the numerical values will depend on the particular machine.

Dual-frequency Operation

To reduce the rf voltage requirement it has been proposed to slow down the magnetic field rise and speed up the fall by switching the resonant frequency at the top and bottom of each magnet cycle. A frequency change by a factor 3 is often proposed, so that the rise occupies three-quarters of the cycle and the energy gain per turn is only two-thirds of that required with a single frequency. However, it will be seen from Figure 2 that a particular reduction in energy gain does not ensure the same fractional reduction in rf voltage requirement. In moving down from the "single frequency" curve to the "dual frequency" curve for constant energy gain one must steer along a curve of constant bucket area, i.e. towards lower values of ϕ_s. At high values of ϕ_s the reduction in V is almost the ideal one-third; thus in the Driver where ϕ_s reaches 56° the voltage is reduced from 3680 kV to 2550 kV, a savings of 31%; but in the case of the Booster, where ϕ_s reaches only 14° where maximum V is required, the slope of the energy gain curve is rather steep and V is reduced only from 750 kV to 624 kV, i.e. 17%. While the voltage reduction for the Driver - over a megavolt - is clearly significant, the 126 kV achievable for the Booster is not felt to be worth the extra complication and expense of a dual-frequency magnet power supply. The reference design therefore assumes single-frequency operation for the Booster.

Added Second Harmonic

An alternative method of modifying the magnet waveform is to add a second harmonic component

$$B = B_0 - B_1 \cos 2\pi f t + B_2 \cos(4\pi f t + \Delta) \qquad (5)$$

In particular J. Crawford[2] has suggested the use of a phase shift $\Delta = \pi/4$ with $B_2/B_1 = 1/4\sqrt{2}$ in order to make the slope of the field ramp as constant as possible (Figure 3). Runs with RAMA[3] show that this does reduce the rf voltage requirement (from 750 kV to 721 kV for the Booster, from 3680 kV to 3200 kV for the Driver) but not as much as the dual-frequency cycle.

A more effective scheme, at least for the Booster, would be to use a phase shift $\Delta = 0$ to flat-bottom the waveform, slowing down the initial field rise. RAMA runs show that $B_2/B_1 = 0.10$ is optimum for the Booster, lowering the maximum rf voltage to 706 kV. For the Driver, where the peak voltage requirement is set by the field-rise requirement at mid-cycle rather than by the bucket area, this scheme is not very effective; the optimum fraction $B_2/B_1 = 0.05$ lowers the maximum voltage only from 3680 to 3670 kV. It is possible that a different value of the phase shift Δ might reduce the voltage requirement further but this has not yet been investigated.

Another consideration, pointed out by the power supply engineers, is that addition of second harmonic is likely to be more expensive than use of a dual-frequency cycle, because it requires provision of additional power supplies rather than additional switches.

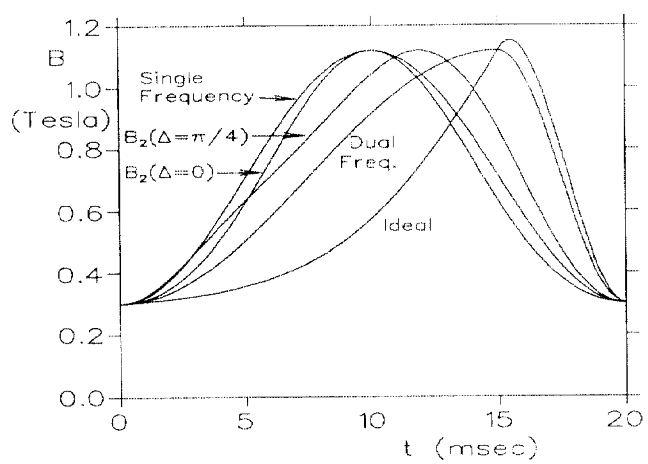

Fig. 3. Various magnetic field waveforms considered for the Booster.

Ideal Magnet Waveform

Rather than trying out various waveforms we may seek to determine the "ideal" waveform from the point of view of obtaining constant bucket area A for a constant rf voltage V. Under these conditions the equation for the bucket area may be written

$$A = \text{const}\sqrt{V\frac{\gamma}{|\eta|}}\alpha(\phi_s) = \text{constant} \qquad (6)$$

giving a relationship between synchronous phase ϕ_s and energy γ. Evaluating the constant at some reference energy γ_i we can express $\alpha(\phi_s)$ directly in terms of energy

$$\alpha(\phi_s) = \alpha_i \sqrt{\frac{\gamma_i^3}{\gamma^3} \cdot \frac{\gamma_t^2 - \gamma^2}{\gamma_t^2 - \gamma_i^2}} \tag{7}$$

For very high transition energies ($\gamma \ll \gamma_t$) as is the case for the KAON Factory, this may be simplified to the form

$$\alpha(\phi_s) = \alpha_i (\gamma_i/\gamma)^{3/2}. \tag{8}$$

Using the approximate formula (4) for $\alpha(\phi_s)$ and solving for $\sin \phi_s$ we find that we have an equation for the rate of change of magnetic field in terms of the proton energy:

$$\frac{C\rho}{V}\dot{B} = \left\{1 - \left[\alpha_i(\gamma_i/\gamma)^{\frac{3}{2}}\right]^{0.79}\right\}^{\frac{1}{0.79}} \tag{9}$$

This can immediately be converted into a differential equation for the momentum $\mu \equiv \beta\gamma$ so that we can express the time, in units of the limiting orbital period τ_c, as an integral over momentum:

$$t - t_i = \tau_c \frac{E_0}{eV} \int_{\mu_i}^{\mu} \left\{1 - \left[\alpha_i \left(\frac{1+\mu_i^2}{1+\mu^2}\right)^{3/4}\right]^{0.79}\right\}^{\frac{-1}{0.79}} d\mu \tag{10}$$

This can be integrated numerically and an example is shown in Figure 3. The curve of $\mu(t)$ is characterized by a very slow initial rise, associated with a singularity in the integral for $\alpha_i = 1$ ($\phi_s = 0$). Because of this the waveform must be started with a small finite slope $\dot{\mu}$ where $\alpha_i = 1 - \delta$. In fact α_i and V are uniquely determined by the choice of A at μ_i through (2) and of t_f at μ_f through (10). Runs with the RAMA code confirm that the waveform does produce constant bucket area A and constant fill fraction for constant rf voltage V (Figure 4). Allowing one-quarter of the cycle for field fall the required voltage is reduced to 500 kV for the Booster and to 2050 kV for the Driver. The maximum rate of field rise $\dot{B}$, and hence the voltage developed across the magnet coils, turns out to be only 6% higher than for the single-frequency harmonic waveform. Table I summarizes the maximum values of V and $\dot{B}$ required for each of the waveforms. Figure 3 shows a cubic reset waveform, matching smoothly to the ideal, to avoid any ringing in the circuit. The overshoot is minimal, raising the maximum field from 1.12T to 1.15T, but without any requirement for good field quality at the higher values.

To implement such waveforms exactly requires programmable power supplies. This is almost certainly too expensive an option for the very large system required in the Driver. In view of the considerable savings in rf voltage, it may, however, be worthwhile considering for the Booster. Programmable supplies are being provided for the fast-cycling AGS Booster, and will be used to generate a linear field ramp preceded by a $t^{\frac{3}{2}}$ segment[4], a waveform having some similarities to that proposed here. Besides minimizing the rf voltage required, our scheme provides other advantages:

- The slow initial rise in V avoids initial increases in both $\Delta\nu_z$ and ν_s.
- The rf frequency rises less steeply than for the harmonic waveforms, easing the power supply requirements for biasing the rf cavity tuners.
- Programmable supplies provide flexibility, allowing different waveforms to be tried, if desired.
- Separate programmable supplies for dipoles and quadrupoles should allow good tracking to be achieved between the two.

Table I. Parameters for Various Magnet Waveforms

	Booster					Driver
Waveform	Risetime (%)	$\hat{V}$ (kV)	$\hat{B}$ (T)	$\hat{\dot{B}}$ (T/s)	$\check{\dot{B}}$ (T/s)	$\hat{V}$ (kV)
Single Frequency	50	750	1.12	129	-129	3680
Dual Frequency	75	624	1.12	86	-257	2550
Added 2nd Harmonic						
$\Delta = \pi/4$	59	721	1.12	86	-165	3200
$\Delta = 0$	50	706	1.12	138	-138	3670
Ideal	62.5	530	1.19	147	-207	
Ideal	75	500	1.15	137	-285	2050

Bringing ϕ_s from 42° at extraction immediately to 0 on injection into the Collector ring will cause 5-10% longitudinal emittance growth – but it is planned to expand the emittance there by a factor 4 anyway.

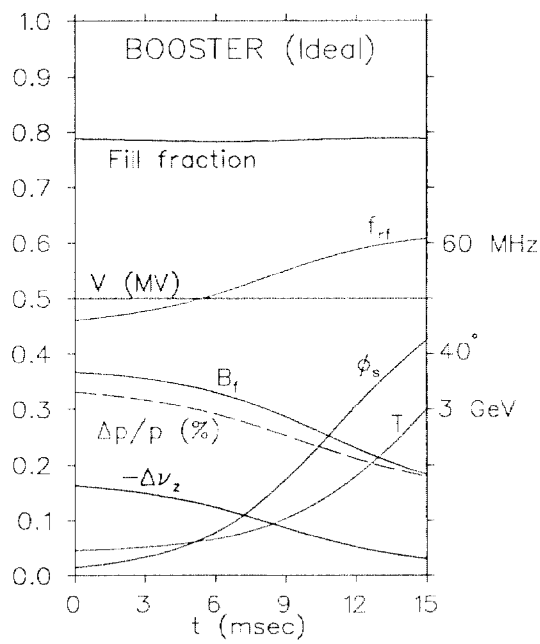

Fig. 4. Bunch parameters during the Booster cycle for the "ideal" magnet waveform giving constant bucket area for constant rf voltage.

Acknowledgements

The authors are very grateful to Pat Stewart and Jana Thomson for preparing this paper, and to Joe Chuma for his assistance with the illustrations.

References

[1] W. Hardt, private communication.

[2] W. Bothe, J. Crawford *et al*, *Magnet Power Supplies*, Proc. XIII EHF Workshop, Legnaro, February 1989, EHF-89-11.

[3] R. Baartman, H. Schönauer, *RAMA: A Computer Code Useful for Designing Synchrotrons*, TRI-DN-86-15 (1986).

[4] AGS Booster Design Manual, BNL (1988).