BEAM TRANSPORT IN BENT CRYSTAL EXTRACTORS

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Bent single crystals have been suggested as a method for the extraction of beams from high-energy particle accelerators. To investigate the efficiency of beam extraction, we have extended our Fokker-Planck transport studies of channeled beam transport to include crystal curvature. Dechanneling fluxes can be determined as a function of bend angle and particle energy. Solved as an initial value problem, the theory also includes the divergence angle of the beam impinging on the crystal.

Introduction

We have recently considered the transport of beams of charged particles in crystalline channels using a Fokker-Planck model in two-dimensional phase space.[1] This work was motivated by an interest in the extension of plasma acceleration schemes to the solid-state plasma[2,3] and therefore concentrated on the effect of large accelerating gradient on the channeling. In the near term, there are other applications of the phenomenon of channeling in crystals in accelerator physics which are of interest.[4] The deflection of a beam by channeling in a curved crystal[5] has several uses in such a context; extraction of a beam (or some part of it) being one. In this paper, we extend the theory of Ref. [1] to include the effect of the crystal curvature. The procedure is straightforward. This gives us a Green's function from which the distribution function in the two-dimensional phase space appropriate to planar channeling[1] can be found. This is done in the usual way by convolution over a given initial distribution. The measured quantity typically is the relative flux of particles channeled. This is obtained by integrating the distribution function over the phase area occupied by the channeled particles. We then evaluate the channeled flux for particular sets of parameters.

Fokker-Planck Solution

In this part, we discuss the solution of the Fokker-Planck equation for planar channeling in a curved crystal. This is of necessity brief and will heavily rely on Ref. [1] for details.

For a particle moving through a bent crystal under channeling conditions, there is, in addition to the force from the channel potential, a centrifugal force acting. In the case of constant curvature treated here, the force is:

$$F_x = -\left(KX + \gamma mc^2/R_b\right), \tag{1}$$

where K is the "spring constant" of the channel potential here assumed to be harmonic and R_b is the radius of curvature of the crystal. The usual relativistic factor, or the particle energy in rest energy units is γ. As in Ref. [1], it is convenient to normalize momenta to mc and energy to mc^2 where m is the particle mass. The radius of curvature of the bend is related to the deflection angle, χ, by $R_b = L/\chi$, where L is the length of the crystal. In normalized units, the equation of motion then becomes:

$$a_c^2 \frac{d^2X}{ds^2} + \bar{K}X + \epsilon = 0, \tag{2}$$

where $\bar{K} = Ka_c^2/\gamma mc^2$, X has been normalized to a_c, the channel half width, and s is the (dimensional) path length through the crystal. Now, by defining $\tilde{X} = X + \epsilon/K$, the equation of motion is the same as in Ref. [1] with the accelerating gradient zero. The result for the Green's function quoted there, Eq. (11), can be immediately taken over by replacing η, ζ with $\eta - \eta_0$, $\zeta - \zeta_0$, respectively. This accommodates an initial distribution $f(\eta_0, \zeta_0)$ at $s = 0$. The mapping between $(\tilde{X}, \theta) \rightarrow (\eta, \zeta)$ is given by the solution of a harmonic oscillator equation:

$$\eta = \tilde{X} \sin \omega s + \frac{\theta}{a_c\omega} \cos \omega s \tag{3a}$$

$$\zeta = \tilde{X} \cos \omega s - \frac{\theta}{a_c\omega} \sin \omega s, \tag{3b}$$

where $\omega = 1/a_c\sqrt{K}$ is the betatron frequency in the channel well.

We remark that we have recently considered the case of non-constant curvature. In this case, the general method used in Ref. [1] can be applied but the result cannot be obtained in so simple a way. We will report on the non-constant curvature bending in a subsequent paper.

Dechanneling Flux

We have considered two initial distributions at $s = 0$. Both are uniform in X. The distribution in angle has been taken to be Gaussian in one case and uniform in the other. Spatial uniformity is a very good approximation because the beam width is much larger than the separation between crystallographic planes. Gaussian distributions are conventional for the beam divergence while a uniform distribution makes more sense when making comparisons with some other theories. In both cases, the initial distribution is convolved with the Green's function. To calculate the flux, the resulting distribution is integrated over the phase area occupied by the channel. In general, one is then confronted with a four-dimensional integral to be evaluated numerically. In the case of both of the initial distributions, two integrals can be expressed in terms of known functions. The remaining double integral is done numerically. In the Gaussian case, the channelled flux is:

$$\begin{aligned}
\mathcal{J} &= -\frac{1}{4\pi}\left(\frac{\Delta}{2b}\right)^{1/2} \int_{-\infty}^{\infty} d\theta \exp\left(\theta - \frac{\bar{\theta}}{\theta_{\mathrm{rms}}}\right)^2 \\
&\cdot \left\{ \int_{\eta_2}^{\eta_1} d\eta' \exp\left(-\eta'^2\right) \left[i^1 \operatorname{erfc}\left(\zeta^1_{-1+\epsilon}(-1+\epsilon)\right)\right.\right. \\
&\left. - i^1 \operatorname{erfc}\left(\zeta^1_{-1+\epsilon}(1+\epsilon)\right)\right] \\
&- \int_{\eta_2}^{\eta_1} d\eta' \exp\left(-\eta'^2\right) \left[i^1 \operatorname{erfc}\left(\zeta^2_{-\psi_n}(-1+\epsilon)\right)\right. \\
&\left. - i^1 \operatorname{erfc}\left(\zeta^2_{-\psi_n}(1+\epsilon)\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&- \int_{\eta_1}^{\eta_4} d\eta' \exp\left(-\eta'^2\right) \left[i^1 \operatorname{erfc}\left(\zeta^1_{1+\epsilon}(-1+\epsilon)\right)\right. \\
&\left. - i^1 \operatorname{erfc}\left(\zeta^1_{1+\epsilon}(1+\epsilon)\right)\right] \\
&+ \int_{\eta_3}^{\eta_4} d\eta' \exp\left(-\eta'^2\right) \left[i^1 \operatorname{erfc}\left(\zeta^2_{\psi_n}(-1+\epsilon)\right)\right. \\
&\left.\left. - i^1 \operatorname{erfc}\left(\zeta^2_{\psi_n}(1+\epsilon)\right)\right]\right\}, \qquad (4)
\end{aligned}
$$

where the rms angular width is θ_{rms}, the beam and channel can be misaligned by the angle $\bar{\theta}$ and $\psi_n = \psi_c/a_c\omega$. The quantities a, b, h, and Δ are defined in Ref. [1]. The limits of integration of the inner integral are:

$$
\eta_{1,2} = \left[(\pm 1 + \epsilon)\sin\omega s - \psi_n \cos\omega s - \frac{\theta_{\mathrm{rms}}\theta}{a_c\omega}\right] \Big/ \sqrt{2b} \quad (5a)
$$

$$
\eta_{3,4} = \left[(\mp 1 + \epsilon)\sin\omega s - \psi_n \cos\omega s - \frac{\theta_{\mathrm{rms}}\theta}{a_c\omega}\right] \Big/ \sqrt{2b}. \quad (5b)
$$

Where, i^1 erfc is the first iterated integral of the complementary error function and $\zeta^{1,2}_\alpha(\beta)$ is a function of its argument and θ and η' as defined:

$$
\begin{aligned}
\zeta^1_\alpha(\beta) &= \left(\frac{b}{2\Delta}\right)^{1/2} \left\{ \frac{1}{\cos\omega s}\left[\alpha - \left(\sin\omega s + \frac{h}{b}\cos\omega s\right)\sqrt{2b}\eta' \right.\right. \\
&\left.\left. - \frac{\theta \sin\omega s}{a_c\omega}\right] - \beta \right\} \qquad (6a)
\end{aligned}
$$

$$
\begin{aligned}
\zeta^2_\alpha(\beta) &= \left(\frac{b}{2\Delta}\right)^{1/2} \left\{ \frac{1}{\sin\omega s}\left[-\alpha + \left(\cos\omega s + \frac{h}{b}\sin\omega s\right)\sqrt{2b}\eta' \right.\right. \\
&\left.\left. + \frac{\theta \cos\omega s}{a_c\omega}\right] - \beta \right\}. \qquad (6b)
\end{aligned}
$$

The integrals are done numerically using a two-dimensional Romberg quadrature routine constructed from the one-dimensional algorithms of Ref. [6].

Some of our results, for the parameters of the experiment in Ref. [7], are shown in Fig. 1. Here is shown the bending dechannelled fraction as a function of particle momentum. (The bending radius was fixed in the experiment. Our results are shown as the solid line. Experimental points at room temperature (X) and 128°K (Δ) are shown. The dechanneling is 40%–50% of that experimentally observed.

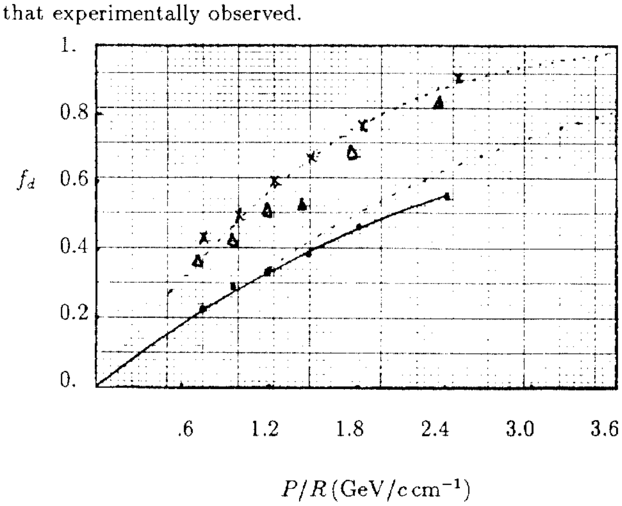

Figure 1: Dechanneled fraction: (X)293°K, (Δ)178°K, ..., Theory, Ref. 8, Fokker-Planck.

There are several possibilities for the difference. The theory assumes that the bending is constant. While this is so in the experiment for the central, bent portion of the crystal, there is a flat segment of crystal before the bend. We have modelled this by taking the initial distribution to be uniform of width equal to the critical angle. To further examine this approximation, we have recently extended the theory to include non-constant (including piecewise constant) curvature. As a consequence of the bending mechanism, the crystal was bent perpendicular to the direction of the beam as well as along it. How this two-dimensional effect manifests itself in the context of a one-dimensional model is not clear to us. This is complicated by the observation that the experimental data appear to agree rather well with one-dimensional statistical equilibrium theory[8] (shown dashed). The relationship between these theories and the Fokker-Planck treatment in the transverse phase space is a subject of current research. We have also calculated the normal dechanneling in the case of 150 GeV protons shown in detail in Ref. [7]. Our dechannelling length of about 6.5 cm is longer than that observed, about 1.5 cm, but quite consistent with the empirical scaling of Ref. [4].

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