Multipolar Correctors in the Regular Cells of the LHC

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Abstract The strong systematic multipolar components which are present in the magnetic field of superconducting dipoles may dramatically deteriorate the dynamical behaviour of the particles circulating in the Large Hadron Collider (LHC), especially at injection energy.

Here we review two possible methods for the compensation of these spurious systematic fields: one in which correcting windings are placed in the dipole gap, and the other in which lumped multipoles are located near the main quadrupoles as well as in the middle of each half-cell (Neuffer approach).

We will compare their performances and demonstrate that both of them improve the stability of the particle motion in the LHC. Furthermore, we will show that different strategies can be followed to set the strengths of the lumped multipolar correctors, all leading to similar, satisfactory results.

Practical considerations have then determined the choice of the lumped scheme for the compensation of the systematic multipolar errors in the LHC.

The Systematic Errors

The systematic components of the spurious multipolar fields in superconducting dipoles are mainly due to persistent currents in the superconductor, iron saturation, and coil deformation under the effect of electromagnetic forces. The relative magnetic field is defined by

$$B_y + iB_x = B_o \sum_{i=1}^{\infty}(b_n + ia_n)\left(\frac{x+iy}{R_r}\right)^{n-1}, \qquad (1)$$

where
B_o is the dipole field in the vertical direction,
R_r is the reference radius,
b_n and a_n are the normal and skew multipolar coefficients,
$2 \times n$ is the number of poles.

Two sets of expected values of systematic multipolar coefficients have been worked out by the CERN LHC Magnet Team over the last few years: the first set was reported in Table 5.4 of Ref. [1], and an extensive study on the effects of such errors and their compensation both at injection and at collision energy is included in Ref. [2]; the second set, rather more pessimistic about the higher order components, has been made available just recently[3], and here we will evaluate the performances of the machine under these new, more stringent, conditions.

	b_3	b_4	b_5	b_7	b_9
1987 data	−4.05	0.05	0.56	−0.03	0.01
1990 data	−4.05	±0.05	0.56	0.13	0.02

Table 1: Multipolar Coefficients for Systematic Errors (in units of 10^{-4} at $R_r = 1$ cm) at injection ($B_o = 0.56$ T)

Only the normal components of systematic errors are taken into account in Table 1, and the quadrupole component, mostly due to the saturation of the yoke, is neglected because it can be substantially reduced by proper shaping of the iron, and finally corrected by the tuning quadrupoles.

The octupolar coefficient, b_4, changes sign every octant, owing to the two-in-one design of the LHC dipoles.

The systematic multipoles of Table 1 mainly produce a large amplitude- and momentum-dependent tune shift which results in a sensible reduction of both linear and dynamic apertures. Octupole errors mostly induce an amplitude-dependent tune shift[4], whilst decapole errors give a stronger contribution to the momentum-dependent tune shift[2,5]. The higher order multipole errors perturb the beam predominantly at the larger amplitudes, mainly affecting the dynamic aperture, as will be seen in the comparison of the results obtained with the two different sets of systematic errors.

LHC Lattice and Compensation Methods

The LHC lattice used here is that of Ref. [6], retuned at $Q_x = 70.28$, $Q_y = 70.31$. It is made of eight arcs and eight insertions, each of them including two dispersion suppressors and one long straight section. An arc contains 49 regular half-cells with four dipoles each, and a dispersion suppressor contains four pseudo-half-cells with three dipoles each. Therefore there are 1760 dipoles in total, and all of them are 9.54 m long. Sextupoles, powered in two families, are included to correct chromaticity and are placed next to the quadrupoles of the regular cells. The injection optics considered consists of four equal insertions with β^* values fixed at $\beta_x^* = \beta_y^* = 6.5$ m, placed in the even straight sections, one dump-dedicated insertion in straight section No. 3, and three identical insertions with $\beta_x^* = \beta_y^* = 4$ m at the interaction point in the other three odd straight sections.

The main purpose of any correction scheme must be to reduce the tune shift to a sufficiently low value over a sufficiently large range of amplitude and momentum.

Our aim here is to find a satisfactory compensation for sextupolar, octupolar, and decapolar components, and to evaluate the effects of the higher order ones (14-pole and 18-pole), possibly indicating a maximum tolerable value.

Our quality criterion, inspired by the operational experience with the p$\bar{\text{p}}$ Collider at the SPS is that:

$$|\Delta Q_{x,y}| \leq 0.005 \qquad (2)$$

$$\begin{array}{llll} \text{for} & a \leq r_o = b/2 & \text{if} & \Delta p/p = 0 \\ \text{or for} & a \leq r_o' = r_o/\sqrt{2} & \text{if} & \Delta p/p = \pm\delta \end{array}$$

where:
$b = 20$ mm is the radius of the vacuum chamber, and at injection $b/2$ corresponds to the amplitude of a particle at 10σ of a 15π mm mrad emittance beam at a maximum-β location in the regular cell,
δ is the bucket half-height; at injection $\delta = 1.25 \times 10^{-3}$.

Correction by Bore-Tube Windings

Additional multipolar windings placed in the bore of each dipole are the most natural way that one can think of to cancel systematic errors, but not always the easiest from the hardware point of view. In particular, in the case of the LHC superconducting magnets, in the dipole bore tube there is not enough space for all the multipolar coils (6-, 8- and 10-polar) one would like to install to achieve a really "local" error compensation; instead, at most one of them may be foreseen per half-magnet, therefore allowing only a "quasi-local" correction.

Amongst all the possible arrangements achievable with only one kind of corrector per half-dipole, we have selected the one in which sextupolar windings are implemented in every dipole, whilst octupolar and decapolar windings are implemented every other dipole, alternatively, and the strength of each multipolar corrector integrated over a cell equals that of the corresponding systematic error.

The compensation for the sextupolar errors is obviously "more local" than for octupoles and decapoles, and indeed, at larger amplitudes the tune shift starts to increase more rapidly; nevertheless, this simple setting, without any further optimization, allows the dynamical behaviour of the machine to be improved enough to largely satisfy the criterion of Eq. (2), and thus to consider this correction method well acceptable.

Correction by Lumped Multipoles

The significant problems presented by the practical implementation of the bore-tube windings in the LHC two-in-one dipoles pushed us to explore a different correction technique, based on lumped correction elements by which, instead, a "global" compensation can be achieved.

Different dispositions of correction elements can be thought of, and, of course, the larger the number of correctors, the better the compensation. The most efficient arrangement with a small number of elements seems to be that proposed by Neuffer[7], and we have chosen to implement it in the regular cells for the compensation of the sextupolar, octupolar, and decapolar systematic field components of the LHC dipoles: to correct each kind of multipolar error to first order three elements per half-cell are used, two of them are placed at the ends of the half-cell and one in the middle. Actually end correctors of contiguous half-cells can be combined, giving a total of four correctors per cell: two, powered together (except for the sextupoles), next to the cell quadrupoles, and two, also powered together, halfway between them.

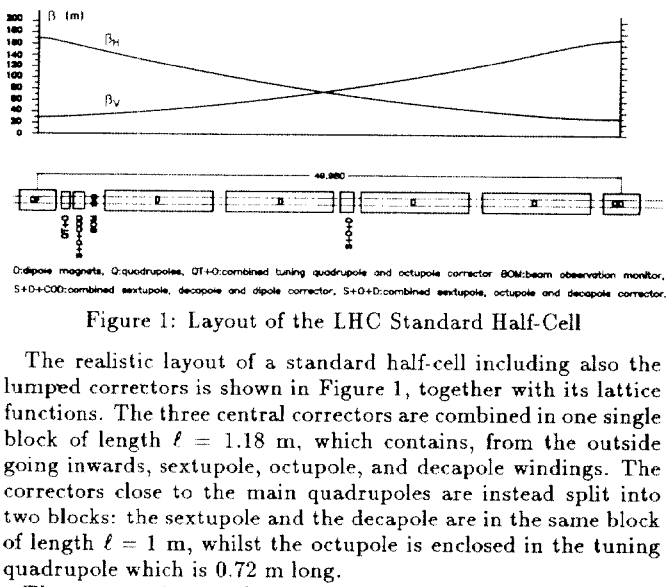

Figure 1: Layout of the LHC Standard Half-Cell

The realistic layout of a standard half-cell including also the lumped correctors is shown in Figure 1, together with its lattice functions. The three central correctors are combined in one single block of length $\ell = 1.18$ m, which contains, from the outside going inwards, sextupole, octupole, and decapole windings. The correctors close to the main quadrupoles are instead split into two blocks: the sextupole and the decapole are in the same block of length $\ell = 1$ m, whilst the octupole is enclosed in the tuning quadrupole which is 0.72 m long.

Three strategies can be employed to set the strengths of the lumped multipoles at their optimum values, two minimizing the amplitude-dependent tune shift, and the third minimizing the momentum-dependent tune shift; in spite of the very different starting points, the three approaches give fairly similar results, producing three sets of correctors whose values are rather close to each other; they all allow the condition of Eq. (2) to be satisfied, and the compensation scheme to be considered adequate.

Figure 2 gives a very impressive representation of the power of the lumped correction method: particles of nominal momentum have been tracked for 100 turns at injection with starting coordinates on a regular mesh in the transverse physical plane X–Y at a focusing quadrupole ($\beta_x = 169$ m, $\beta_y = 30$ m), and the horizontal and vertical tune shifts are plotted against transverse coordinates for (a) the ideal bare LHC (chromaticity corrected), (b) the machine with systematic errors and only chromaticity correction, (c) the machine with systematic errors and lumped multipole correctors. The effect of the systematic errors is evident in the strong distortion of the tune surfaces in Figure 2b; yet, these curves are almost perfectly smoothed out and brought back to the original bare machine shape (Figure 2a) by the action of the lumped correctors (Figure 2c).

The three different optimization methods are shortly described below. We should point out that since the sextupoles next to the main quadrupoles are also chromaticity sextupoles, the first-order compensation of systematic sextupoles is always done at the same time as the correction of chromaticity.

Amplitude-dependent tune shift minimization The first set of correctors, called Simpson Correctors, was obtained[2] by minimizing the amplitude-dependent tune shift by means of a "trial and error" tracking procedure. Particles over a wide range of amplitude were tracked, starting along the diagonal, the horizontal, and the vertical direction in the transverse space X–Y to simulate a round, a flat horizontal, and a flat vertical beam respectively; the optimization was done implementing the systematic error components one by one and tuning their correcting elements: sextupolar errors were corrected with sextupolar and octupolar elements, octupolar errors were corrected with octupolar elements, and decapolar errors were corrected with decapolar correctors; the quadratic sum of the horizontal and vertical tune shifts at half of the vacuum chamber radius along the three directions was minimized, taking care that the behaviour of the tune shifts was monotonic along any direction and that up to 3/4 of the vacuum chamber their values were still below 0.01. On the strengths of the correctors the constraint was applied that the integrated gradients of the horizontal and vertical octupoles (decapoles) were identical, and, according to the Simpson's Rule, they were just half of the central octupole (decapole) gradient. As a first guess the corrector strengths were fixed in such a way that their integrated gradient over one cell was equal and opposite to the integrated gradient of the relative systematic multipolar field. The dependence of tune shift on momentum was not considered, but only checked a posteriori: as will be shown in the next section, this scheme of correction is efficient also for off-momentum particles.

An alternative set of correction multipole strengths, called here Normal Forms Correctors, was calculated using a method which employs normal forms techniques to compute the quasi-invariants of the non-linear motion and to minimize the tune shifts. By this method, which is extensively reviewed in Ref. [8], the constraint on the ratio of the integrated gradients of the central and lateral octupole (decapole) is removed, whilst the constraint on the equality of the lateral elements, which in principle could be released as well, has been retained owing cost considerations. Although the minimization is only done on the lower order coefficients of the tune shifts, this method has proved to give reasonably good compensation for the tune shifts over a large range of amplitudes and momenta.

Momentum-dependent tune shift minimization A completely different approach was used to compute the strengths of the Non-Linear Chromaticity Correctors[5]; the contribution to chromaticity from the systematic sextupole errors is mainly linear with a small quadratic part, the contribution from octupole errors is mainly quadratic, and that from decapole errors is mainly cubic. Then, sextupole correctors, including the central one, have been used to set the linear part of chromaticity to zero, whilst octupoles have been used to cancel its second-order derivative and decapoles to cancel its third-order derivative. Since only two free parameters are necessary for each kind of corrector, the condition that the lateral octupoles (decapoles) had the same strength was retained. This procedure was applied successfully around $\Delta p/p = 0$, but the correctors were found to minimize rather well the linear and non-linear chromaticities also at very large momenta. Furthermore, even if the amplitude dependence of the tunes was not taken into account in the optimization, this scheme was recognized as being very efficient also at large amplitudes.

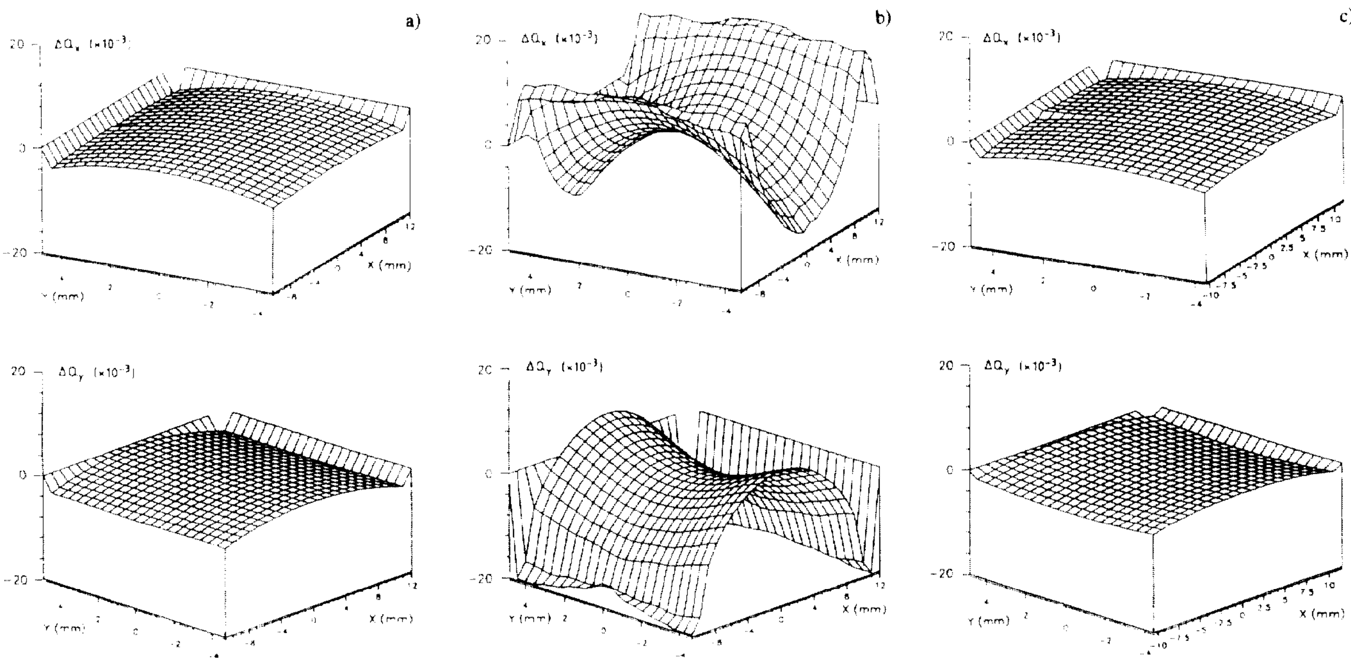

Figure 2: Tune shift vs. X and Y in the LHC: a) bare, b) with systematic errors, and c) with errors and lumped multipole correctors.

Results

On- and off-momentum particle tracking has been used to compare the performances of the local and the global correction methods, the latter with its three sets of lumped correctors. In Table 2 linear apertures based on tune shift and on smear,

$$Sm_I = \frac{\sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(I_i - < I >)^2}}{< I >}, \quad I = I_x, I_y \text{ (C–S inv.)}, \quad N = \text{No. of turns}, \tag{3}$$

and the short-term dynamic aperture are displayed for these four configurations of the LHC, and compared with an ideal bare machine and with the machine with systematic errors and no compensation other than for chromaticity, for the two sets of errors available.

Two Sets of Errors — Different Correction Schemes APERTURE with Different Criteria — Injection — Round Beam Amplitudes in mm ±0.25 mm at a Focusing Quadrupole (β_x = 169 m)						
	LINEAR APERTURE				DYNAMIC AP.	
CRITERION	$\|\Delta Q_{x,y}\| \leq 0.005$		$Sm_{x,y} \leq 0.10$		400 turn over-flow	
$\Delta p/p\|$ ($\times 10^{-3}$)	0	1.25	0	1.25	0	1.25
Ideal Bare Machine	19.4	18.6	13.1	12.5	48.25	44.25
1987 Systematic Errors						
Errors + Chrom. C.	7.67	>	6.89	>	17.50	9.75
+ Coil Correction	18.1	8.45	12.1	9.62	19.50	16.25
+ Normal Forms Corr.	20.0 [1]	7.41	15.6	9.10	20.00	14.75
+ Simpson Correction	19.0 [1]	12.1	13.6	9.62	19.00	15.50
+ Non-Linear Chrom. C	14.3	13.7	17.3	13.5	20.00	16.00
1990 Systematic Errors						
Errors + Chrom. C.	8.06	>	12.7	1.95	15.50	10.00
+ Coil Correction	14.2	6.76	13.4	7.02	19.00	10.00
+ Normal Forms Corr.	10.5	7.15	14.7	7.28	16.00	9.00
+ Simpson Correction	14.0	7.41	13.5	6.89	15.50	11.00
+ Non-Linear Chrom C.	12.6	7.67	15.6	8.19	17.00	10.00

[1] linear ap. threshold is not reached, particles are lost before; dynamic ap. is shown.
>tune-shift or smear values are above threshold for all amplitudes.

Table 2: Linear and Dynamic Apertures.

The change of the octupole error sign from arc to arc in the 1990 data of Table 1 is beneficial, as the average effect of b_4 on the machine becomes smaller. We see that all the correction schemes allow recovery of most of the linear aperture of the ideal bare machine and fully satisfy the criterion of Eq. (2), but they cannot do as much for the dynamic aperture. The detrimental effect of the new b_7 and b_9 is also evident: they strongly lower the power of all the correcting methods, substantially equalizing their performances; both linear and dynamic aperture are deteriorated and the effect is particularly severe for the off-momentum particles.

The correction has been tested also at collision energy[2]: the tracking results show that the method is still applicable and that, owing to the smaller dimensions of the beam, the problems are less critical. The strengths of the correctors at high energy are those determining the specification displayed in Table 3.

	Sextupole		Octupole		Decapole	
	ℓ(m)	$B^{(2)}(Tm^{-2})$	ℓ(m)	$B^{(3)}(Tm^{-3})$	ℓ(m)	$B^{(4)}(Tm^{-4})$
Lateral	1.00	4500	0.72	100000	1.00	32×10^6
Central	1.18	650	1.18	75000	1.18	60×10^6

Table 3: Recommended Gradients for Lumped Correctors

References

[1] G. Brianti and K. Hübner eds., "The Large Hadron Collider in the LEP Tunnel", CERN 87-05, 1987.

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[8] W. Scandale, F. Schmidt and E. Todesco, "Multipolar Correctors for the LHC: an Analytical Approach with Normal Forms", these Proceedings.