MULTIPOLAR CORRECTION FOR LHC: AN ANALYTICAL APPROACH WITH NORMAL FORMS

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Abstract

We briefly review the formalism of symplectic maps for nonlinear betatron oscillations [1] and the theory of non resonant normal forms [2],[3] in two dimensions. We then develop a technique for correcting the nonlinearities of the LHC lattice due to the systematic errors in the superconducting magnets by inserting multipolar correctors near the main quadrupoles and in the middle of each half cell (Neuffer lay-out). Using these analytical tools we minimize the amplitude-dependent tuneshift up to the second order in the non linear invariants; interference terms are taken into account.

A realistic model of the LHC is considered, including insertions, chromaticity correction and asymmetry of the cells. Two distinct cases are analyzed: normal dipoles and normal quadrupoles errors. We study different correction schemes, checking their effectiveness with analytical methods as well as with tracking simulations: the correction is good up to the dynamic aperture and, for some schemes, also for off momentum particles.

One of the main advantages of the strategy outlined is the analytic computation of the global minimum without the need of any first guess. Moreover, the choice of the amplitude as the perturbative parameter allows to order the contributions to the tuneshift in a way that is suitable to the high order effect that dominate the LHC. Correction schemes can be computed also via the numerical tracking approach [4]: we checked for a realistic case that both methods lead to the same solution within a very good approximation.

Normal form estimate of the tuneshift

The betatron oscillations of a single particle can be described using the formalism of symplectic maps also when the magnetic lattice contains large magnetic field imperfections [1]. Each lattice element has multipole components (systematic and random errors, correctors) whose strength is given by the integrated gradients of the field:

$$K_l = \frac{1}{B_0\rho_0}\int_{s_i}^{s_{i+1}} \frac{\partial^l B_y}{\partial x^l}(x,y;s)|_{(0,0;s)}ds \qquad (1.1)$$

All the maps relative to the single elements are composed and truncated at an order N to get the final map over one turn $\mathcal{F}$. The non resonant normal form approach consists in conjugating $\mathcal{F}$ with a map $\mathcal{N}$ which is an amplitude-dependent rotation using a symplectic transformation Φ. Both $\mathcal{N}$ and Φ are power series in the normal form coordinates; contrary to the classical perturbative theory where the small parameters are the gradients of the field, in this approach the perturbative parameters are the nonlinear invariants ρ_1, ρ_2. A theorem [2] guarantees that both the normal form and the conjugating function can be computed to every perturbative order provided that the linear tunes ν_x, ν_y are not resonant.

The tuneshift reads as a power series in the nonlinear invariants: as the coefficients of this serie satisfy some restrictions imposed by the symplecticity conditions, one has the following expression:

$$\begin{aligned}\delta\nu_x &= 2g_2^2\rho_1 + g_1^2\rho_2 + 3g_3^3\rho_1^2 + 2g_2^3\rho_1\rho_2 + g_1^3\rho_2^2 + O(\rho^3)\\ \delta\nu_y &= g_1^2\rho_1 + 2g_0^2\rho_2 + g_2^3\rho_1^2 + 2g_1^3\rho_1\rho_2 + 3g_0^3\rho_2^2 + O(\rho^3)\end{aligned} \qquad (1.2)$$

where g_j^{i+1} are real coefficients; it can be easily computed [5] that the first two tunehsift orders $i = 1,2$ depend on the integrated gradients K_l according to:

$$g_j^2 \propto (K_2)^2, K_3 \qquad j = 0,..,2 \qquad (1.3)$$

$$g_j^3 \propto (K_2)^4, (K_2)^2K_3, K_2K_4, (K_3)^2, K_5 \qquad j = 0,..,3 \qquad (1.4)$$

Due to the choice of the amplitude as small parameter the contributions of the gradients to the tuneshift are ordered in a way which is different from the standard classical approach [6]. It must be pointed out that the amplitude is the natural parameter for the correction problem as one wants to minimize the tuneshift over a certain domain in the amplitudes. Moreover in the LHC case this ordering is more appropriate as the higher order effects of the sextupoles are dominating the tuneshift both on the first and the second order of ρ_1 and ρ_2 [5].

Lattice model

We consider a realistic model of the LHC with a four fold symmetry which includes detuned insertions (β^* = 4.0 m alternating with β^*=6.5 m) and cell asymmetry [7]. The correction of the errors in the dipoles is carried out using four elements for each cell according to the Neuffer scheme [6]: besides the usual chromaticity correctors M_F and M_D placed near the quadrupoles we insert in the middle of each half cell a central corrector M_C (fig. 1). Each element has sextupole, octupole, decapole and in some schemes dodecapole multipoles.

This cell lay-out allows a very effective correction of the first order effect in the gradients using the Simpson rule [6], [8]. Such a rule is not the best choice to compensate the amplitude-dependent tuneshift in the LHC case, since the tuneshift due to the higher order effect of the sextupole is dominant [5].

The chromaticity correction is carried out up to the first order in the momentum error: this gives a linear relation between the gradients K_{2C}, K_{2F} and K_{2D}. We use the central sextupole K_{2C} and the higher order multipoles to minimize the amplitude dependent tuneshift, while K_{2F} and K_{2D} correct chromaticity according to the relation:

$$\begin{cases} K_{2F} = c_{1F} + c_{2F}K_{2C} \\ K_{2D} = c_{1D} + c_{2D}K_{2C} \end{cases} \tag{2.1}$$

where c_{1F}, c_{1D} are proportional to the chromaticity of the machine with the errors, while c_{2F}, c_{2D} are proportional to the chromaticity caused by the central corrector.

The values of the errors both in the dipoles and in the quadrupoles are taken from references [9],[10] and are listed in tab. 1.

Tab. 1 - Normal integrated gradients of the systematic errors.

	Dipoles	Foc. quad.	Def. quad.
K_2	$-2.90\cdot10^{-2}$	-	-
K_3	$1.07\cdot10^{-1}$	-	-
K_4	$4.81\cdot10^{2}$	-	-
K_5	-	$-2.65\cdot10^{4}$	$2.65\cdot10^{4}$
K_6	$-7.73\cdot10^{6}$	-	-
K_7	-	-	-
K_8	$1.44\cdot10^{12}$	-	-
K_9	-	$3.08\cdot10^{11}$	$-3.08\cdot10^{11}$

Correction method

The correction of the amplitude dependent tuneshift is carried out using an order by order method. We start to minimize the first order, fixing the sextupoles and the octupoles, and then we switch to the second order, fixing the decapoles and the dodecapoles (if any). In fact for each order $i = 1, 2$ we have a set of $i+2$ coefficients $g_0^{i+1}, ..., g_{i+1}^{i+1}$ (1.2) depending on the multipole gradients up to the $4i+4$ pole. There are two possibilities: if for a given tuneshift order i we have at least $i+2$ free parameters among the multipole gradients K_l the coefficients can be set to zero by solving the system:

$$\begin{cases} g_{i+1}^{i+1}(K_l) = 0 \\ \dots \\ g_0^{i+1}(K_l) = 0 \end{cases} \tag{3.1}$$

Otherwise, we need to build a norm for the tuneshift $\delta\nu|_i$ due to the order i and minimize it. We used the norm which comes from the sum of the square of the tuneshift on both planes integrated over the sum R of the invariants:

$$t_{2i}(\rho_1, \rho_2; K_l) = (\delta\nu_x|_i)^2 + (\delta\nu_z|_i)^2 \tag{3.2}$$

$$\|g_j^{i+1}\| = \frac{1}{R^{2i}} \int_0^R t_{2i}(\rho_1, R-\rho_1; K_l)d\rho_1 \tag{3.3}$$

If we follow the order by order method $\|g_j^{i+1}\|$ is a polynomial of low order in the gradients K_l and therefore can be minimized analytically. For this reason the normal form method has the advantage of automatically finding the global minimum, while with the tracking methods it is not possible to distinguish a local from a global minimum.

Correction schemes for the dipole errors

We analyzed four different correction schemes for the dipole errors, neglecting the multipoles in the quadrupoles; in all the cases the 'focusing' and 'defocusing' sextupoles are fixed by the chromaticity relation (2.1), so that only the central sextupole gradient K_{2C} is left free. The relative values of the octupoles, decapoles, and eventually dodecapoles vary in the following way:

Scheme **s**) Free parameters for the amplitude dependent tuneshift correction are:

$$K_{2C}, K_{3C}, K_{4C} \tag{4.1}$$

The octupole and the decapole are fixed using partly the Simpson rule:

$$K_{3F} = K_{3D} = \frac{1}{2}K_{3C} \qquad K_{4F} = K_{4D} = \frac{1}{2}K_{4C} \tag{4.2}$$

(we disregard the relation between the sum of the errors and the sum of the correctors). Having only two parameters for the first order and one for the second one we cannot compensate the tuneshift exactly but instead have to minimize it using the norm (3.3).

Scheme **a**) Free parameters for the amplitude dependent tuneshift correction are:

$$K_{2C}, K_{3C}, K_{3F}, K_{4C}, K_{4F} \tag{4.3}$$

'Defocusing' octupoles and decapoles are set to the same values of the 'focusing' ones according to the symmetry condition:

$$K_{3D} = K_{3F} \qquad K_{4D} = K_{4F} \tag{4.4}$$

Having three free parameters for the first order we can compensate it exactly; the second order must be minimized using the norm function as only two parameters are available.

Scheme **b**) Free parameters for the amplitude dependent tuneshift correction are:

$$K_{2C}, K_{3C}, K_{3F}, K_{4C}, K_{4F}, K_{5C}, K_{5F} \tag{4.5}$$

'Defocusing' octupoles, decapoles and dodecapoles are set to the same values of the 'focusing' ones according to the symmetry condition:

$$K_{3D} = K_{3F} \qquad K_{4D} = K_{4F} \qquad K_{5D} = K_{5F} \tag{4.6}$$

As we have inserted dodecapoles in the correctors we can compensate exactly the tuneshift of both orders.

Scheme **c**) Free parameters for the amplitude dependent tuneshift correction are:

$$K_{2C}, K_{3C}, K_{3F}, K_{3D}, K_{4C}, K_{4F}, K_{4D} \tag{4.7}$$

As we break the symmetry condition (4.4) we do not need to insert dodecapoles to compensate exactly the first orders of the tuneshift. In this scheme both orders need to be minimized at the same time [5]: this leads to a very complicated nonlinear system which must be solved by numerical methods.

The effectiveness of the schemes was tested both with normal forms and tracking [11]; the main result is summarized in tab. 2 which gives the region in amplitude where the tuneshift is less than $5 \cdot 10^{-3}$. Amplitudes are normalized to the horizontal beta function in the focusing quadrupoles. We checked also the off momentum case, setting the energy error to half bucket heigth.

Tab. 2 - Width of the linear zone.

On momentum: $\Delta p/p = 0$					
$\delta\nu = .005$	**nc**	**s**	**a**	**b**	**c**
A [mm]	7.2	16.4	17.1	18.0	15.9

Off momentum: $\Delta p/p = \pm 1.25 \cdot 10^{-3}$					
$\delta\nu = .005$	**nc**	**s**	**a**	**b**	**c**
A [mm]	2.6	11.0	10.1	5.1	3.3

One remarks that in the on momentum case all the schemes provide a very good correction which allows to gain a factor 2 in the linear aperture with respect to the bare machine chromaticity corrected **nc**. Schemes **b** and **c** which were computed to compensate exactly also the second tuneshift order do not show a dramatic improvment with respect to **a**. In the off momentum case we gain a factor 4 with schemes **s** and **a**, while less than a factor 2 with **b** and **c**. For this reason both **s** and **a** are preferable as they provide a simpler and better overall correction. It must be pointed out that all the schemes work also at high amplitudes as they have a healing effect on the higher orders; this was checked using normal forms [5]. Moreover it seems that in some cases the correction of the amplitude-dependent tuneshift leads to the same solution that one gets from the minimization of the off momentum tuneshift; an analysis with normal forms should give an analytic explaination to this hypothesis.

Quadrupole errors

Due to the symmetry of the quadrupoles, the nonlinear multipole errors in these elements are 12^{th} and 20^{th} poles (see tab. 1). As in this paper we deal with the amplitude-dependent tuneshift up to the second order in the nonlinear invariants we neglect the effect of the 20^{th} pole, while we consider the contribution of the dodecapole to the coefficients g_j^3, $j = 0, .., 3$ which is linear in the gradient K_5 (see 1.4) and proportional to the third power of the beta function according to the following formula:

$$g_3^3 = \frac{K_5}{1152}\beta_x^3 \qquad g_2^3 = -\frac{K_5}{128}\beta_x^2\beta_y$$
$$g_1^3 = -\frac{K_5}{128}\beta_x\beta_y^2 \qquad g_0^3 = \frac{K_5}{1152}\beta_y^3 \qquad (5.1)$$

In this approximation there are no interfering terms between the dodecapole and the lower order multipoles: for this reason we treat these errors separatly from the dipole ones. The sign of the error is changing from focusing to defocusing elements; nevertheless, as the contribution is weighted by the beta function, one has that the focusing element mainly gives tuneshift on the horizontal plane, and the defocusing on the vertical plane. For this reason there is no overall compensation between the quadrupoles errors and an accurate estimate of their effect is needed.

We first computed the tuneshift due to the dodecapoles in the cells: one sees that while for a round beam it is negligible, in the case of a flat beam the situation is more critic ($\delta\nu \approx 10^{-2}$), requiring a correction. The effect of the errors in the quadrupoles in the insertion is almost five times bigger, due to the high value of the beta function: we conclude that a correction is needed for all the quadrupoles in the lattice. As the best solution is to place corrector coils inside the elements (local correction), the computation of the scheme is trivial and does not require the application of the normal form technique developed in this paper.

Conclusions

We have applied the normal form theory to evaluate the effect of the systematic errors both in the dipoles and in the quadrupoles on the amplitude-dependent tuneshift. We have seen how this analytical approach cas be used to understand which multipoles are more dangerous for the stability of the motion, taking into account high order interfering terms which for the LHC case are very strong.

The correction method outlined, which has been tested succesfully both with the tracking [4] and with the classical perturbation theory approach [5], has some important advantages. The mimimization procedure is completely automatic and not restricted to a certain ratio of emittances as it attacks the coefficients of the tuneshift function. Moreover we tested that the schemes are effective also for the higher orders (and therefore at high amplitudes) and for off momentum particles. A minimization procedure which corrects directly the off momentum tuneshift will be considered in the next future.

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