TRANSITION CROSSING WITH THE SPACE CHARGE – THE JOHNSEN AND UMSTÄTTER EFFECTS

S.A. Bogacz

Accelerator Physics Department, Fermi National Accelerator Laboratory*,

P.O. Box 500, Batavia, IL 60510, USA

A longitudinal phase-space simulation (ESME) of the transition crossing is carried out (including various collective and single particle effects contributing to the longitudinal emittance blow up). The simulation takes into account the longitudinal space-charge force (bunch length oscillation), the transverse space-charge (the Umstätter effect) and finally the dispersion of the momentum compaction factor (the Johnsen effect). As a result of this simulation one can separate relative strengths of the above mechanisms and study their individual effects on the longitudinal phase-space evolution, especially filamentation of the bunch and formation of a "galaxy-like" pattern. Finally, a simple scheme of the γ_t-jump is implemented as a cure

Introduction

The colliding mode operation of the present generation of high energy synchrotrons and the accompanying r.f. manipulations, make considerations of individual bunch area of paramount importance. Thus, a longitudinal emittance blow up in one of a chain of accelerators, while not leading to any immediate reduction in the intensity of the beam in that accelerator, may cause such a reduction of beam quality that later operations are inhibited (resulting in a degradation in performance).

In this paper we employ a longitudinal phase-space tracking code (ESME)[1] as an effective tool to simulate transition crossing in a circular accelerator. One of the obvious advantages of the simulation compared to existing analytic formalisms, e.g. based on the Vlasov equation[2], is that it allows consideration of the collective effects in a self-consistent manner with respect to the changing accelerating conditions. Furthermore this scheme allows to model nonlinearities of the longitudinal beam dynamics, which are usually not tractable analytically.

Included in the simulation is the investigation of the γ_t-jump as a possible cure aimed at eliminating or limiting emittane growth across transition. The machine-dependent parameters, which are considered here, are derived from the proposed Fermilab's Main Injector.

Longitudinal Phase Space Tracking with the Space Charge

Briefly summarized, the tracking procedure used in ESME consists of turn-by-turn iteration of a pair of Hamilton-like difference equations describing synchrotron oscillation in θ–ε phase-space ($0 \le \theta \le 2\pi$ for the whole ring and $\varepsilon = E - E_o$, where E_o is the synchronous particle energy). Knowing the particle distribution in the azimuthal direction, $\rho(\theta)$, and the revolution frequency, ω_o, after each turn, one can construct the longitudinal wake field induced by the coherent space charge force[3]

$$V_i(\theta) = e\omega_o \sum_{n=-\infty}^{\infty} \rho_n\, Z_{s\text{-}c}(n\omega_o) e^{in\theta},$$

where

$$Z_{s\text{-}c}(n\omega_o) = \frac{nZ_o}{2\beta\gamma^2} \left\{1 + 2\ln\frac{b}{a}\right\} \quad (1)$$

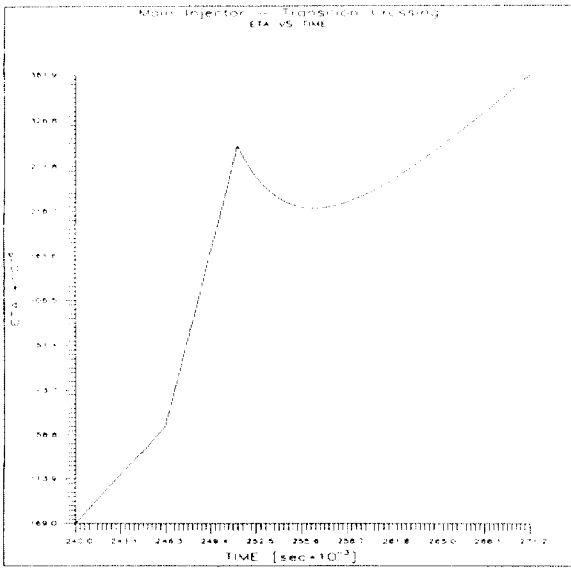
Here, a and b are the radii of the beam and the smooth vacuum pipe, respectively.

As a result of the transverse space charge forces each particle suffers a horizontal betatron tune shift, which is proportional to the particle density, $\rho(\theta)$, at the given longitudinal position θ. This tune shift translates directly into the change of γ_t. Close to the transition, when η goes through zero, even very small corrections to γ_t play dominant role and they govern the longitudinal beam dynamics. One of the features of ESME code is that each particle has its own γ_t, which allows us for straightforward implementation of the Umstätter effect (described above). Similarly, to account for the dispersion of the momentum compaction factor (Johnsen effect), different parts of the bunch (particles with different momentum offset) are allowed to cross transition at different times. Both contributions to the γ_t shift are summarized below[4]

$$\Delta\left(\frac{1}{\gamma_t}\right)^2 = 2hr_p R \frac{1}{\beta^2\gamma^7 a^2}\rho(\theta) - \alpha_1 \frac{\Delta p}{p} - 2j(t)\frac{1}{\gamma^3} \quad (2)$$

The last term in the above equation represents some external γ_t-jump accomplished by firing a pulsed quadrupole magnet. One purposely taylors j(t), so that the transition crossing happens much faster and no significant emittance blowup has time to develop. For the purpose of this simulation the last γ_t manipulation is implemented according to the following η-program .

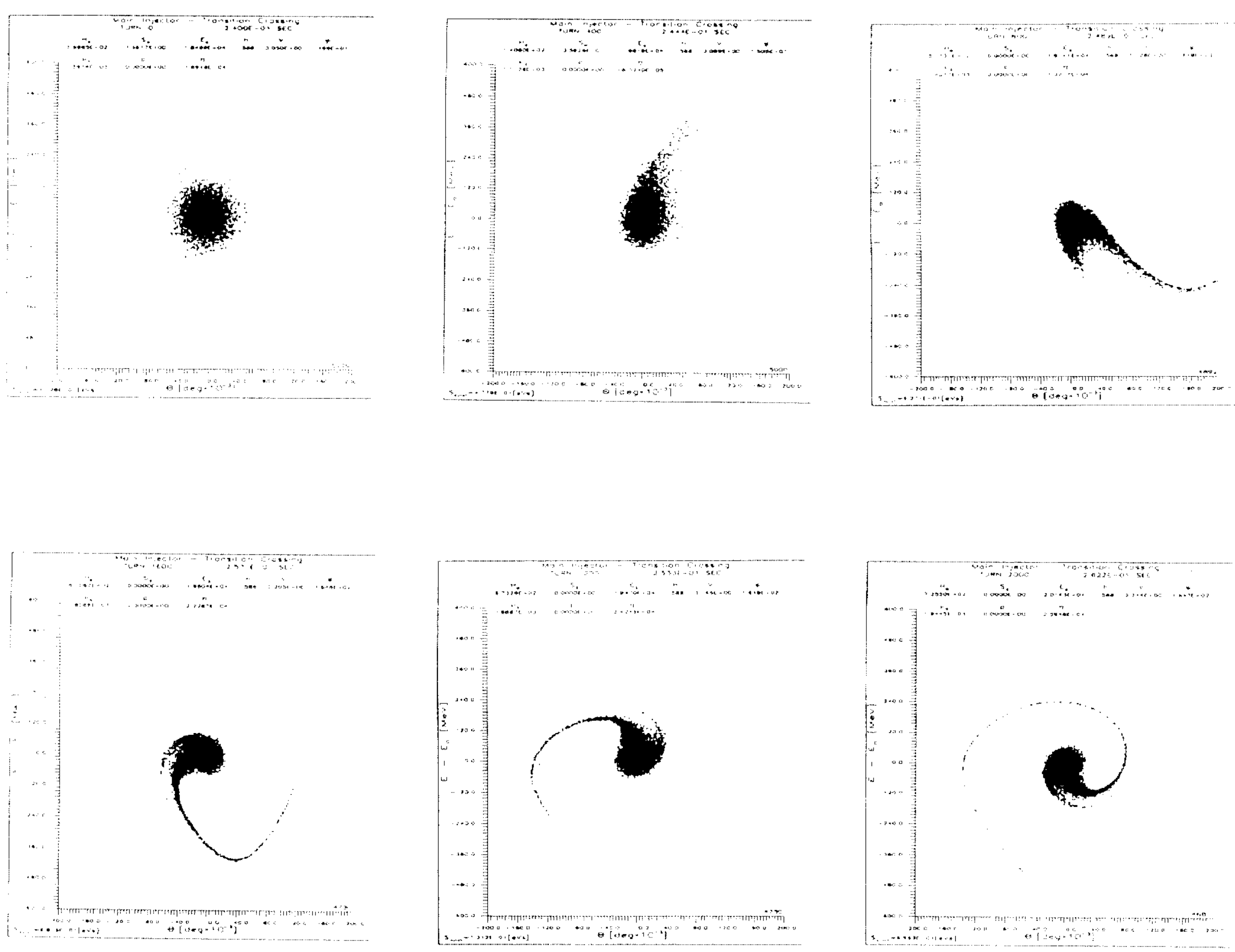
* Operated by the Universities Research Association under contract with the U.S. Department of Energy



ESME Simulation

As a starting point for our simulation a single bucket in $\theta-\varepsilon$ phase-space is populated with 5000 macro-particles according to a bi-Gaussian distribution matched to the bucket so that 95% of the beam is confined within the contour of the longitudinal emittance of 0.4 eV–sec. Each macro-particle is assigned an effective charge to simulate a bunch intensity of 6×10^{10} protons.

The dispersion of the momentum compaction factor, α_1, is assigned a value of 5×10^{-3} and all three features described by Eqs. (1) and (2) are used in the simulation. The simulation is carried out over a symmetric (with respect to the transition) time interval of 2700 turns. The following sequence of the longitudinal phase space snap-shots taken every 400 turns illustrates dilutions effects due to extensive filamentation of the beam at transition.



Here we will present in parallel fashion results of the transition crossing without and with the γ_t-jump labeled by a) and b) respectively. To visualize the position and shape of individual bunches as they evolve in time one can compose a "mountain range" diagram by plotting θ-projections of the bunch density in equal increments of revolution number and then stacking the projections to imitate the time flow. The resulting mountain range plot for both cases are given below.

References

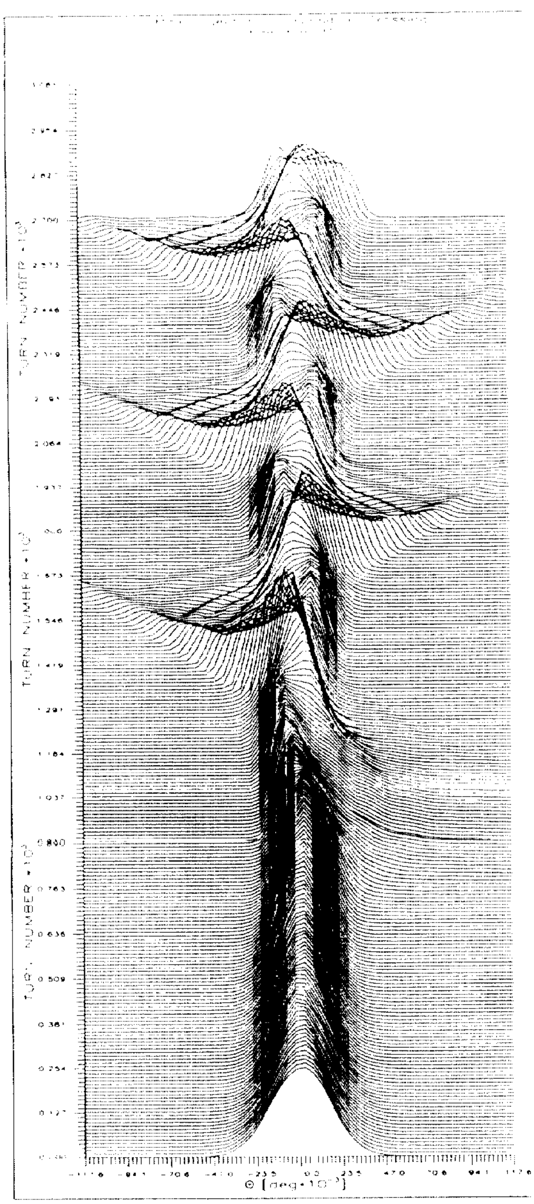
[1] J.A. MacLachlan, FERMILAB TM-1274 (1984)

[2] S. Krinsky and J.M. Wang, Particle Accelerators, **17**, 109 (1985)

[3] J.A. MacLachlan, FERMILAB FN-446 (1987)

[4] A. S ørenssen, Particle Accelerators, **6**, 141 (1975)

a



b