PHASE ANALYSIS OF RESONANT EFFECTS WITH SYNCHROTRON RADIATION IN ELECTRON ACCELERATORS

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Abstract

The radiation of electrons and their quantum fluctuations lead to structure modification of the phase diagrams. Specific features of such diagrams are the absence of separatrix, appearing of limiting stable cycle and quasiseparatrix with slots in region of saddle fixed point, if the sextupole field is sufficient. Qualitative phase analysis makes clear the picture of acceleration process in electron synchrotrons, may be used for calculations of accelerator dynamic aperture, formation of the beams with small phase volume and realization of the slow extraction process.

Introduction

The phase plane method consists in constructing of the phase diagrams with shifts from resonance. Phase diagrams for cyclic electron accelerators differ from the similar diagrams for proton machines. The reason of this is synchrotron radiation, leading to increasing or decreasing of betatron oscillations, depending from the main ring magnet structure choice. For illustration the shortcut Bogolubov-Crylov equations, taking account of synchrotron radiation and their quantum fluctuations, have been obtained.

1. $3\nu=m$ Resonance with Synchrotron Radiation

In the electron synchrotron the radial betatron oscillations are described by the equation [1]:

$$x''+x(1-n)/R^2= \mathcal{F}_x(x,\psi,w;s) \quad , \qquad (1)$$

$$\mathcal{F}_x(x,\psi,w;s)= -2\psi' w/E_s- \psi w'/E_s-n_1x^2/(2R^3)-$$

$$-x'[\Gamma_1+\Gamma(1-(1-2n)\psi/R)]$$

$X=x+\psi\Delta E/E_s$- radial deviation of the non-equilibrium particle with the energy $E_s+\Delta E$, ψ- the dispersion function of the synchrotron main ring, $\Gamma/R= w/(cE_s)$- radiation energy losses of electron per unit of path, $\Gamma_1=1/E_s\ dE/ds$- electron energy increasing per unit of path because of acceleration, $w(s)=\sum_k e_k\delta(s-s_k)-\overline{\sum_k e_k\delta(s-s_k)}$, e_k-the energy of k-th quantum, radiated by electron in the direction of its moving with coordinate s, upper line denotes average energy over all radiated quanta, $n=-R/H\partial H/\partial x$, $n_1=R^2/H\ (\partial^2H/\partial R^2)$, R- radius of the path curvature of particle with energy E_s in magnetic field H, R_0- average radius of the synchrotron main ring Because of statistical independence of individual acts of quantum radiation

$$\overline{w(s)}= 0 \ , \qquad (2)$$

$$\overline{w(s)w(s')}=55/(24\sqrt{3})\ r_0\Lambda\gamma^5\delta(s-s')/\ R^3$$

$$r_0= e^2/(mc^2), \quad \Lambda= \hbar/(mc).$$

Equation (1) takes into consideration only linear terms with w and Γ. Its solutions near resonance $3\nu= m+\delta$ have the form

$$x= \alpha\varphi+ \alpha^*\varphi^*= \sqrt{\varepsilon\beta}\cos\Phi,$$
$$x'= \alpha\varphi'+ \alpha^*\varphi^*{}'= -\sqrt{\varepsilon/\beta}\,(\alpha\cos\phi + \sin\phi), \qquad (3)$$

$$a=|a|e^{i\vartheta}\ , \quad \phi= \vartheta+ s\nu_x/R_0+ \lambda_s, \quad \varepsilon= 4|a|^2W$$

where α,β - Twiss parameters, $\beta= |\varphi|^2/W$, $\lambda_s=\lambda_0+ \int ds'/\beta+\nu_x s/R$ -Floquet functions phase, $\delta= \nu_x-m/3 +\text{æ}\Delta p/p$ -shift from resonance for particle, having momentum $p+\Delta p$, æ- radial chromaticity of the synchrotron main ring, W- Wronskian.

Complex amplitude is described by following equation

$$da/ds= i\varphi^*/(2W)\ \mathcal{F}_x(x,\psi,w;s) \qquad (4)$$

According to (2), quantum fluctuations give non-zero effects in case if terms, describing them, have quadric forms. These terms are linear in equation (4) and equal to zero at averaging over all radiated quanta.

As it is known, canonical variables are amplitude squared $|a|^2 = \varepsilon/(4W)$ and phase ϑ. Terms with $w(s)$ introduce quadrically in equation for $|a|^2$ and it will allow to take them into consideration. We form combinations

$$a\,da^*/ds \pm a^* da/ds = \begin{cases} d|a|^2/ds \\ -2i|a|^2 d\vartheta/ds \end{cases} \qquad (5)$$

$$= \begin{cases} 1/W\ \mathrm{Im}(a^* \mathcal{F}_x \varphi^*) \\ \quad +1/(2W^2)\ \mathrm{Re}\,(\mathcal{F}_x(s)\varphi^*(s)\int_0^s \mathcal{F}(s')\varphi_x(s')ds') \\ -[1/W\ \mathrm{Re}(a^* \mathcal{F}_x \varphi^*) \\ -1/(2W^2)\ \mathrm{Im}(\mathcal{F}_x(s)\varphi^*(s)\int_0^s \mathcal{F}(s')\varphi_x(s')ds')] \end{cases}$$

Let's transform the second term of first equation into

$$J = \mathrm{Re}(\mathcal{F}_x(s)\varphi^*(s)\int \mathcal{F}_x(s')\varphi_x(s')ds'$$
$$= 1/2\ \frac{d}{ds}\left|\int_0^s \mathcal{F}_x(s')\varphi^*(s')ds'\right|^2 \qquad (6)$$

Leaving the term, describing quantum fluctuations, we get

$$\overline{\langle J\rangle} = 1/2\ \frac{d}{ds} \qquad (7)$$
$$\left|\int_0^s w(s')/E_s(\varphi^{*\prime}\psi - \varphi^*\psi')ds' - w(s)/E_s\varphi^*\psi\right|^2$$

Averaging equation (7) for betatron oscillations phases and, then, statistical averaging over all radiated quanta give finally the following equation

$$\overline{\langle J\rangle} = 55/(48\sqrt{3})r_0\Lambda\gamma^5/E_s^2\ \langle|\varphi^*\psi' - \varphi^*\psi'|^2\rangle \qquad (8)$$

If we make similar operations for another terms in (6), we get shortcut Bogolubov-Crylov equations in canonical variables

$$\begin{cases} d\varepsilon/ds = A - \langle\xi_x\rangle\varepsilon + \varepsilon\, d(\ln\gamma)/ds - \\ \qquad -|F_m|/(2R_0)\ \varepsilon^{3/2}\sin(3\Phi - \arg F_m)\ , \\ d\Phi/ds = \delta/R_0 - |F_m|/R_0\ \varepsilon^{1/2}\cos(3\Phi - \arg F_m), \end{cases}$$

$$A = 55/(24\sqrt{3})\ r_0\Lambda K\gamma^5/(R_0\beta_{max})\ ,$$

$$K = \beta_{max}R_0/W\langle[\psi'|\varphi'|^2 - |\varphi|^{2\prime}\psi\psi' + |\varphi|^2\psi'^2]\rangle,$$

$$\langle\xi_x\rangle = 2/R_0\langle\Gamma[1-(1-2n)\psi/R]\rangle\ , \Gamma = 2/3r_0R_0\gamma^3/R^2$$

Factor K may be reduced to the form $K=(\psi_{max}/R)^2$, if radius of curvature R is constant for all dipole magnets of synchrotron. Term A in the first equation determines radial increasing of the oscillation amplitude because of quantum fluctuations. Second term describes decreasing ($\langle\xi_x\rangle > 0$) or increasing ($\langle\xi_x\rangle < 0$) of radial oscillation amplitude due to classical part of synchrotron radiation. $\langle\xi_x\rangle$ may be regulated with the help of special damping systems. $d(\ln\gamma)/ds$ corresponds to adiabatic damping of oscillations with energy increasing. This term is absent in the case of the beam accumulation and beam slow extraction. We neglect it further.

2. Analysis of the System of Equation Solutions.

If resonances are absent ($F_m=0$), equations (9) describe solution of radial oscillations amplitude changes, because of synchrotron radiation

$$\varepsilon = 1/\langle\xi_x\rangle[A - (A - \langle\xi_x\rangle\varepsilon_0)e^{-\delta\langle\xi_x\rangle(\Phi-\Phi_0)/R_0}] \quad (10)$$

In limit case $\Phi\to\infty$ by $\langle\xi_x\rangle > 0$, solution of equation (10) determines the constant amplitude $\varepsilon_{st} = A/\langle\xi_x\rangle$ because of classical radiation damping and quantum fluctuations. It corresponds to the stable limit cycle on the phase plane ($\sqrt{\varepsilon},\Phi$).

Terms with $|F_m|=0$ are the main ones in (9), if the amplitude of betatron oscillation are small. Then the phase plane diagram represents itself untwisting spiral ($\varepsilon=0$ is the point of "unstable-focus" type) because of quantum fluctuations. If the amplitude is somewhat more, a stable limit cycle may take place ($\langle\xi_x\rangle > 0$). The resonance term is main for the large betatron oscillation amplitude and phase diagram must have characteristic resonant structure. These quality considerations are confirmed by the phase diagramms constructing for the different shifts from resonance.

The accounts are executed for $|F_m|=0.8292\,sm^{-1/2}$, $R_0=34.49$ m, $\delta=0.005$, characteristic for Yerevan electron synchrotron with the limit energy 6 Gev.

If the radiation absent $A=0$, $\langle\xi_x\rangle=0$ and the shift from resonance $\delta=0.005$ is present, equations (9) describe the

resonance $3\nu_x=m$. Fig.1 illustrates a standard situation for proton machines. Saddle points for $\delta\Rightarrow 0$ approaches to the phase plane center along direction $\Phi_{st}=k\pi/3$, k=0,2,4 and the stability region is decreased. If $\delta=0$,the separatrix is degenerated in rays $\Phi=(2k+1)\pi/6$, k=0,1,2,3,4,5.
Taking account of the synchrotron radiation leads to instability of movement over all phase plane. The characteristic triangular structure of the phase diagramms remains, if resonant term is large enough, but the separatrix will destroy and the slots appear in the angles of the characteristic triangular. we call this structure a *quasi-separatrix*. The stability region does not exist now.

Depending of chosen magnet elements, installed in the synchrotron ring, the radiation damping ($\langle\xi_x\rangle>0$) or antidamping of radial oscillations ($\langle\xi_x\rangle<0$) may exist. Mapping points can flow out and flow into inner region of the quasi separatrix.

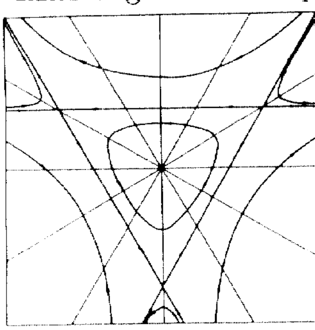
Fig.1

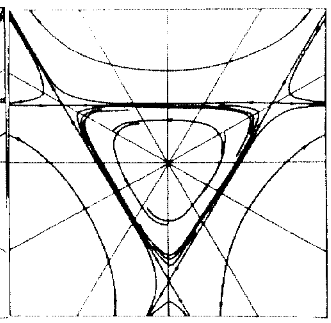
Fig.2

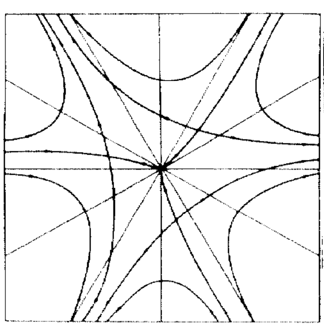
Fig.3

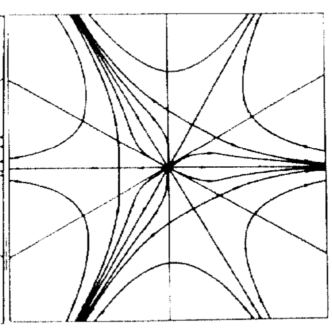
Fig.4

Phase diagrams, corresponding to radiation damping of radial oscillations $\langle\xi_x\rangle<0$, $A\neq 0$ for shifts δ=0.005, 0.0005, 0.0 are presented in figures 2,3,4. The separatrix will be destroyed at the angles of characteristic triangular (saddle points) with coordinates determinate by solutions

$$\varepsilon^3-\varepsilon_{st}\varepsilon^2-(\varepsilon_q^{3/2}-\varepsilon_{cl}^{1/2}\varepsilon)=0 \qquad (11)$$

$$tg(3\Phi-\arg F_m)=(\varepsilon_q^{3/2}-\varepsilon_{cl}^{1/2}\varepsilon)/(\varepsilon_{st}^{1/2}\varepsilon)$$

where $\varepsilon_{st}=(4\delta/|F_m|)^2$, $\varepsilon_q=(2R_0A/|F_m|)^{2/3}$, $\varepsilon_{cl}=(2R_0\langle\xi_x\rangle/|F_m|)^2$,$\sqrt{\varepsilon_{st}}$-coordinates of saddle points in absence of radiation $A=0,\langle\xi_x\rangle=0$, $\sqrt{\varepsilon_q},\sqrt{\varepsilon_{cl}}$- coordinates of saddle points at shift δ=0 for quantum and classical increasing (or decreasing) of radial oscillations respectively.

There are two increasing mechanisms of radial oscillations amplitude: (i)its slow increasing in inner region of quasi-separatrix, because of radiation $\langle\xi_x\rangle<0$ up to some threshold value near by saddle points (ii)then, fast increasing of amplitude due to resonance

If $\langle\xi_x\rangle>0$ and $A\neq 0$, limit stable cycle exists in the inner region of quasi-separatrix (fig.5). In this case three mechanisms have influence on the development of radial oscillations amplitude: (i)slow increasing from zero up to the amplitude, corresponding to the limit stable cycle, (ii)classical radiation damping, leads to slow decreasing of amplitude in the region between limit stable cycle and neighboring quasi-separatrix branches. mapping points flow into inner region through slots. (iii)the resonance stipulates fast increasing of amplitude in external region of quasi separatrix.

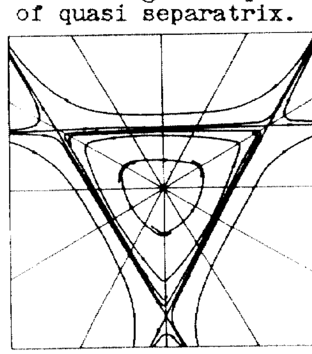
Fig.5

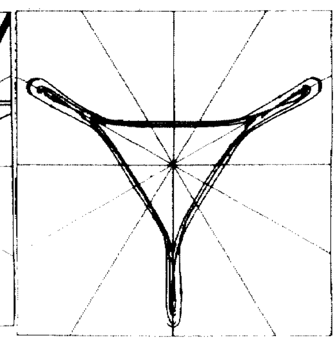
Fig.6

The regular cubic nonlinearity in proton machines stipulates additional time independent solutions (center type points). In electron synchrotron these fixed points can be stable ore unstable foci, depending of sigh $\langle\xi_x\rangle$, fig.6.

Conclusion.

We can conclude ,that mapping points flow out the inner region of quasi-separatrix through slots near fixed points, form a small phase volume of beam outside of this region. It may be used for realization of the electron slow extraction out of the synchrotron.

Phase diagramms for any m-resonance have a similar peculiarity of forming.

Reference.

[1] Kolomensky,A.A.,and Lebedev,A.N.,1966: Theory of Cyclic Accelerators, North Holland.