BEAM BEHAVIOR UNDER A DEFLECTING MODE SYNCHRONIZED WITH AN ACCELERATING MODE IN RF CAVITIES

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Abstract

Under a deflecting mode synchronized with an accelerating mode in rf cavities, electron beams exhibit peculiar behavior in a circular accelerator or storage ring. The electric and magnetic fields of the deflecting mode produce the effects of beam pinching and bunch lengthening. The pinching effect enhances the brightness of the synchrotron radiation. Furthermore, beam loading under the deflecting mode gives rise to a self-pinching effect and affects the Robinson's stability.

Introduction

When a deflecting mode cavity is used together with an accelerating mode cavity under the condition that rf phases of both modes are synchronized with each other in a circular accelerator or storage ring, synchro-betatron interactions dominate over the beam behavior and they can vary the emittances of synchrotron and betatron oscillations. In this paper, the variations of the emittances during one revolution are derived in the Hamiltonian formulation[1] of the synchro-betatron motions.

The closed orbit distortion at the deflecting mode cavity induces significant beam loading effects. We investigate two kinds of these effects, that is, the self-pinching effect and Robinson's stability.

Influence of the Deflecting Mode

The synchro-betatron motions are influenced by the electric and magnetic fields of the deflecting mode. The influence of the deflecting mode is investigated in the Hamiltonian formulation of synchro-betatron motions. The synchrotron and betatron motions are described in the phase spaces (σ, δ) and (x, p_x), respectively. Here σ is the longitudinal displacement from the bunch center in the forward direction; δ is the energy deviation defined as $\delta = (E - E_0)/E_0$, E is the energy of the particle and E_0 is the energy of the reference particle; x is the horizontal displacement from the closed orbit of the particle with the energy deviation δ, and p_x is the canonically conjugate momentum of x. The quntity p_x is defined as

$$p_x = \beta_0^2 \frac{P_x}{P_0} - p_\eta \delta_P, \tag{1}$$

where

P_x: horizontal kinetic momentum;
P_0: kinetic momentum of the reference particle;
β_0: Lorentz factor of the reference particle;
$\delta_P = (P - P_0)/P_0$: momentum deviation
(P : kinetic momentum of the particle);

$$P_\eta = \beta_0^2 (d\eta/ds), \tag{2}$$

with η an energy dispersion function and s a longitudinal coordinate along the orbit of the reference particle.

The Hamiltonian of the synchro-betatron motions under the deflecting mode can be written as[1]

$$H = \frac{p_x^2}{2\beta_0^2} + \frac{\beta_0^2}{2}(\kappa^2 + g_0)x^2 + \frac{\delta^2}{2\beta_0^2}\left(\frac{1}{\gamma_0^2} - \kappa\eta\right)$$

$$- \frac{\delta_P}{2\beta_0^2(1+\delta_P)}(p_x + P_\eta \delta_P)^2$$

$$- \beta_0 \left(\frac{q\hat{E}(X,s)}{kE_0}\right) \cos\left(\frac{ks}{\beta_0} + \phi_s + \phi + \Delta\theta_x\right), \tag{3}$$

where

κ : curvature of the reference orbit;

$$g_0 = (\kappa/B_Z)(\partial B_Z/\partial X)_{X=0}, \tag{4}$$

X : horizontal displacement from the reference orbit;
B_Z: bending magnetic field;
$\hat{E}(X,s)$: longitudinal electric field amplitude;
k : wave number of the oscillating field;
q : electric charge of the particle;
ϕ_s : phase constant;

$$\phi = -(k/\beta_0)\sigma, \tag{5}$$

$$\Delta\theta_x = \frac{k}{\beta_0^2}\left(1 - \frac{\delta}{\beta_0^2\gamma_0^2}\right)(P_\eta x - \eta p_x), \quad \text{and} \tag{6}$$

γ_0: Lorentz factor of the reference particle.

The term $\Delta\theta_x$ in Eq.(3) refers to the phase slip due to the betatron motion.

In order to present the influence of the deflecting mode on the particle motions, the canonical equations are derived from the Hamiltonian described above to evaluate the changes of x and p_x after passing through one of the cavities, Δx and Δp_x, as follows:

$$\Delta x = \frac{p_x}{\beta_0^2}\Delta s - \eta \Delta\delta_P, \tag{7}$$

$$\Delta p_x = \phi_H - P_\eta \Delta\delta_P, \tag{8}$$

where

$$\Delta\delta_P = \frac{\Delta\delta}{\beta_0^2}\left(1 - \frac{\delta}{\beta_0^2\gamma_0^2}\right), \tag{9}$$

$$\Delta\delta = T_E\left(\frac{qV(X)}{E_0}\right)\sin(\phi_s + \theta_E - \phi + \Delta\theta_x), \tag{10}$$

$$\phi_H = \beta_0 T_H\left(\frac{qV_H(X)}{E_0}\right)\cos(\phi_s + \theta_H + \phi - \Delta\theta_x), \tag{11}$$

$$V(x) = \int_0^{\Delta s} \hat{E}(X,s)ds, \quad V_H(x) = Z_0\int_0^{\Delta s} \hat{H}(X,s)ds, \tag{12}$$

$$T_E = \sqrt{\overline{C_E}^2 + \overline{S_E}^2}/V(X), \qquad T_H = \sqrt{\overline{C_H}^2 + \overline{S_H}^2}/V_H(X), \tag{13}$$

$$\theta_E = \tan^{-1}(S_E/C_E), \quad \theta_H = \tan^{-1}(S_H/C_H), \tag{14}$$

$$C_E = \int_0^{\Delta s} \hat{E}(X,s)\cos\frac{ks}{\beta_0}ds, \quad S_E = \int_0^{\Delta s} \hat{E}(X,s)\sin\frac{ks}{\beta_0}ds, \tag{15}$$

$$C_H = Z_0\int_0^{\Delta s} \hat{H}(X,s)\cos\frac{ks}{\beta_0}ds, \quad S_H = Z_0\int_0^{\Delta s} \hat{H}(X,s)\sin\frac{ks}{\beta_0}ds, \tag{16}$$

$\hat{H}(X,s)$: vertical magnetic field amplitude;
Z_0: characteristic impedance in a vacuum; and
Δs: cavity length.

Conveniently, we designate V_H as the magnetic voltage while V is termed the accelerating voltage. The factors $T_E \exp(j\theta_E)$ and $T_H \exp(j\theta_H)$ are complex transit time factors of the electric voltage $V(x)$ and the magnetic voltage $V_H(x)$, respectively.

Furthermore, the canonical equations of the synchrotron motion give the changes of σ and δ after passing around the ring as follows:

$$\Delta\sigma = -\left(\frac{2\pi h\alpha_m^*}{k\beta_0}\right)\delta - \frac{\eta}{\beta_0^2}\left(1 - \frac{\delta}{\beta_0^2\gamma_0^2}\right)\phi_H - \left(\frac{\xi x - \eta y}{\beta_x\beta_0^4\gamma_0^2}\right)\Delta\delta, \tag{17}$$

$$\Delta\delta = T_E\left(\frac{gV(X)}{E_0}\right)\sin(\phi_s-\theta_E+\phi+\Delta\theta_x), \tag{18}$$

where h is a harmonic number,

$$\alpha_m^* = \alpha_m - \frac{1}{\gamma_0^2},\ \zeta = \alpha_x\eta+\beta_x p_\eta, \tag{19}$$

α_m is a momentum compaction factor, and α_x and β_x are Twiss parameters of the betatron motion.

Variations of Emittances

The emittances of the synchrotron and betatron motions are important for obtaining the beam size. The betatron emittance ε_x and the synchrotron emittance ε_σ are defined as

$$\varepsilon_x = \frac{x^2+y_x^2}{\beta_x},\ \varepsilon_\sigma = \frac{\sigma^2+y_\sigma^2}{\beta_\sigma}, \tag{20}$$

where

$$y_x = \alpha_x x+\beta_x p_x,\ y_\sigma = \alpha_\sigma\sigma+\beta_\sigma\delta, \tag{21}$$

and α_σ and β_σ are Twiss parameters of the synchrotron motion.

Using Eqs.(7)−(21), the variations of the emittances ε_x and ε_σ during one revolution are averaged over several betatron and synchrotron oscillation periods to give

$$\left\langle\frac{\Delta\varepsilon_x}{\varepsilon_x}\right\rangle_{\beta,s} = -AJ_0(a)-B(J_0(a)+J_2(a)), \tag{22}$$

$$\left\langle\frac{\Delta\varepsilon_\sigma}{\varepsilon_\sigma}\right\rangle_{\beta,s} = A(J_0(a)+J_2(a))-CJ_2(a), \tag{23}$$

where

$$A = \frac{k\eta}{\beta_0^2}\left(\frac{qV_H}{E_0}\right)\left[T_E\sin(\phi_s+\theta_E)-T_H\sin(\phi_s-\theta_H)\right], \tag{24}$$

$$B = \frac{k^2\eta\alpha_\sigma\varepsilon_\sigma}{2\beta_0^5\gamma_0^2}\left(\frac{qV_H}{E_0}\right)T_H\cos(\phi_s-\theta_H), \tag{25}$$

$$C = \frac{k\eta\alpha_\sigma}{\beta_\sigma\beta_0^3\gamma_0^2}\left(\frac{qV_H}{E_0}\right)\left[\frac{2}{k}T_H\cos(\phi_s+\theta_H) - \frac{\varepsilon_x k(\eta^2+\zeta^2)}{\beta_x\beta_0^6}T_E\cos(\phi_s+\theta_E)\right], \tag{26}$$

$$a = \frac{k}{\beta_0}\sqrt{\varepsilon_\sigma\beta_\sigma}, \tag{27}$$

$J_n(a)$ (n=0,2) is the n-th order Bessel function, and a is the amplitude of the synchrotron oscillation in the rf phase dimension.

When the particle energy is so high that $\gamma_0 \gg 1$, Eqs.(22) and (23) approximately become

$$\left\langle\frac{\Delta\varepsilon_x}{\varepsilon_x}\right\rangle_{\beta,s} = -AJ_0(a), \tag{28}$$

$$\left\langle\frac{\Delta\varepsilon_\sigma}{\varepsilon_\sigma}\right\rangle_{\beta,s} = A(J_0(a)+J_2(a)). \tag{29}$$

For positive A with $a < j_{01} = 2.41$ (j_{01}: the first zero of $J_0(a)$), the betatron emittance decreases and the synchrotron emittance increases.

The electric and magnetic fields have longitudinal distributions differing from each other along the particle orbit. From Eqs.(12)−(16), therefore, the complex transit time factors $T_E\exp(j\theta_E)$ and $T_H\exp(j\theta_H)$ also differ from each other. That is why the quantity A has a non-zero finite value under the deflecting field. Thus we find different longituinal distributions of the electric and magnetic fields of the deflecting mode are essential for varying the synchrotron and betatron emittances.

Enhancement of Brightness

In the high energy region where the radiation damping becomes effective on synchrotron and betatron oscillations, the effect of the deflecting field on the beam is expected to modify the damping times of the synchrotron and betatron oscillations so that the beam size becomes smaller to enhance the brightness of the radiation.

The damping partition numbers of synchrotron and betatron oscillations, J_σ and J_x, can be expressed as

$$J_\sigma = J_{\sigma0}-J_D(J_0(a)+J_2(a)), \tag{30}$$

$$J_x = J_{x0}+J_D J_0(a), \tag{31}$$

where $J_{\sigma0}$ and J_{x0} are damping partition numbers in the absence of the deflecting field, and the terms including J_D are additional terms due to the deflecting mode. The quantity J_D is written

$$J_D = \frac{E_0}{U_0}A, \tag{32}$$

where U_0 is an average synchrotron radiation energy loss per turn.

The area S of the transverse section of the beam is proportional to the product of horizontal and vertical beam sizes. Supposing that the vertical emittance is proportinal to the horizontal emittance, the area S is proportional to the horizontal emittance ε_x at the location where the energy spread little affects the beam size. Where S_0 is S with no deflecting mode, the ratio of S to S_0 is

$$\frac{S}{S_0} = \frac{J_{x0}}{J_{x0}+J_D J_0(a)}. \tag{33}$$

The energy spread δ_S can be written as

$$\frac{\delta_S}{\delta_{S0}} = \frac{a}{a_0} = \left[\frac{J_{\sigma0}}{J_{\sigma0}-J_D(J_0(a)+J_2(a))}\right]^{-1/2}, \tag{34}$$

because δ is inversely proportional to the square root of J_σ. Here δ_{S0} and a_0 are δ_S and a with no deflecting mode, respectively. The synchrotron amplitude a is self-consitently evaluated for a given value of J_D using Eq.(34).

Figure 1 shows the J_D-dependence of S/S_0 and δ_S/δ_{S0} with D=0 and 0.5, where $D = 1 - J_{x0}$ and a_0=0.1 radian. The solid line indicates D=0 and the broken line indicates D=0.5. The area S decreses by a factor of about 3 to 5, and turns to increasing when J_D exceeds 2 or 2.5, because of the growth of the synchrotron oscillation amplitude. The energy spread δ_S gradually increases with J_D, and rapidly increases when J_D exceeds $J_{\sigma0}$. When the energy spread δ_S increases too much, however, the particle runs away from the rf bucket, and the beam size and the energy spread become meaningless. From the above results, it is concluded that the brightness of the radiation is enhanced about 3 and 5 times for D=0 and 0.5, respectively, although the energy spread is 3−5 times enlarged.

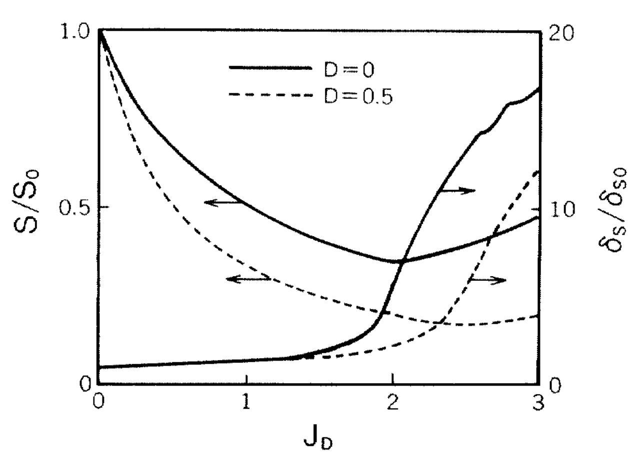

Figure 1. J_D−dependence of S and δ_S.

Beam Behavior under Beam Loading Effect

Self-Pinching Effect

Eqations (28) and (29) can be valid even when a finite closed orbit distortion exists. The quantity $\widetilde{V}_H$ is the complex magnetic voltage including an rf phase shift and written as

$$\widetilde{V}_H = j\left(\frac{IZ_\perp}{1+\beta}\right)X_0\cos\Psi e^{j\Psi}, \tag{35}$$

where I is the beam current; $Z_\perp$ the coupling impedance, β the cavity coupling coefficient to the external circuit; X_0 the horizontal displacement of the bunch center; and Ψ is the tuning angle[2] of the deflecting mode cavity.

The rf phase θ is so set that the accelerating voltage is directly proportional to $\sin\theta$. The beam accepts a maximum decelerating voltage with $\Psi=0$, so that the synchronous phase θ_s is $\Psi-\pi/2$ for positive ηX_0 and $\Psi-\pi/2$ for negative ηX_0. Then the constant A in Eq.(24) becomes

$$A = -\frac{qIZ_\perp}{2E_0(1+\beta)}k\eta X_0 T\left[\cos(2\Psi-\alpha)+\cos\alpha\right], \tag{36}$$

where

$$T = \sqrt{T_E^2+T_H^2-2T_E T_H\cos\Delta\theta}, \tag{37}$$

$$\alpha = \tan^{-1}\left(\frac{T_H\sin\Delta\theta}{T_E-T_H\cos\Delta\theta}\right), \tag{38}$$

and $\Delta\theta=\theta_H-\theta_E$. Usually T_E is greater than T_H and $\Delta\theta$ is positive, so that $0<\alpha<\pi/2$. Then the self-pinching condition of positive A is that $\Psi<\alpha-\pi/2$ for positive ηX_0 and that $\Psi>\alpha-\pi/2$ for negative ηX_0. When $\eta X_0<0$ and $\Psi=\alpha/2$, we get the maximum self-pinching rate A_{max} :

$$A_{max} = k(-\eta X_0)\frac{qIZ_\perp}{2E_0(1+\beta)}T(1+\cos\alpha). \tag{39}$$

Robinson's Stability

The deflecting mode affects the beam to give rise to Robinson damping or anti-damping in a different way from the accelerating mode. Suppose that the bunch center is passing through the rf cavity with horizontally displaced from the symmetry axis by the amount X_c which is given by

$$X_c = X_0+\eta\delta_m\sin(\omega_s t), \tag{40}$$

where δ_m is the energy oscillation amplitude divided by the energy of the reference particle, ω_s is synchrotron angular frequency, and t is the time. Following Wilson's formulation in the accelerating mode[3], we get the complex magnetic voltage defined in Eq.(35)

$$\widetilde{V}_H = jIZ_\perp\widetilde{u}_H/(1+\beta). \tag{41}$$

where

$$\widetilde{u}_H = X_0\cos\Psi e^{j\Psi}+\frac{\eta\delta_m}{2}(a\sin\psi-jb\cos\psi)e^{j\alpha_+}, \tag{42}$$

$$\psi = \omega_s t+\alpha_-, \tag{43}$$

$$a = \cos\Psi_+ +\cos\Psi_-,\quad b=\cos\Psi_+ -\cos\Psi_-, \tag{44}$$

$$\alpha_\pm = (\Psi_+\pm\Psi_-)/2,\quad \tan\Psi_\pm = -(\xi\pm\mu),\quad (|\Psi_\pm|<\pi/2), \tag{45}$$

$$\xi = -\tan\Psi,\quad \mu = 2Q_L\frac{\omega_s}{\omega_0}, \tag{46}$$

ω_0 is the accelerating angular frequency; and Q_L the loaded Q. The Panofsky-Wenzel theorem[4] relates the electric and magnetic voltages as follows

$$\frac{\partial\widetilde{V}_b}{\partial X} = jk\widetilde{V}_H. \tag{47}$$

Then we get the beam-induced voltage for $|X_0|\gg\eta\delta_m$

$$\widetilde{V}_b = k\left(\frac{IZ_\perp}{1+\beta}\right)\left[-X_0^2\cos\Psi e^{j\Psi} + \frac{\eta\delta_m X_0}{2}(b\cos\psi-ja\sin\psi)e^{j(\alpha_+ +\frac{\pi}{2})}\right]. \tag{48}$$

The complex quantity $\widetilde{V}_b$ traces out an ellipse with a semi-major axis a and a semi-minor axis b in the complex plane as illustrated in Fig.2 for positive ηX_0. The symbols δ_{max} and δ_{min} mean the maximum and minimum energy deviations divided by the energy of the reference particle, which are replaced by each other for negative ηX_0. For positive ηX_0, therefore, high energy particles accept a slightly lower accelerating voltage compared with the reference particle. For negative ηX_0, low energy particles accept a slightly lower accelerating voltage. Therefore, the positive ηX_0 contributes to the Robinson damping, while the negative ηX_0 contributes to the Robinson anti-damping.

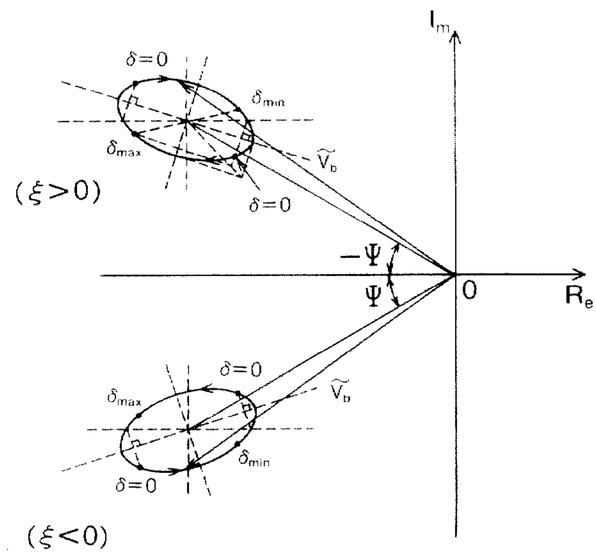

Figure 2. Behavior of the beam induced voltage $\widetilde{V}_b$ for positive ηX_0. The symbols δ_{max} and δ_{min} mean the maximum and minimum energy deviations divided by the energy of the reference particle, which are replaced by each other for negative ηX_0.

Conclusions

Beam behavior was studied under the deflecting mode. The electric and magnetic fields can vary the synchrotron and betatron emittances inversely to each other. Different longitudinal distributions give different transit time factors, which differentiate the effects of the electric and magnetic fields. The fields induce a beam pinching effect enhancing the synchrotron radiation brightness. Furthermore, beam loading under the deflecting mode gives rise to a self-pinching effect and affects the Robinson's stability.

References

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