LONGITUDINAL HEAD-TAIL INSTABILITY IN THE CERN-SPS COLLIDER

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Abstract

The first observation of a single bunch longitudinal head-tail instability in the CERN SPS collider is reported. An evaluation of the growth rate for the dipole mode is presented, which agrees with the experimental results.

Introduction

A slow growing longitudinal instability has been observed in the CERN-SPS collider, along the 26 GeV/c injection energy "flat bottom". The 7 ns long single bunches captured and held by the 100 MHz RF system oscillate in the dipole and quadrupole modes, with growth times of the order of a few seconds (fig. 1). The observations made exclude long range instabilities, like coupled-bunch or Robinson (m=1) types. They rather suggest a single bunch effect excited by short range wake fields. A head-tail instability can exist in the longitudinal plane as well as in the transverse plane provided chromatic effects are important. This is actually the case close to transition energy (γ_{tr} = 23.4).

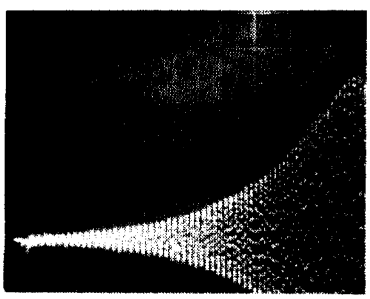
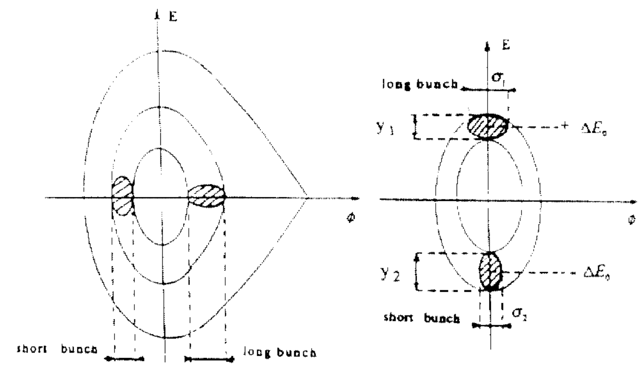


Fig.1 Typical growing dipole oscillation (output of RF-bunch phase detector sampled) 2s/div 10^{11} protons per bunch.

The Longitudinal Head-tail Instability Mechanism

Like the well known transverse head-tail instability, its longitudinal counterpart is a single bunch effect driven by short range wake fields. The regenerative action comes from the oscillation phase shift asymmetry experienced by the particles when they travel from head to tail of the bunch and vice-versa. In the transverse plane the betatron frequency is energy dependent, through the machine chromaticity. Therefore, particles in the upper half of the longitudinal phase plane (travelling from head to tail, say) which have a higher than average energy, oscillate at a different betatron frequency than those in the lower half plane. The resulting phase shift asymmetry gives the necessary damping (or antidamping) effect (see for instance [1]). In the longitudinal phase plane, it is well known that the phase shift of the synchrotron oscillation in the upper half of the longitudinal phase plane is simply π, as it is in the lower half plane. Consequently there should be no instability. However, this is only true to the first order of the energy deviation. As will be shown in the following, when second order energy effects are taken into account, the longitudinal phase space is not exactly symmetric with respect to the phase axis, the "instantaneous synchrotron frequency" being slightly different in the two half planes. This leads again to a phase shift asymmetry, for the longitudinal oscillation, between head to tail and tail to head, and hence to possible instabilities. As a second order effect, the instability is expected to be weak, unless the "longitudinal chromaticity" (variation of synchrotron frequency vs energy) is large. This is necessarily the case at transition crossing when the particle's rotation in phase space changes from clockwise to anticlockwise.

Another way of looking at the dipole longitudinal head-tail instability is due to Hereward [2][3]. Assume the bunch is making coherent dipole oscillations. As the energy of the center of the bunch oscillates, the length σ of the bunch also oscillates at the same frequency. This is because the bucket is not exactly symmetric. Take, for instance, the case $\phi_s \neq 0$ (fig. 2a).



a) $\phi_s \neq 0$ b) second order effect in ΔE_0

Fig.2 Bucket asymmetries: a) energy axis; b) phase axis
Note that bunch area is conserved (Liouville)

The trajectories are not symmetric with respect to the energy axis, hence the bunch is longer when on the right side than when on the left, because the spacing of the trajectories is larger on the right than on the left. These bunch length oscillations are clearly in quadrature with energy oscillations and are not really interesting here. On the contrary, an asymmetry of the bucket with respect to the phase axis (fig. 2 b) will give a bunch length oscillation which is in phase with the energy oscillation.

The instability mechanism can then be seen as follows: a bunch having an energy lower than equilibrium gets shorter and will probably lose more energy in the impedance responsible for the short range wake fields. The result will be an amplification of the original oscillation, the energy loss being in phase with the energy deviation. On the contrary, the bucket asymmetry given by a non zero stable phase or a higher harmonic RF can only make the short range wake fields change slightly the coherent synchrotron frequency.

For a bunch with a deviation ΔE from the equilibrium energy, the bunch length change $\Delta\sigma$ gives an additional energy loss per turn:

$$\delta E = e\frac{dU}{d\sigma}\Delta\sigma \quad (1)$$

U being the average decelerating voltage induced in the offending impedance. The rate of change of energy of the bunch is therefore:

$$\Delta\dot{E} = -\delta\dot{E} = -f_0 e\frac{dU}{d\sigma}\Delta\sigma \quad (2)$$

$$\Delta\dot{E} = -f_0 e\frac{dU}{d\sigma}\frac{\sigma}{E}\chi\,\Delta E \quad (3)$$

where f_0 is the revolution frequency,

and $\chi = (\Delta\sigma/\sigma)/(\Delta E/E)$ characterizes the bucket asymmetry.

Inserting the extra term (3) in the usual synchrotron equations with $\phi_s = 0$ gives an imaginary component of the synchrotron frequency:

$$\Delta\Omega = \frac{1}{2} f_0 e \frac{dU}{d\sigma}\frac{\sigma}{E}\chi \tag{4}$$

and therefore an exponential growth of the oscillation amplitude with an e- folding time $\tau_e = -1/\Delta\Omega$. In this expression dU/dσ is characteristic of the machine impedance and χ of the chromatic properties of the lattice.

Chromatic Effects

To evaluate χ let us consider a bunch oscillating as in fig.2b, between + ΔE_o and $-\Delta E_0$ (oscillation of the bunch center). As phase space behaves like an incompressible fluid, with constant flux measured between two trajectories, the relative variation of bunch length $(\sigma_1 - \sigma_2)/(\sigma_1 + \sigma_2)$ is equal to the relative variation of the speed along the mid-trajectory:

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} = \frac{|\dot\phi_1| - |\dot\phi_2|}{|\dot\phi_1| + |\dot\phi_2|} \tag{5}$$

$|\dot\phi_{1,2}|$ being the velocity along the mid-trajectory measured when $\Delta E = \pm\Delta E_o$.

Expanding the relation:

$$\dot\phi = h(\omega - \omega_0) = h\left(\frac{\beta c}{R} - \omega_0\right) \tag{6}$$

up to second order in energy deviation $\Delta E/E$, we obtain [4]:

$$\dot\phi = -\frac{h\omega_0}{\beta_0^2}\left[\eta\frac{\Delta E}{E} + \frac{1}{\beta_0^2}\left(\alpha_1\alpha_2 - \alpha_1\eta + \frac{\beta_0^2}{2\gamma_0^2}\left(3 - \frac{\eta}{\beta_0^2}\right)\right)\left(\frac{\Delta E}{E}\right)^2\right] \tag{7}$$

where $\omega_0 = 2\pi f_0$, h is the harmonic number, R the orbit radius, $(\eta = \gamma_{tr}^{-2} - \gamma^{-2})$ and α_1 and α_2 are defined, following Johnsen [5], by the relation:

$$R = R_0\left(1 + \alpha_1\left[\frac{\Delta p}{p} + \alpha_2\left(\frac{\Delta p}{p}\right)^2\right]\right) \tag{8}$$

$$(\alpha_1 = \gamma_{tr}^{-2})$$

Inserting equation 7 into 5 gives, for $\Delta E = \pm\Delta E_0$

$$\frac{\sigma_1 - \sigma_2}{\sigma_1 + \sigma_2} = \frac{|\dot\phi_1| - |\dot\phi_2|}{|\dot\phi_1| + |\dot\phi_2|} = \frac{\alpha_1\alpha_2 - \alpha_1\eta + \frac{\beta_0^2}{2\gamma_0^2}\left(3 - \frac{\eta}{\beta_0^2}\right)}{\eta\beta_0^2}\frac{\Delta E_0}{E} \tag{9}$$

which, for relativistic particles $(\beta_0 = 1)$ and reasonable lattices $(|\eta| << 3)$ leads to:

$$\chi = \frac{\alpha_1(\alpha_2 - \eta) + \frac{3}{2}\gamma_0^{-2}}{\eta} \tag{10}$$

The parameter α_2 can be related to the variation of γ_{tr} for a momentum deviation Δp. For a particle having an energy $p = p_0 + \Delta p$ which circulates on a different orbit $R = R_0 + \Delta R$, we calculate γ_{tr} using the relation:

$$\gamma_{tr}^{-2} = \frac{dR/R}{dp/p} = \left(\frac{dR/R}{dp/p_0}\right)\frac{p}{p_o} = \left(1 + \frac{\Delta p}{p_0}\right)\frac{dR}{R}/\frac{dp}{p_0} \tag{11}$$

dR and dp being differentials taken in the vicinity of ΔR and Δp, and p_0 the momentum on the central orbit.

With $\Delta p/p$ replaced by $(\Delta p/p + dp/p)$ in equation (8), one obtains:

$$\frac{dR}{R} = \frac{R_0\left[\alpha_1\left(1 + 2\frac{\Delta p}{p_0}\alpha_2\right)\frac{dp}{p_0}\right]}{R_0\left[1 + \alpha_1\frac{\Delta p}{p_0}\right]} \tag{12}$$

limited to first order in $\Delta p/p$ and dp/p. It follows:

$$\gamma_{tr}^{-2} = \left(1 + \frac{\Delta p}{p_0}\right)\frac{dR}{R}/\frac{dp}{p_0} = \alpha_1\left(1 + 2\alpha_2\frac{\Delta p}{p_0}\right)\left(1 - \alpha_1\frac{\Delta p}{p_0}\right)\left(1 + \frac{\Delta p}{p_0}\right) \tag{13}$$

which, for $\alpha << 1$ gives finally:

$$\frac{\gamma_{tr} - \gamma_{tr_0}}{\gamma_{tr_0}} = -\left(\alpha_2 + \frac{1}{2}\right)\frac{\Delta p}{p_0} \tag{14}$$

For the CERN SPS in collider mode, the variation of γ_{tr} is given by the simulation program MAD [6]. For the horizontal chromaticity setting used during the observations, one obtains from equation (14) $\alpha_2 \approx -0.7$. In this situation χ is a positive quantity at 26 GeV/c $(\gamma_{tr} = 23.4)$, and one expects an instability if $dU/d\sigma$ is negative (equation 4). This is normally the case: short bunches tend to lose more energy than long ones.

Energy loss in RF cavities

For the relatively long bunches considered, the energy loss is essentially due to the accelerating 200 MHz RF cavities. The contributions of resistive wall and broad band impedance (its resistive component) have been calculated and found to be negligible compared to the contribution of the fundamental mode of the cavities.

To evaluate the energy loss, we assume a reasonable cosine squared bunch shape (with only 100 MHz cavities operating) represented by:

$$I(\tau) = \frac{q}{\sigma}\left(1 + \cos 2\pi\frac{\tau}{\sigma}\right) \tag{15}$$

where q is the charge in the bunch.

The total voltage included is given by the convolution integral:

$$V(t)=\frac{R}{Q}\omega_r\int_{\sigma/2}^{\sigma/2} I(\tau)\cos\omega_r(t-\tau)d\tau \qquad (16)$$

where R/Q is the usual geometric parameter of the cavity and ω_r its resonant frequency. After straight forward but tedious calculations, one obtains finally:

$$V(t)=\frac{q}{\sigma}\frac{R}{Q}\left\{\sin\omega_r\left(\frac{1}{1-k^2}\right)\left(t+\frac{\sigma}{2}\right)+\frac{1}{\left(\frac{1}{k}-k\right)}\sin\frac{2\pi t}{\sigma}\right\} \qquad (17)$$

where $k=\omega_r\sigma/2\pi$.

The total energy loss W is given by:

$$W=\int_{-\sigma/2}^{\sigma/2} V(t)I(t)dt \qquad (18)$$

I(t) being an even function, only the first term of (17) contributes to the integral which finally reduces to:

$$W=q^2\frac{R}{Q}\omega_r\frac{1-\cos 2\pi k}{\left[2\pi k\left(1-k^2\right)\right]^2} \qquad (19)$$

Fig. 3 shows the corresponding decelerating voltage $U(\sigma)=W/q$, from which we obtain for 7 ns long bunches and q = 10^{11} protons:

$$\frac{dU}{d\sigma}\sigma \simeq -107\ \text{kV}$$

The instability growth time can then be calculated from equation 4. One obtains $\tau_e \simeq 7$ seconds, very close to the measured values (5 to 6 seconds, fig.1).

Conclusion

Single bunch dipole instabilities observed along the 26 GeV injection flat bottom are clearly of the longitudinal head-tail type. To our knowledge, this is the first observation of such instabilities. In the SPS collider the conditions required for this type of instability to occur are met: high density bunches close to transition energy for a long period of time and high impedance at a relatively low frequency.

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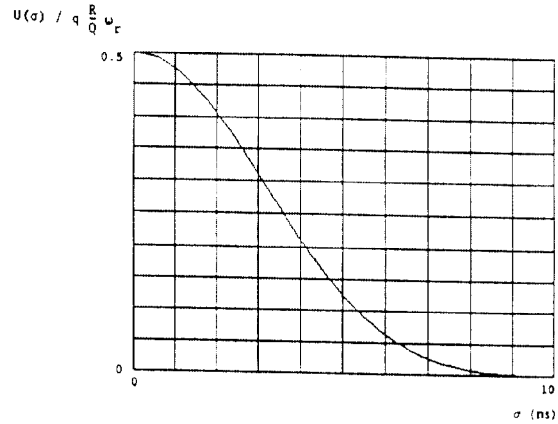

Fig. 3 Normalized decelerating voltage for a cosine squared bunch

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