MULTIBUNCH INSTABILITY INVESTIGATION FOR THE ELETTRA CAVITIES

E. Karantzoulis and A. Wrulich
Sincrotrone Trieste, Padriciano 99 , 34012 Trieste, Italy

Introduction

ELETTRA is a Synchrotron light source under construction in Trieste (Italy) which is optimized for photon energies in the ultraviolet to soft X-ray region with good tunability and high brilliance/flux from insertion devices. The radiation performance is achieved by means of a storage ring for electrons/positrons in the energy range from 1.5 to 2.0 GeV with a full energy linac as injection system to avoid ramping of energy in the storage ring.

The main task of its RF-system is to replace the energy losses due to radiation as well as to provide a large momentum acceptance to confine Touscheck scattered particles. The main parameters of the RF-system are listed in table 1.

Table 1. Main parameters for the storage ring RF-system.

Energy	2.0	1.5	GeV
Revolution frequency	1.1566		MHz
Harmonic number	432		
RF frequency	499.654		MHz
Total radiation losses+)	380	127.6	keV/turn
Peak effective voltage	1.8	1.7	MV
Synchronous phase	169.76	175.69	degree
Number of cavities	6	4	

+) includes radiation losses due to insertion devices

The cavity has been designed by the ELETTRA RF-group [1] and two prototypes have already been built. It is made as a single cell unit with a smooth shape (like a superconducting cavity) to prevent multipactoring and to facilitate the damping of parasitic modes. The tuning of the cavity is achieved by means of an external mechanical tuner acting on the length of the unit. The tuning range needed to compensate the reactive component of beam loading is ±100 kHz, which corresponds to ±0.1 mm change in the cavity length which is still within the mechanically elastic range [2]. Tuning may also be achieved by temperature control since for a temperature change of 1 °C, the fundamental mode is shifted by 9 kHz.

To satisfy all the requirements from the users the storage ring will operate in two different modes, in the single (or few) bunch mode with high current per pulse and in the multibunch mode with a large number of bunches each containing a relatively modest number of particles. For our investigation here we have assumed that all RF-buckets are filled and the machine contains 432 bunches with a total current of 400 mA.

For such a high number of bunches, coupled bunch mode instabilities are likely to occur while single bunch effects are less important. In this report we investigate only coupled bunch instabilities due to higher (parasitic) modes of the cavity. The modes of the prototype cavity have been measured and a comparison has been performed with numerical calculations using OSCAR2D [3] and URMEL-T [4]. Table 2 shows the longitudinal modes and table 3 the transverse modes respectively. It can be seen that the agreement between calculation and measurement is rather good for all modes below the cut off frequencies.

Table 3. Transverse dipole cavity modes.

mode	frequency (MHz)		quality factor		R (KΩ)
	URMEL-T	measured	URMEL-T	measured	URMEL-T
1	748	743.1	55000	44000	1.60
2	751	748.7	57000	40000	6.20
3	1120	1120.4	48000	39500	9.40
4	1231	1221.3	93000	89500	0.04
5	1276	1248.0	67000	37000	4.60
6	1313	1306.6	69000	58000	0.85
7	1599	1561.1	52000	38000	0.03
8	1676	1637.8	84000	32000	7.20
9	1718	1712.6	86000	62000	0.70
10	1755	1720.1	120000	38000	1.70
11	1830	1776.7	65000	36000	4.50
12	1837	1817.7	96000	42000	8.30

Table 2. Longitudinal cavity modes.

MODE	frequency (MHz)		quality factor		R (KΩ)
	OSCAR2D	measured	OSCAR2D	measured	OSCAR2D
1	500	499.9	45000	43000	3840
2	945	944.2	46000	43500	1600
3	1061	1060.5	61000	57000	30
4	1415	1420.7	54000	49500	400
5	1513	1509.3	63500	53500	400
6	1608	1614.0	74000	53000	1600
7	1864	1875.3	55500	42000	400
8	1941	1946.8	83000	64000	200
9	2074	2087.2	61000	24000	40
10	2112	2120.0	89000	30000	2200
11	2323	2314.7	53500	13000	100
12	2396	2422.4	113000	22500	900
13	2441	2494.1	98500	1300	100
14	2517	2579.9	61000	1600	700
15	2693	2703.7	88000	7300	800
16	2772	2809.3	61000	1900	1300
17	2853	2867.6	96000	3200	500
18	2896	2909.5	125000	1800	8
19	2925	2933.7	90000	3300	5
20	3079	3078.5	64500	6600	100

The parasitic mode frequencies are however not identical from cavity to cavity, but are modified by various effects such as manufacturing errors and differences in the temperature distribution of the cooling system as well as temperature drifts. Due to these and other uncertainties, a more accurate analysis of coupled bunch instabilities should take into account that the frequencies are only known within a certain interval. A realistic approach is therefore to consider the tuning range needed to compensate the reactive component of beam loading. Beam loading generates a frequency shift of the fundamental mode up to 82 kHz and accordingly also the frequencies of the parasitic modes are modified.

To get reliable values for the frequency shift of the parasitic modes over the tuning range of the cavity, measurements have been performed on the prototype. The frequency of the fundamental mode has been shifted by means of the mechanical tuning system which modifies the length of the cavity. In this way the fundamental mode has been shifted by -100, -50, +50, +100, kHz from its nominal value and the frequencies of the parasitic modes have been measured. For this measurement the prototype cavity was terminated by two cylindrical tubes of 100 mm diameter and 90 mm length. The cut off frequency of the longitudinal mode was found to be 2290 MHz whereas the cut off frequency for the transverse modes was 1757 MHz. The results are presented in tables 4 and 5.

Table 4. Frequency shifts of the longitudinal modes.

frequency shift (KHz)		MODE	frequency shift (KHz)	
-100	-50	accelerating	+50	+100
-75	-40	2	+40	+81
+46	+26	3	-19	-39
+229	+126	4	-120	-235
-22	-10	5	+15	+32
+402	+212	6	-206	-406
0	0	7	0	0
+230	+126	8	-110	-230
+95	+40	9	-65	-130
+808	+385	10	-445	-895
+185	+85	11	-105	-215
+230	+100	12	-130	-260
+380	+180	13	-220	-450
+158	+120	14	-47	-140
+1175	+560	15	-655	-1315
+139	+95	16	-84	-170
+231	+152	17	-120	-244
+432	+289	18	-219	440
0	0	19	0	0
+481	+327	20	-235	-476

Table 5. Frequency shifts of the transverse (dipole) modes.

frequency shift (KHz)		MODE	frequency shift (KHz)	
-100	-50	accelerating	+50	+100
+288	+143	1	-145	-294
0	0	2	0	0
-75	-37	3	+36	+72
+350	+162	4	-175	-350
+390	+192	5	-202	-410
+2012	+1025	6	-950	-1875
+89	+41	7	-47	-99
+1837	+925	8	-900	-1800
+537	+275	9	-262	-537
+248	+124	10	-124	-248
+352	+175	11	-174	-349
-22	-11	12	+11	+24

It is interesting to note that the corresponding higher order mode frequency shifts are linear with respect to the change on the fundamental but with differing sign and magnitude.

Longitudinal Coupled Bunch Instabilities

To calculate frequency shifts the following expression has been used [5,6,7] :

$$\Delta\Omega_{mn} = j\,\frac{m}{m+1}\,\omega_s\,\frac{\sqrt{\pi}}{2\,M\,h}\left(\frac{R}{\sigma}\right)^3\frac{I}{V_{RF}\cos\varphi_s}\;\frac{\sum\limits_{p=-\infty}^{+\infty}\frac{Z(\omega_p)}{\omega_p}\,\omega_0\,h_m(\omega_p)}{\sum\limits_{p=-\infty}^{+\infty} h_m(\omega_p)}$$

with the growth rate defined as:

$$\frac{1}{\tau} = j\,\Delta\Omega_{mn}$$

where m is the bunch shape mode, M the total number of bunches, n the coupled bunch mode number (n=0,1,2,...M-1), ω_s the synchrotron frequency, I the total current, σ the rms bunch length, R the machine radius, V_{RF} the total peak voltage and φ_s the synchronous phase angle (see table 1). The frequencies of the coupled bunch modes are given by $\omega_p = \omega_o(pM+n+mQ_s)$, with Q_s the synchrotron tune, p an integer and ω_o the revolution frequency. '$h_m(\omega_p)$' represents the envelope of the power spectrum approximated by Hermitian modes and Z (ω_p) is the impedance.

In the numerical evaluation of $\Delta\Omega_{mn}$ the cavity frequencies have been scaled linearly according to

$$F_i' = F_i + \left(\frac{\Delta F\ HOM_i}{\Delta F\ 500\ MHz}\right)\Delta F\ 500\ MHz$$

where i is the order of parasitic mode given in table 2 and the slopes are taken from table 4. Although the measurements have been performed well beyond the longitudinal mode cutoff (2295 MHz) for the evaluation of the growth rates only the modes below cut off have been taken into account. A typical output plot of the code written to perform these calculations is shown in figure 1 for dipole, quadrupole and sextupole modes.

The horizontal axis represents the relative frequency shift on the fundamental mode whereas the vertical axis shows the growth rates.The curves are drawn for three different bunch shape modes and represent the highest growth rates. Modes with lower growth rates but still unstable are not indicated here. Figure 2 shows the corresponding graph for 2 GeV.

The numbers attached to the curves refer to the cavity mode numbers as listed in table 2.The fact that the fundamental frequency of the prototype cavity is 0.05 % different from the actual required frequency does not affect the results; an adjustment of the frequency by scaling did not change them.

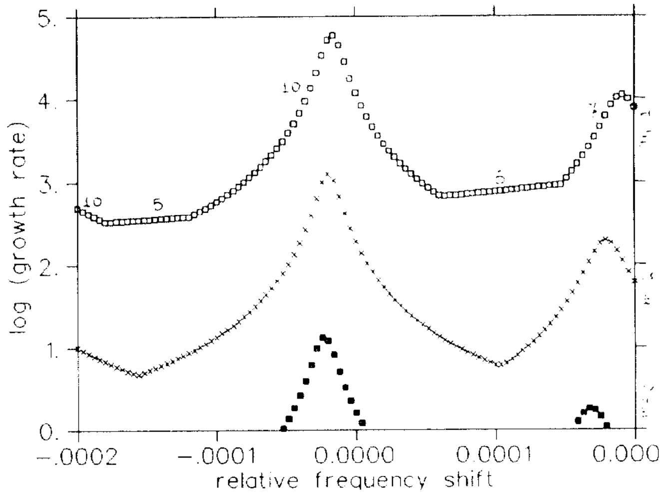

Figure 1. Maximum growth rates for longitudinal coupled bunch instabilities (1.5 GeV).

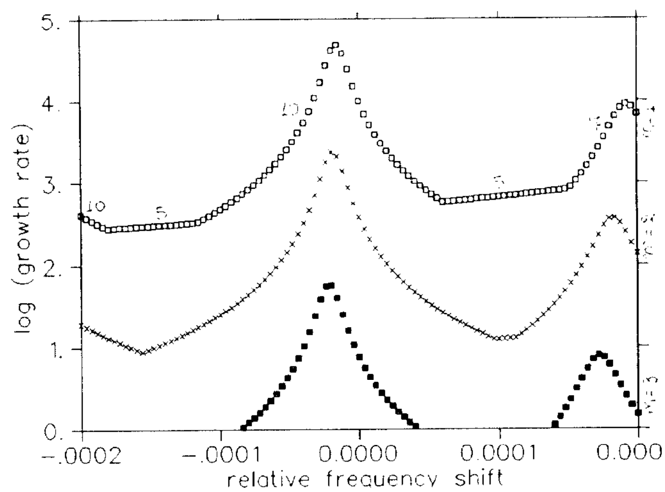

Figure 2. Maximum growth rates for longitudinal coupled bunch instabilities (2.0 GeV).

Transverse coupled bunch mode instabilities

The frequency shifts are given by [8]:

$$\Delta\Omega_{mn} = j\,\frac{1}{m+1}\,\frac{e}{E}\,\frac{c\,\beta_y\,I}{4\sqrt{2}\,M\,\sigma}\;\frac{\sum\limits_{p=-\infty}^{+\infty} Z_\perp(\omega_p)\,h_m(\omega_p-\omega_\xi)}{\sum\limits_{p=-\infty}^{+\infty} h_m(\omega_p-\omega_\xi)}$$

The symbols have the same meaning as for the longitudinal case and β_y is the vertical beta function at the cavity position, $\omega_\xi = \xi Q_y\omega_o/\alpha_c$ the chromatic frequency with the chromaticity defined as $\xi = \Delta Q/Q/\Delta p/p$ and α_c, the momentum compaction. Note that in the transverse case $\omega_p = \omega_o\,(pM + n + mQ_s + Q_y)$.

The transverse coupled bunch instabilities have been calculated for the dipole and quadrupole modes, by shifting the frequencies in a similar manner as in the longitudinal case, where now the maximum frequency shifts have been taken from the measured data presented in table 5 and the nominal frequencies, the quality factors of the modes Q and the shunt impedances have been taken from the measured data given in table 3. Again only modes with frequencies less than the transverse cut off frequency (1757 MHz) have been taken into account. Figure 3 shows the fastest growth rates for compensated chromaticity and for 1.5 GeV.

Figure 4 shows the corresponding graph for 2.0 GeV. Again, the numbers labeled to the curves indicate the cavity mode numbers.

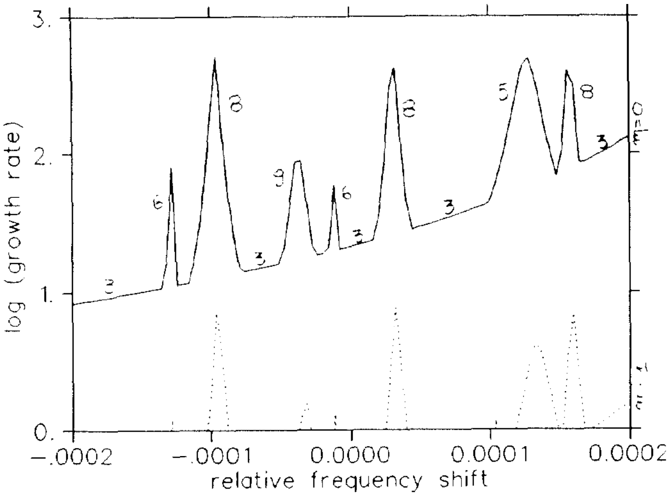

Figure 3. Maximum growth rates for transverse coupled bunch instabilities (1.5 GeV).

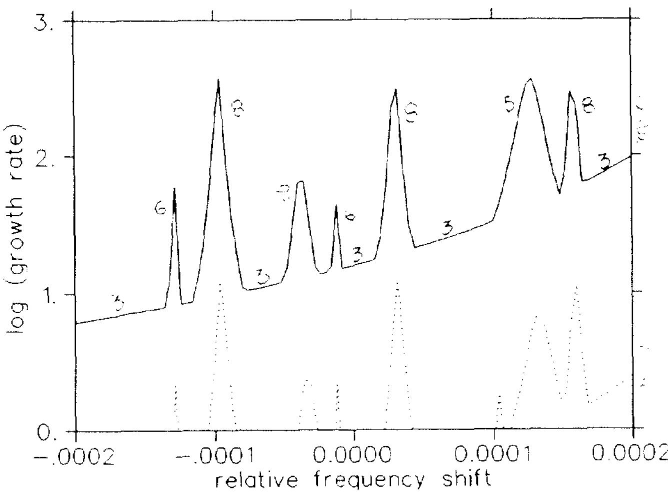

Figure 4. Maximum growth rates for transverse coupled bunch instabilities (2.0 GeV).

Statistical analysis

The analysis above has been performed for one cavity only where the coupled bunch instability thresholds have been derived over a frequency width which corresponds to the tuning range of the cavity. The situation can considerably change when more than one cavity is used (in ELETTRA there are between 4 and 6) and the modes are superimposed in an incoherent way.

Since the differences in resonance frequency from cavity to cavity are *a priori* not known, a reasonable way to superimpose them is to use a statistical analysis. A similar method has been used [9,10] to calculate threshold impedances for the HERA accelerators. In our case we are not interested in impedance thresholds, since we have obtained them from measurements, but on the instability thresholds for coupled bunch instabilities.

The calculation has been performed in the following way: the frequencies of the fundamental modes have been chosen randomly from a certain frequency interval, i.e. ±100 kHz in our case, around the fundamental mode. Taking into account the measured scaling behaviour of the parasitic modes (table 4 and 5), the higher order mode spectra of all cavities have been superimposed and the growth rate for the most dangerous bunch mode has been derived. In this way the evaluation has been performed over the whole frequency interval and then was compared with the radiation damping and/or Landau damping rate. The calculation is repeated many times and the probability of an instability that exceeds the damping rate evaluated.The results of these calculations are presented in tables 6 and 7. (Only the unstable coupled bunch modes with the highest probability are shown.)

Table 6. Instability probability (over 10,000 statistics) for longitudinal coupled bunch oscillations.

% Instability Probability for m=1, with:			
Parasitic cavity mode number	Coupled bunch mode number	4 Cavities (1.5 GeV)	6 Cavities (2 GeV)
2	384	49.4	99.3
4	364	52.2	73.6
5	9	100	100
6	99	80.5	91.4
7	325	100	100
8	387	87.6	96.8
10	105	99	99.9

Table 7. Instability probability (over 10,000 statistics) for transverse coupled bunch oscillations.

% Instability Probability for m=0, with:			
Parasitic cavity mode number	Coupled bunch mode number	4 Cavities (1.5 GeV)	6 Cavities (2 GeV)
3	319	13.5	16.5
5	209	21.3	26.3
8	305	8.1	11.9

In the longitudinal case the growth of couple bunch instabilities is counteracted mainly by radiation damping (~6 ms for 2 GeV) whereas for the transverse case the Landau damping time resulting from the betatron frequency spread is dominant with a value of approximately 5 ms.

Conclusions

From our analysis it is evident that a parasitic mode suppressor scheme is needed. Almost all longitudinal modes are unstable whereas with compensated chromaticity the betatron spread generated by the sextupoles is sufficient to damp almost all transverse modes.

For ELETTRA it has been decided to damp the parasitic modes by a broad band attenuator. Very good preliminary results have been achieved by the ELETTRA RF-group experimenting with a broad band wave guide damper [11].

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References

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