SPACE CHARGE EFFECTS IN LOW ENERGY SYNCHROTRONS *

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We have studied space charge effects in low energy proton synchrotrons using a simulation code developed specifically for supercomputers. The tracking of the beam with a few thousand macro particles is completely self-consistent, thus making it possible to follow the charge distribution as well as the beam size as a function of time. A proper substitution of the kick for the continuous space charge force has been considered.

The simulation is performed for a model lattice with and without half integer resonances. In the lattice with the resonances, the phase space density limit is observed as expected. However, the limiting value is not directly related to the Laslett tune shift of individual particles. If we look at individual particles, there is a bare tune region where the real tune of some particles modified by space charge is within the resonance width. In the lattice without half integer resonances, the rms emittance growth is associated with higher order resonances driven by the beam itself. Only the tail of the beam is affected by the resonance, and the phase space density at the core remains almost unchanged.

Introduction

In high energy colliders, such as the proposed Superconducting Super Collider (SSC), small emittance beams with sufficient intensity must be transported from an ion source through a succession of accelerators without deterioration in the beam quality to achieve the adequate luminosity. One of the limiting factors in achieving a high luminosity is in the lowest energy booster synchrotron. For example, the experimental results in the Fermilab Booster show that the beam emittance blows up almost by a factor of two with the ordinary beam intensity [1]. The emittance growth has mainly two characteristics. One is the almost linear dependence of the final emittance on the beam intensity beyond a certain intensity value. The other is that the emittance growth occurs within a very short time after the beam is injected. From these results, space charge effects are believed to be the most probable source of the emittance growth.

Space charge effects in circular machines were first studied by L.J.Laslett who derived the so-called Laslett tune shift formula [2]. The formula gives the expected tune depression of particles as a function of the beam energy, emittance, and intensity.

$$\Delta\nu = -\frac{r_p n_t}{2\pi\beta^2\gamma^3\epsilon^{un}B_f}, \tag{1}$$

where r_p is the classical proton radius, n_t is the total number of particles in the ring, β and γ are the Lorentz factors, ϵ^{un} is the unnormalized beam emittance, and B_f is the bunching factor. In the case of the Fermilab Booster, the bare tune in the two transverse planes is about $\nu_{x,y} = 6.85$, and the resonance which would be first hit by the loaded tune is the half integer resonance at $\nu_y = 6.50$. Therefore the maximum possible tune shift will be $\Delta\nu_y = -0.35$. By fitting the experimental data using a straight line on the relation between the final emittance and the beam intensity and assuming that the charge distribution is Gaussian, the Laslett formula give the maximum tune shift $\Delta\nu_y = -0.37$. The agreement is remarkable.

However, the model is naive, and there are several questionable points. One is the dynamical behaviour of the beam or the self-consistent picture of the beam evolution. As the detailed observation shows, once the emittance growth starts, the distribution itself changes, and it is a function of the beam intensity as well [3]. Because of this, there are some doubts as to whether the distance between the bare tune and the half integer resonance corresponds the maximum tune shift and whether it indicates the phase space density limit based on the constant distribution. The other question is the effect of resonances. The tune of each particle is spread out between the bare tune and the maximum loaded tune, and this spread is at least ten times wider than the typical width of half integer resonances. In addition, half integer resonances are affected by the nonlinear field of space charge which makes the tune dependent on the amplitude of the particle. This so-called detuning effect prevents particles from continuously diverging in the amplitude once they hit the resonances. Furthermore, we know very little about the effects of higher order resonances.

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In this paper, we will discuss the self-consistent model and the transient beam behaviour of the space charge dominated beam. We have done a multi-particle computer simulation to reproduce the beam evolution as it is in the real machine and to study the phenomena more clearly than in the experiments. *Time* is taken as the independent variable, and this makes it easy to substitute the kick for the continuous space charge force.

Lattice and Initial Beams

As a test lattice, we use the SSC Low Energy Booster (LEB) [4]. This is the first booster synchrotron in the SSC accelerator complex. The beam is accelerated by linear accelerators up to 600MeV and injected into the LEB. The machine lattice has a race-track shape with two dispersion-free long straight sections. The superperiodicity is two. The design value of the normalized initial beam emittance is $0.50\pi.mm.mrad$, and 10^{10} particles are in one rf bucket. There are 72 rf buckets in the ring, and each bucket is filled with the same number of particles.

As the initial charge distribution, three different distributions: Gaussian, Waterbag, and K-V, are assumed in the transverse planes. The longitudinal spatial charge distribution is uniform and the momentum distribution is Gaussian.

In order to find the matched beam conditions, numerical integrations of the envelope equations are performed using the Runge-Kutta method. According to the envelope equations, matching conditions with the space charge force depends only on the rms emittance and the intensity. By using this fact, I.Hofmann has introduced the concept of the equivalent beam [5]. If two beams have the same rms emittance and the same intensity, we call these two beams "equivalent" regardless of the difference in the particle distribution.

As a natural extension of the equivalent beam, we will define the rms tune shift. The rms tune shift is defined by the rms linear part of the space charge force. It is equal to the Laslett tune shift when the distribution is K-V because the space charge force is linear. The equivalent beam has the same rms linear part of the space charge, thus the same rms tune shift.

Definition of the Beam Quantities

RMS emittance and its numerical uncertainty

We will use the rms vertical emittance ϵ_y as one of the parameters to measure the beam growth,

$$\epsilon_y = \sqrt{\left(\frac{\sum y_i^2}{n}\right)\left(\frac{\sum y_i'^2}{n}\right) - \left(\frac{\sum y_i y_i'}{n}\right)^2}, \tag{2}$$

where y_i and y_i' are the vertical position and divergence of the beam, respectively, and n is number of macro particles.

The standard deviation of the rms emittance as a function of the number of macro particles is [6]

$$\frac{\sigma_{\epsilon_y}}{\epsilon_y} = \sqrt{\frac{2}{n}}. \tag{3}$$

We call this the numerical uncertainty of the rms emittance.

Core, middle, and tail widths

As independent measures of the growth of the beam, we will define three kinds of widths based on the beam profile in the real space (x, y). By using several widths of a profile, one can study the evolution of the core, middle, and the tail of the beam separately. We will take the 38% width, which means 38% of the total number of macro particles are enclosed within the width, for looking at the behaviour of the beam core, the 95% width for the beam tail, and the 68% width for the middle which should be similar to the rms emittance when the distribution is Gaussian.

Phase space density and tune shift

The charge distribution is further analyzed by looking at the phase space density. To calculate phase space density, we normalize the coordinates in transverse phase space. The phase space is then divided into 30 to 40 rings of varying amplitudes. The number of particles is counted for each ring to find the phase space density as the ratio of the number of particles per phase space area in each ring. We assume that the phase space density is a function of amplitude only.

Actually what we are interested in is not the phase space density itself, but the tune shift as a function of the amplitude. The tune shift is calculated by summing the number of particles within a certain amplitude and applying locally the Laslett tune formula for the uniform distribution. Obviously, the tune shift near the center is calculated by a smaller number of macro particles resulting in a larger uncertainty. We assume that the tune shift calculated by n macro particles has the uncertainty,

$$\sigma_{\Delta_\nu} = \frac{\Delta_\nu}{\sqrt{n}}. \qquad (4)$$

Beam in the Lattice with Half Integer Resonances

RMS emittance

When the initial distribution is Gaussian, the rms emittance grows if the bare tune is chosen below $\nu_y = 11.66$ as shown in Fig.1. From now on, we call the tune below which the emittance starts to increase "critical tune". It is most likely that, for this case, the half integer resonance is causing the emittance growth. However, it should be noticed that the critical tune is much lower than expected. Since the maximum tune shift is $\Delta\nu_y = -0.33$ for this distribution, some particles in the beam with small amplitudes presumably have already reached the resonance when the bare tune is about $\nu_y = 11.83$. But we cannot see any growth between $\nu_y = 11.83$ and the critical tune.

Core, middle, and tail widths

In addition to the rms emittance, we can examine the behaviour of the width of the core, middle, and tail as shown in Fig.2. The width becomes wider as the tune decreases below $\nu_y = 11.66$ and there is no growth above that tune value. Again, we should notice that the growth starts at a much lower tune value than expected. Furthermore, we can see that the growth is larger for the core than for the tail, an indication that the charge distribution is no longer the same once the growth starts.

Phase space density and tune shift

The larger growth of the core suggests that the density near the core is affected more than the other parts. This can be seen in Fig.3 where the phase space density and the tune shift are shown as a function of the particle amplitude. When the tune is below $\nu_y = 11.63$ and the rms emittance increases by more than a factor of two, the phase space density and the corresponding tune shift are limited to certain values. The maximum tune shift near the center is about equal to or somewhat less than the distance between the bare tune and the half integer resonance.

Dependence on the initial distribution

In order to see the relationship of the critical tune and the maximum tune shift, we have examined the emittance growth for different initial distributions: Waterbag and K-V. Beams are equivalent and have the same rms tune shift $\Delta\nu_y = -0.17$ but the maximum tune shifts are different. The dependence of the critical tune on the choice of the distribution is weak. In all three distributions, the critical tune is near $\nu_y = 11.66$. These results imply that the critical tune is mostly determined by the rms tune shift and not by the Laslett tune shift.

Beam in the Lattice without Half Integer Resonances

RMS emittance

As shown in Fig.4, the rms emittance becomes large near $\nu_y = 11.60$ and comes down to almost the initial level at $\nu_y = 11.55$. The maximum growth near $\nu_y = 11.60$ is not as large as that caused by the half integer resonances.

The results do not change when the sextupoles, which drive $3\nu_x = 35$ and $\nu_x + 2\nu_y = 35$, are excited at even 10 times the realistic value in LEB. The third order resonances, $3\nu_x = 35$ and $\nu_x + 2\nu_y = 35$, are definitely not the source of the emittance growth.

Core, middle, and tail widths

Figure 5, for the squared width of the core, middle, and tail of the beam, shows that only the tail has the growth similar to that of the rms emittance. The growth of the core and middle is relatively small. In contrast to the emittance growth due to the half integer resonances, the growth is not related to the phase space density limit but caused simply by the distortion of the tail.

Phase space density and tune shift

The results are more clearly shown in Fig.6. Only large amplitude particles in the beam are affected at the bare tune region where the rms emittance becomes large, and there is practically no change in small amplitude particles. In addition, from the figures of the phase space density and the tune shift, we notice that the distortion of the distribution occurs mostly at the amplitude which has the loaded tune near $\nu_y = 11.50$. Although there is no half integer resonance at $\nu_y = 11.50$, this tune value still seems to generate an emittance growth.

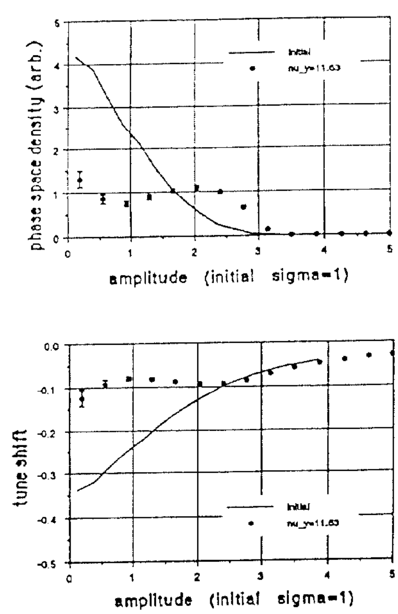

Fig.3: Phase space density (top) and tune shift (bottom) vs. particle amplitudes when bare tune $\nu_y = 11.63$. Solid lines show the initial values and dotted marks show the final ones.

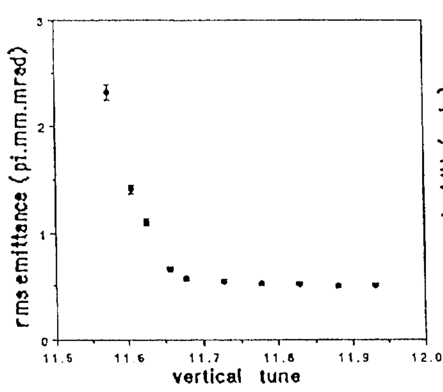

Fig.1: Vertical rms emittance vs. vertical bare tune with half integer resonances.

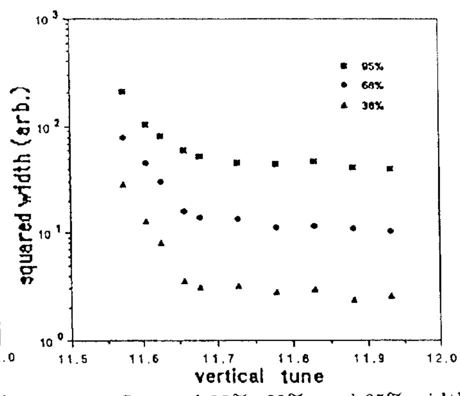

Fig.2: Squared 38%, 68%, and 95% width of the vertical profile vs. vertical bare tune.

Single particle motion

To study the possible resonance structure in phase space, we have tracked 10 test particles and plotted this motion in the normalized phase space. The maximum amplitude is equal to 3σ of the distribution. The horizontal amplitude is zero. We assume that the charge of these particles are infinitesimally small so that they are tracked in the same way as macro particles, but they do not contribute to the calculation of the charge distribution and the space charge force. The tracking of test particles together with macro particles enables us to see a single particle motion in the self-consistent field which is dynamically changing.

Figure 7 shows the Poincaré map of the first 32 turns of these test particles when the bare tune is $\nu_y = 11.60$. Particles with the amplitude near 3σ and the corresponding loaded tune of about $\nu_y = 11.50$ are trapped in islands of the fourth integer resonances, obviously causing the rms emittance growth. Particles with small amplitudes are not distorted and this is consistent the results for the core width.

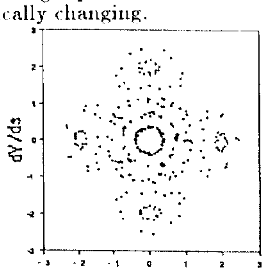

Fig.7: Poincare map of test particles.

Lattice with higher superperiodicity

Since there is no driving force for fourth integer resonances in the external guiding magnets, the resonance found here must be considered to be excited by the beam itself. An important question is whether the driving term created by the beam has the same harmonic structure as the lattice. To check the role of the lattice harmonics, we have constructed a lattice twice as large by connecting the two rings together and studied the effect near $\nu_y = 23.5$. Since this large machine has superperiodicity four, $4\nu_y = 94$ resonance is not a structure resonance. The beam intensity is one-half of the original value so that the maximum tune shift is the same as before. The emittance growth disappears in this big lattice. In fact, a similar effect of the lattice harmonics has been suggested by G.Parzen [7]. However, his simulation with only a few particles is not as convincing as the present study [8].

Summary and Discussions

In the lattice with half integer resonances, the limit of the phase space density and the limit of the tune shift exist. It is obvious that the maximum tune shift is limited by the distance from the bare tune to the resonance. Consequently, as the bare tune is taken closer to the resonance, a larger rms emittance growth is observed.

However, the limit appears only when the bare tune is much closer to the resonance, below $\nu_y = 11.63$, than expected. Since the beam initially has the maximum tune shift $\Delta\nu_y = -0.33$, we expected that the maximum tune shift would be limited below the bare tune $\nu_y = 11.83$. Contrary to this expectation, between $\nu_y = 11.83$ and 11.63, the maximum tune shift is still maintained at the initial value, and some particles have the loaded tune even below the resonance. Therefore, the resonance free region on the tune space cannot be estimated simply from the maximum tune shift according to the Laslett formula.

Interestingly, the resonance free region on the tune space is almost the same for different initial charge distributions, provided that beams are taken to be equivalent. This suggests strongly that we should take the rms tune shift, which depends only on the rms emittance, rather than the tune shift of individual particles, as the measure of the distance between the bare tune and the resonance. The rms tune shift $\Delta\nu_y = -0.17$ agrees much better with the distance between the critical tune and the resonance line than the maximum tune shift.

We conclude that the Laslett tune shift of individual particles is not relevant to the effects of half integer resonances. Instead, the rms tune shift representing the beam as a whole is the one that determines the resonance free region in the tune space. The phase space density limit appears when the bare tune is taken close enough to the resonance such that the distance from the bare tune to the resonance becomes less than the rms tune shift. Because of this, one scheme for reducing the maximum tune shift by injecting the beam uniformly in phase space, the so-called painting scheme, should not be effective if the resulting rms emittance is the same. For the decision of the optimum linac energy when it is used as the injector to the first booster synchrotron, the rms tune shift should be examined rather than the Laslett tune shift.

The rms emittance growth is observed even when there is no half integer resonance in the ring. However, in contrast to the growth caused by the half integer resonance, the growth in this case is not related to the existence of a phase space density limit. The charge distribution within the beam does not change much except in the tail part. The maximum tune shift is not affected by anything in the tune region we have surveyed.

From the observation of the phase space trajectory of a single particle, we have found that the rms emittance growth is generated by the fourth integer resonance. The driving force of the resonance is the space charge force of the beam itself. By choosing the machine superperiodicity properly, we can eliminate the emittance growth. The results for a lattice with the superperiodicity four show that the rms emittance growth is not visible on the fourth integer resonance $4\nu_y = 94$.

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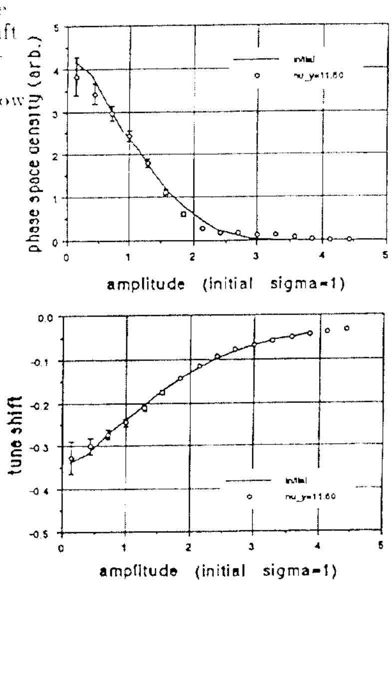
Fig.6 (right): Phase space density (top) and tune shift (bottom) vs. particle amplitudes when bare tune $\nu_y = 11.60$. Solid lines show the initial values and dotted marks show the final ones.



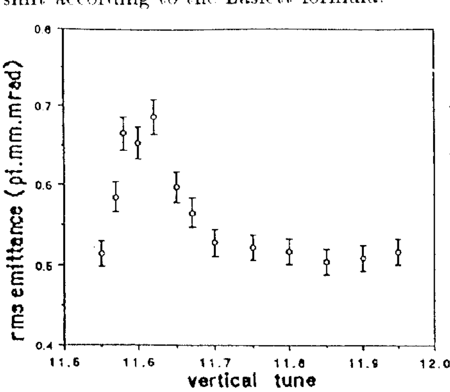


Fig.4: Vertical rms emittance vs. vertical bare tune without half integer resonances.

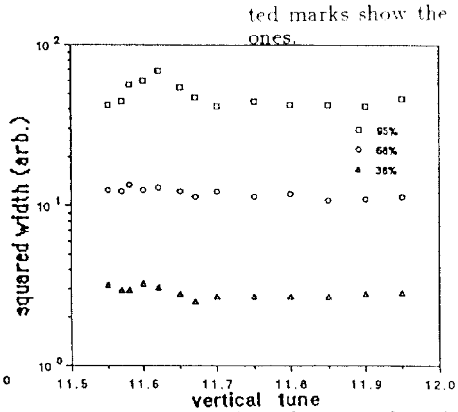


Fig.5: Squared 38%, 68%, and 95% width of the vertical profile vs. vertical bare tune.