BEAM TRACKING OF A SMALL STORAGE RING

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Abstract

A new tracking program by a numerical integration method has been developed for designing small storage rings using superconducting bending magnets.In this program, the Lorentz equations formulated in the curved coordinate are transformed to a simple system of equations without any approximation. As a result, the calculating speed is improved. The accuracy of this program is examined by calculating tunes and chromaticities using a lattice like LEAR at CERN. The results show that this program can simulate both on- and off- momentum particles accurately. And the global error is estimated to be within the tolerance.

This program is applied to a lattice of a small racetrack SR ring with a pair of superconducting bending magnets with a field index and a pair of quadrupole magnets. The ring parameters calculated by a linear optics program are significantly different from the results obtained by this exact formulation. Beam tracking by this program show that the ring has a sufficient dynamic aperture.

Introduction

There are many computer programs for single-particle beam dynamics in circular particle accelerators. Most programs [1][2] make approximations in order to gain a calculating speed. For example, they use the linear trajectory equations of motion, they assume magnetic fields in bending magnets as isomagnetic, a model of fringing fields of bending magnets is simple, and nonlinear elements, such as sextupoles, octupoles, or higher multipoles, are treated in the impulse approximation.

These approximations are adequate for large rings where the betatron oscillation is negligible compared to the bending radius and where bending magnets are isomagnetic. But these approximations are not sufficient for a small storage ring with around ten meter circumference.

The new tracking computer code PROVIDENCE was developed for designing small storage rings. In this program, the equations of motion are precisely integrated with accurate 3-D magnetic fields.

This program is applied to an actual small storage ring, and the betatron tunes are compared with ones obtained by a conventional program. The closed orbit distortion and dynamic aperture of this ring are also calculated by this program.

In this paper, the formulation of the PROVIDENCE, an examination of the program using a LEAR-like lattice, and the calculated results will be discussed following.

Method

The equations of motion are transformed to a simple form so as to fit to the numerical calculation. In the PROVIDENCE these equations are precisely integrated by using adjusted step sizes. The real 3-D magnetic fields of a bending magnet are accurately simulated. These features of this program will be described in the following sections.

Formulation

The Frenet-Seret coordinate system[3] is used. The distance s along the reference orbit is taken as the independent variable of the equations of motion. In this paper, the reference orbit means a path of an on-momentum particle through idealized magnets with no fringing fields. The x and y coordinates measure the horizontal and vertical transverse deviations of the actual orbit from the reference orbit.

In the Frenet-Seret coordinate system, the transverse equations are obtained from the Lorentz equation[3]:

$$x'' + \frac{\ddot{s}}{\dot{s}^2}x' - \frac{1+x/\rho}{\rho} = \frac{e}{m\dot{s}}\left(y'B_s - \left(1+\frac{x}{\rho}\right)B_y\right) \quad (1)$$

$$y'' + \frac{\ddot{s}}{\dot{s}^2}x' = \frac{e}{m\dot{s}}\left(-x'B_s + \left(1+\frac{x}{\rho}\right)B_x\right). \quad (2)$$

From conservation of energy, the following equation holds

$$\dot{s}^2 = \frac{v^2}{x'^2 + y'^2 + (1+x/\rho)^2}, \quad (3)$$

where ρ is the radius of curvature, $\dot{}$ and $'$ denote the differentiation with t and s, respectively, and v is the velocity of a particle.

By using Eq.(3), we finally obtain the following equations:

$$x'' = \frac{C_1}{a}\left(ay'B_s + x'y'B_x - (x'^2+a^2)B_y\right) + \frac{1}{\rho a}(2x'^2 + a^2) \quad (4)$$

$$y'' = \frac{C_1}{a}\left(-ay'B_s - x'y'B_y + (x'^2+a^2)B_x\right) + \frac{1}{\rho a}(2x'y'). \quad (5)$$

where a and C_1 are defined as follows:

$$a \equiv 1 + \frac{x}{\rho} \quad (6)$$

$$C_1 \equiv \frac{e}{mv}\sqrt{x'^2 + y'^2 + (1+\frac{x}{\rho})^2}. \quad (7)$$

Since these equations have simple forms which fit to the numerical integration, the CPU time is significantly reduced. The calculating speed is as fast as a two-dimensional tracking without this transformation.

Algorithm

Equations (4) and (5) are integrated with a fourth-order Runge-Kutta algorithm. A speed of the numerical integration would be slow if we used a constant integration step size which was small in order to obtain the accurate results. Our program has a function of adjusting the variable step size to the local truncation error.

It is known that all explicit high order ($n > 2$) integration methods are not symplectic.[4] Therefore, the tolerance of the truncation error must be selected so as not to accumulate the error to affect the results. In our calculations, the tolerance of the truncation error was set to a value of $1 \times 10^{-12}m$ for x and y, and 1×10^{-12} for x' and y'.

Magnetic Fields

The magnetic fields B_s, B_x, B_y in Eqs.(4) and (5) are calculated by the three-dimensional spline interpolation. The coefficients of the spline functions are in advance calculated from fields at sampled points computed with a three-dimensional magnetic code.

Accuracy of the PROVIDENCE

The accuracy of this program is examined by calculating tunes and chromaticities using a lattice like LEAR at CERN. LEAR($\rho = 4.0m$) is an antiproton ring of which chromaticities were investigated in terms of both calculation and measurement in detail. The experimental results agree well with the calculations using the approach based on the non-linear differential equations.[5] The accuracy of this program was indirectly examined by comparing the differences between values calculated with the equations and with PROVIDENCE using a LEAR-like lattice, since the data of the field distribution of LEAR was not obtained. The differences obtained were as small as 0.02 in the horizontal plane and 0.06 in the vertical plane. The tunes were also in good agreement. These results show that the PROVIDENCE could simulate both on- and off- momentum particles accurately.

Results and Discussion

The program was applied to a small storage ring shown in Fig.1. The energy and the bending radius are $800MeV$ and $0.593m$, respectively. The global error was estimated by tracking a particle 1000 turns with this lattice. The result was that the differences between the theoretical and the calculated values were within $6.0 \times 10^{-13}m$. They are enough small.

Bending Field and Closed Orbit

The bending magnet of this ring consists of a pair of banana-shaped coils with a field index and an iron core. The magnetic field of this magnet was computed by a 3D magnetic field analysis code (TOSCA). The calculated field component B_y on the reference orbit is shown in Fig.2. Figure 3 shows the horizontal displacement of the closed orbit from the reference orbit calculated with the PROVIDENCE. This difference must be taken into consideration in designing good field regions of superconducting bending magnets.

Betatron tunes

The betatron tunes of this ring were calculated by the PROVIDENCE and compared with the results by a linear optics code. The betatron tunes were calculated with the fast fourier transformations by the PROVIDENCE. Table 1 shows the horizontal and vertical tunes with the following various methods: (1) matrix calculation using the isomagnetic bending field (normal calculation), (2) matrix calculation using the bending field that was divided into 280 segments in order to consider non-isomagnetic field, (3) numerical integration method (PROVIDENCE) using dipole and quadrupole expansions of B_x, B_y, (4) the PROVIDENCE using dipole and quadrupole expansions of B_x, B_y, B_s, and (5) the PROVIDENCE using the magnetic field

Table 1: Comparison of Betatron Tunes

No	ν_x	ν_y
1:isomagnetic	1.45	0.53
2:segments	1.36	0.49
3:B_x, B_y	1.38	0.50
4:B_x, B_y, B_s	1.38	0.44
5:true values	1.38	0.43

Only the tunes of No4 are nearly equal to ones of No5 which would be true values. These results show that the differences of the betatron tunes between the results of the PROVIDENCE and the linear optics code are attributed to ignoring the B_s and identifing the closed orbit with the reference orbit. This comparison suggests that the existing linear optics codes are not suitable for small storage rings with superconducting bending magnets.

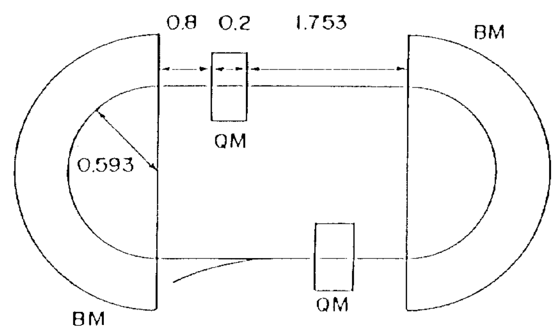

Figure 1: A schematic of SR ring

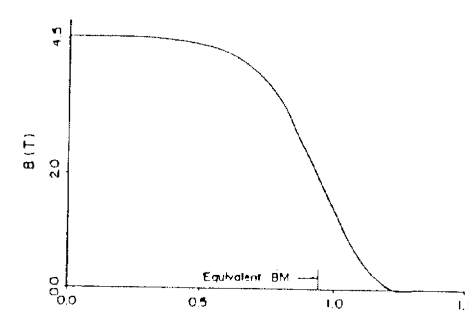

Figure 2: Field distribution of B_y along the reference orbit

Beam Tracking

The dynamic aperture(D.A) of this ring was calculated by beam tracking. The dynamic aperture is defined as a region where particles can be stable without hitting a given vacuum chamber. The chamber sizes(C.S) used are $\pm 50mm$ in the horizontal plane and $\pm 20mm$ in the vertical plane.

The required good field region is obtained by calculating

$10\sigma + COD$,where σ is a beam size calculated with the SYNCH. Then each bending magnet is divided into 40 segments along the reference orbit to consider nonisomagnetic fields of superconducting bending magnets. The COD was calculated with the PROVIDENCE, and alignment errors were assumed as follows: (1) 0.5mm misalignment and 0.5mrad tilt of quadrupole magnets, (2) 1.0mm misalignment, 0.5mrad tilt, and 0.2mrad rotation of bending magnets [1] , and (3) 1.0×10^{-3} relative field strength error between two bending magnets.

The good field region(G.F.R) [2] at the center of the bending magnet was calculated to be $-5.2mm < x < 21.0mm$ and $-16.9mm < y < 16.9mm$.

Figure 4 shows the phase spaces at the center of the bending magnet. The initial point of tracking is $x = 20mm$ and $y = 20mm$ which locates on around the boundary of the required good field region. On account of multipole fields, the form of ellipse of y-coordinate spreads especially. Figure 5 shows the dynamic aperture at the center of the bending magnet. The dynamic aperture region is wider than the required good field region.

Conclusions

The new tracking program PROVIDENCE by the numerical integration method was developed. The calculating speed was significantly improved by simple formulations. And the global error was estimated to be within the tolerance. The accuracy of this program was examined by calculating tunes and chromaticities using a lattice like LEAR. The results showed that the PROVIDENCE could simulate particles accurately.

This program was applied to a small storage ring with superconducting bending magnets. The betatron tunes calculated with this program were significantly different from the values calculated with a linear optics code. The difference comes from the consideration of B_s and from the large difference between the closed orbit and the reference orbit. The beam tracking by this code showed that this ring has a sufficient dynamic aperture.

References

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[3] K.G.Stefen, *High Energy Optics*, **Vol.17**, Interscience Publishers, New York, 1965.

[4] D.Ruth,**IEEE Trans. on Nucl Sci,Vol. NS-30, NO4**, August 1983.

[5] J.Jaeger, D.Moehl,**PS/DL/LEAR/NOTE81-7**, 1981.

[1] These error levels are higer than conventional values for the normal bending magnet. But even such values are difficult to attain with superconducting magnet.

[2] The required good field region in x-coordinate is not symmetric due to the difference between the closed orbit and the reference orbit

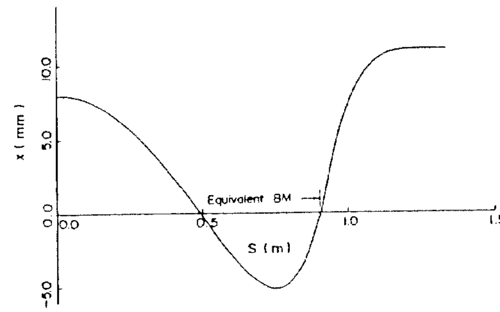

Figure 3: Displacement of the closed orbit from the reference orbit

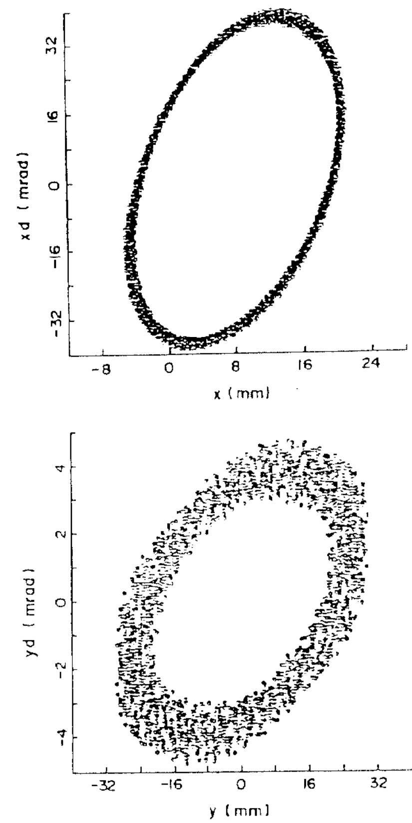

Figure 4: Phase spaces at the center of the bending magnet

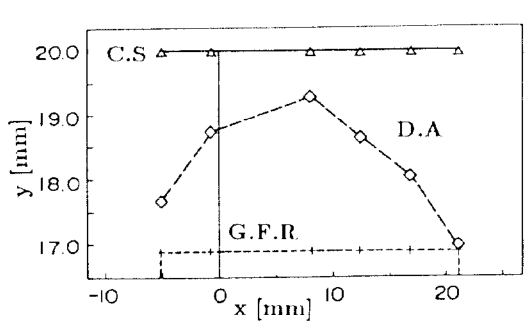

Figure 5: Dynamic aperture at the center of the bending magnet