NUMERICAL MODELING OF TIME DOMAIN 3-D PROBLEMS IN ACCELERATOR PHYSICS

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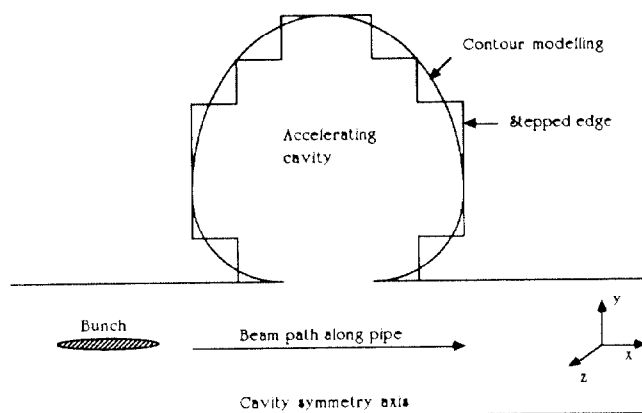*Batavia, Illinois 60510*

Figure 1: Stepped Edge v.s. Contour Geometries

Figure 2: Contour Plot of E_z in the Cavity

Abstract: Time domain analysis is relevant in particle accelerators to study the electromagnetic field interaction of a moving source particle on a lagging test particle as the particles pass an accelerating cavity or some other structure. These fields are called wake fields. The travelling beam inside a beam pipe may undergo more complicated interactions with its environment due to the presence of other irregularities like wires, thin slots, joints and other types of obstacles. Analytical solutions of such problems is impossible and one has to resort to a numerical method. In this paper we present results of our first attempt to model these problems in 3-D using our finite difference time domain (FDTD) code.

Historical Background

In the mid 1960's, Yee [1] introduced a computationally efficient means of directly solving Maxwell's time-dependent curl equations finite differences, now designated as FDTD . Here in the U.S. work was dominated by the development of a code for the study of open (scattering) problems. This code successfully addressed the problem of truncating the FDTD grid in a computationally efficient and effective manner so as to simulate an infinite grid. This added elaboration is not needed in closed problems. The FDTD code initially developed by Taflove [2] has been successfully applied to a wide variety of electromagnetic scattering and interaction problems [3-4], to the modeling of thin wires and cracks, moving objects, and recently to conformably model curved surfaces [5-8]. The stepped edge approximation of a curved surface can introduce significant errors in the calculation of wake fields and impedances. Such errors can be reduced by increasing the mesh size, therefore increasing the cost of computing. Here instead, the Ampere's and Faraday's contours' are deformed near a media interface so as to conform to the interface. This is illustrated briefly in [9] and more detailed in [8]. Two sample results in 2-D taken from paper [9] are shown in Figures one and two.

The extension of the conformal modeling of curved surfaces to 3-D follows the same basic idea. A scattering example of the conformable FDTD method, a sphere illuminated by a plane wave, is shown in Figure three. This figure compares E-plane near field data. This previous
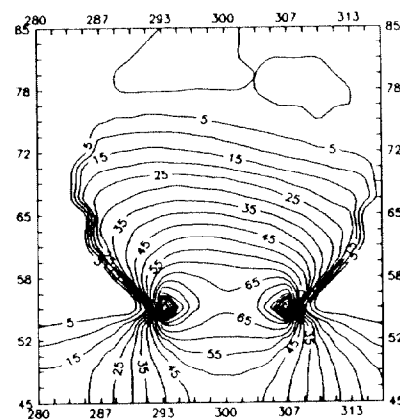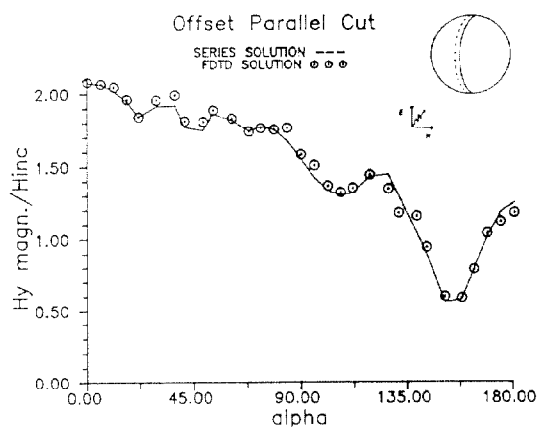
Figure 3: E-plane cut of the near field of a plane wave illuminated sphere

experience with FDTD to study scattering problems and our present involvement in accelerator physics problems was a perfect opportunity for a new application of the FDTD code.

The FDTD code was modified such that the excitation of the computational grid was by moving charges rather than a plane wave. Initial 2-D results were very satisfactory and presented at the PAC-89 conference in Chicago [9]. The logical next step following our 2-D test runs was to perform similar tests in 3-D. In this paper we present our first results in an attempt to apply the FDTD code to 3-D accelerator problems. Again results obtained are in good agreement with the analytical predictions.

Discussion and Results

Two geometries are selected for our 3-D tests: a stepped edge cylindrical pipe, and a square pipe. Runs are made with the pipes empty and with the pipes containing two parallel plates that act like separators. The purpose of these test runs is to build a confidence in our code capability to model 3-D problems. The geometries selected

Figure 4: Problem Geometries

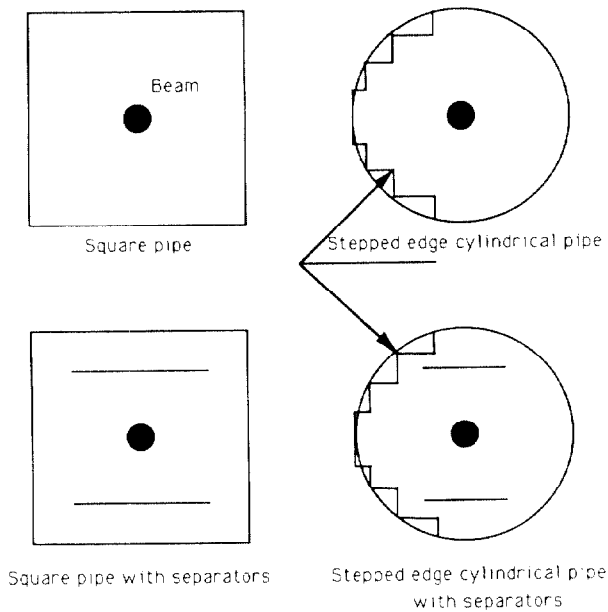Figure 5: Distribution of E-field magnitude across a stepped edge cylindrical beam pipe

at this stage are not meant to reproduce a real machine. The problem geometries are shown in Figure four.

The stepped edge cylindrical pipe provide us with a way of validating our 3-D FDTD code. Contrary to the 2-D model where a line charge move in space the 3-D model considers a charge with a given distribution moving in space. This moving charge distribution corresponds to a current density distribution given by $\rho \vec{v}$ (frequently called convection current) and shows up in Ampere's law:

$$\oint_C \vec{H} \cdot d\vec{l} = \iint_S \sigma \vec{E} \cdot d\vec{S} + \frac{\partial}{\partial t} \iint_S \vec{D} \cdot d\vec{S} + \iint_S \rho \vec{v} \cdot d\vec{S} \quad (1)$$

The movement of charge is modeled by exciting the FDTD grid at approriate spatial and temporal locations, given the desired path and shape of the particle bunch. Here we assume the charge distribution to be gaussian travelling at the speed of light. For a single particle travelling at the speed of light the fields lines are squashed into a plane perpendicular to the axis of motion, and decay as r^{-1} [10]. The case of a gaussian pulse is different but for a sufficiently narrow pulse (comparable to a delta function) we can expect a behavior similar to that of a single particle. The analysis at this point is not self-consistent. In other words the beam is assumed to be rigid and does not change shape in response to a field acting back on it.

Case of a Stepped Edge Cylindrical Pipe: Results with the stepped edge cylindrical smooth pipe are shown in Figures five and six. Figure five is a snapshot of the E-field magnitude distribution across the beam pipe taken at the same location of the beam in the pipe. Figure six clearly shows the r^{-1} decay of the field as we move along a line perpendicular to the axis of motion.

Figure seven shows same type of results taken across a stepped edge cylindrical pipe containing the separators. The separators are perfectly conducting plates that have no thickness. The effect of the plates is seen in the tail of the pulse.

Case of a Square Pipe: For the purpose of further testing the FDTD code applicability to different type of problems we performed similar calculations on a square pipe. Figure eight shows a snapshot of the E-field magnitude across a square pipe taken again at the same
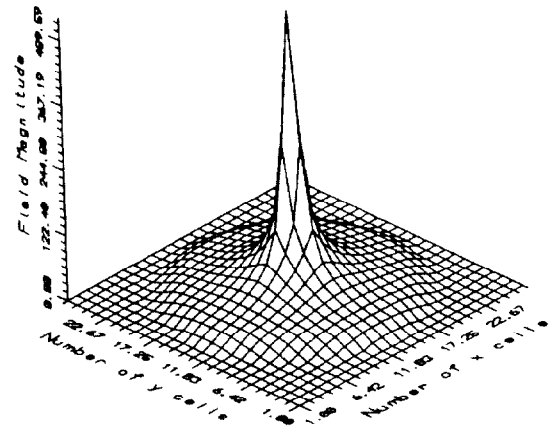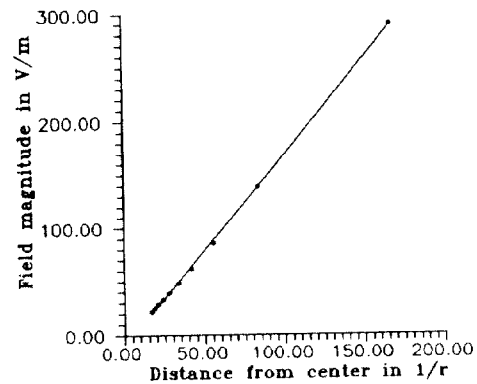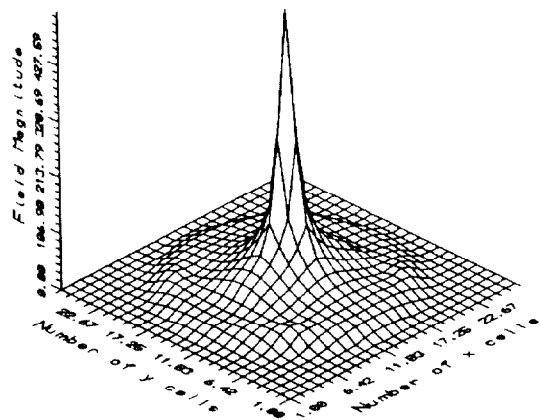

Figure 6: Variation of E-field magnitude versus r^{-1}

Figure 7: Distribution of E-field magnitude across a stepped edge cylindrical pipe containing the separators
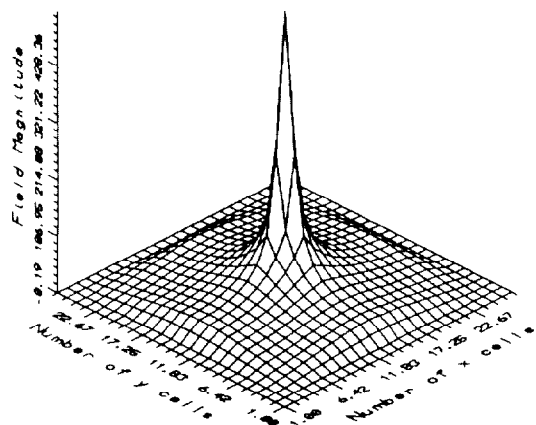
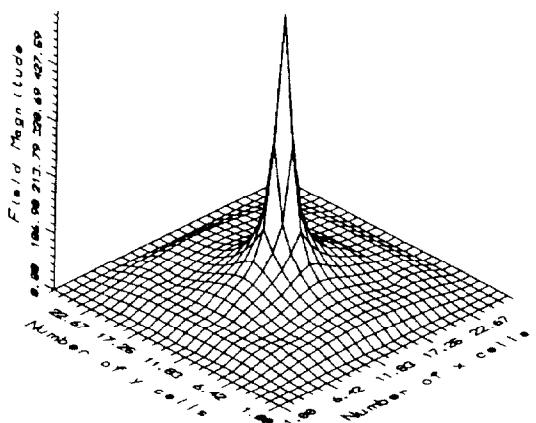Figure 8: Distribution of E-field magnitude across a square pipe

Figure 9: Distribution of E-field magnitude across a square pipe containing the separators

location of the beam. We noticed the field distribution away from the center from the beam get squarish , a result we would somehow expect.

Result for the case of a square pipe with separators is shown in Figure nine. Again the effect of the separators is noticeable around the tail of the pulse.

Summary and Future Investigations

In this paper we have demonstrated the capability of the FDTD code to model simple accelerator physics problems in 3-D. The 3-D conformable FDTD modeling of a beam pipe is currently under investigation. Thereafter, beam monitors and cavities will be added to the pipe. Ferrite loading of the cavities is also possible, given the demonstrated capability of the FDTD code to handle arbitrary media. Coupled with previous two and three dimensional scattering validations, the 3-D FDTD simulation of beam pipes, cavities, wire monitors and resistive wall monitors in a conformable manner is a direct extension of the present results.

References

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