EXTENSION OF PROGRAM TRANSPORT FOR LINACS.

A.J. Riche ,CERN,CH – 1211 Geneva 23, Switzerland.

1. Abstract

The program TRANSPORT has been extended , using an updating procedure to include 3 effects : coupled motion in accelerating sections equipped with solenoids, sections with superposed quadrupoles, and the effect of a quasi – linear variation of the electric field. Pseudo – periodic focusing parameters, used for matching purposes, can be systematically calculated. Plots of these parameters and the envelopes resulting from the matching, help to quickly judge the results. Applications are shown for the LEP injector linac (LIL).

1. Introduction

Programs such as TRANSPORT (1) are very suitable for beam optics so that there is a strong motivation to increase the field of their application when necessary, instead of writing new codes. For calculating the optics of the LIL linac, we had to represent the combined effect of quadrupole focusing or solenoid focusing, with the longitudinal acceleration. Moreover, the electric field in the structures varies linearly along z up to a point from where it is constant to the end . This field configuration, which provides the maximum energy for the particles, is due to the combination of a quasi – constant gradient section and a SLED – type RF input pulse. Using this more exact representation of the actions of the fields on the beam, the matching has been recalculated, and the FODO channel itself was adjusted because the periodicity in the focusing had partially vanished, as a consequence of modifications since the original design was made. Many of the parameters, such as geometrical lay out, are frozen: distances between quadrupoles are now constraints for the new matching. We will briefly outline the new conditions for the machine, and how TRANSPORT was updated to give systematically the local pseudo – periodic parameters for matching purposes. Both implementations of the program are completed by plotting facilities for the results, such as envelopes or periodic coefficients, a synthetic examination of the results being extremely useful when making the many runs involved in the matching process.

2. The new operators

1. Acceleration in linac structures :

The electric field component ξ in the z direction is:

$$\text{for } 0 \le z \le L \;:\; \xi = \xi_1 + \frac{d\xi}{dz} z \;;\; \text{and for } z \ge L \;:\; \xi = \xi_2 \quad (1)$$

The longitudinal space is segmented for approximating the electric field by its average on each part. This partition is mandatory if elements such as solenoids or quadrupoles are superposed on the accelerating structure.

2. Combined longitudinal electric and magnetic fields.

Haissinski (2) derived the transfer matrix for relativistic particles , in the paraxial approximation. Equations and elements of solution are here briefly resumed for convenience. Using complex coordinates, the distance of the particle to axis and its variation with z are:

$u = re^{i\phi}$, $v = \frac{du}{dz}$

The force in the transverse plane, due to the longitudinal and radial B fields is :

$$F = -ivecB - i\frac{u}{2}ec\frac{dB}{dz} \quad (2)$$

The time derivative of the transverse momentum $p_\perp = mv_\perp$ is

$$\frac{dp_\perp}{dt} = e\xi v + E\frac{dv}{dz} \quad (3)$$

Equating these expressions gives the transfer matrix for an segment dz,

$$dR = \begin{pmatrix} 1 & dz \\ -\frac{iecdB}{2E} & 1 - \frac{iecBdz + dE}{E} \end{pmatrix} \quad (4)$$

For a Heaviside step B_1 to B_2 of the longitudinal value of B the matrix is:

$$R_n = \begin{pmatrix} 1 & 0 \\ -iec\frac{(B_2 - B_1)}{2E} & 1 \end{pmatrix} \quad (5)$$

For a slice with constant B, constant electric field, and energies E_1 and E_2 at the ends, the matrix is:

$$R_c = \begin{pmatrix} 1 & i\frac{E_1}{ecB}(e^{-2i\omega} - 1) \\ 0 & \frac{E_1}{E_2}e^{-2i\omega} \end{pmatrix}, \quad with \;\; \omega = \frac{c}{\xi}\frac{B}{2} * Log\frac{E_2}{E_1} \quad (6)$$

Using these transfer matrices R, the beam matrix σ is transported from a point to another according to $\sigma_2 = R\sigma_1 R^t$.

The elements of σ are the means of the second order momenta : <x x>, <x dx/dz>, <x y>... TRANSPORT gives results at the limits of the elements. Solenoids are represented by a central part, where B is constant,with 2 edges of the Heaviside type, through which the longitudinal field varies abruptly between zero and B. The partition of a solenoid in successive segments still keeps the condition B=0 at the point situated at the limit between two segments. There, the mechanical angular momentum $p_\phi = r\frac{d\phi}{dt}\frac{c^2}{E}$ equals the canonical momentum.The canonical angular momentum $rp_\phi + crA_\phi$ is invariant. A_ϕ is the component of the vector potential; rA_ϕ the flux.

We refer to (4) and (5) for discussion and theory of emittance invariants. It is possible, in updated TRANSPORT, to avoid closing the field by a Heaviside step at a point or to include the source in the field (immersed source). The envelopes (given by the terms σ_{11} or σ_{33} of the beam matrix) do not depend on the fact that the points where they are calculated are at zero field or within the field. The problem of the source is different . Since the initial canonical angular momentum is conserved, the envelopes obtained with a source immersed in a field may differ considerably from those obtained with a source placed at zero field, as illustrated by updated TRANSPORT runs given in (6), and a figure in this report.

3. Quadrupole combined with longitudinal electric acceleration (3).

Projecting the movement on one of the axis of the transverse plane, say y, we obtain for the force :

$$- ec\dot{y}g \tag{7}$$

where g is the gradient, and for the time derivative of the transverse momentum:

$$\frac{dp_\perp}{dt} = pc\frac{d^2y}{dz^2} + e\xi\frac{dy}{dz} \tag{8}$$

If p_1 is the initial momentum, and if the variables are changed by

$$Gn = \frac{eg}{cp_1} \quad ; \quad \xi n = \frac{e\xi}{cp_1} \quad ; \quad u = 2\left(\frac{Gn}{\xi n} * \frac{1+\xi n * z}{\xi n}\right)^{\frac{1}{2}} \tag{9}$$

we get from the equation of the dynamics :

$$\frac{d^2y}{du^2} + \frac{1}{u}\frac{dy}{du} + y = 0 \tag{10}$$

The solution is a linear form of Bessel functions of first and second kind:

$$y = AN_0(u) + BJ_0(u) \quad ;$$

$$\frac{dy}{dz} = \left(\frac{Gn}{1+\xi n * z}\right)^{\frac{1}{2}}\left(-AN_1(u) - BJ_1(u)\right) \tag{11}$$

A and B are determined by the initial conditions. An analog solution holds for the other transverse coordinate.

Matching.

Pseudo – periodic focusing structures.

A periodic pattern of focusing elements such as a FODO pattern is the natural and economical solution: identical quadrupoles are used, powered by the same supply, and the variation of the energy is balanced by increasing the distances between the quadrupoles along the linac. For a given linac, these distances, once optimized are, of course, fixed. From a transfer matrix R, supposed to be periodic, the usual Twiss parameters are calculated for one of the transverse direction, as

$$\cos(\mu) = \frac{trace(R)}{2} \quad ; \quad \beta = \frac{R_{12}}{\sin(\mu)} \quad ; \quad \alpha = \frac{R_{11} - R_{12}}{2\sin(\mu)} \tag{12}$$

The beam is represented by a distribution in an ellipsoid. Its equation is :

$$X^T \sigma^{-1} X = 1 \tag{13}$$

where the components of X are the coordinates and their z derivatives. Consider a simple motion decoupled in the 3 coordinates. The beam matrix for one of the transverse directions is

$$\sigma = \begin{pmatrix} \beta\varepsilon & -\alpha\varepsilon \\ -\alpha\varepsilon & \gamma\varepsilon \end{pmatrix} \tag{14}$$

Here, the elements of the beam matrix ,related to the parameters of the beam, are writen using α, β, $\gamma = (1+\alpha^2) / \beta$, and the emittance ε , i.e., the Twiss parameters derived from the transport matrix R for a period.
The beam is perfectly matched at a point if the parameters of the beam are equal to the corresponding Twiss parameters of the transfer matrix R. With periodic system, the condition, fulfilled at one point, will automatically hold at any other point. If periodicity is more or less suppressed by some important perturbation, the matching is spoiled, the beam envelope distorted, the parameters at the end are changed and the transmission may suffer if the transverse amplitudes of the particles with z, reach the edges of the available aperture.

Taking the example of LIL, there are two reasons which make the matching difficult.
The energy was changed by a factor 1.3 from the original design of the focusing system, and the e− and e+ beams travel through the same focusing system. The matching is made for the positrons, for which the emittance is high and losses not welcome. The electron beam is mismatched. In LIL, the quadrupoles of the FODO array are grouped in 3 families, powered independently and 5 matching quadrupoles are set in front of the FODO, with independent currents.

Matching procedure.

We proceed by successive approximations: starting with a set of 3 currents chosen for the FODO families, we calculate the local pseudo – periodic matrices thanks to the part of the program update specially developed for that purpose .
Transfer matrices R of successive semi – periods are stored. By multiplying 2 consecutive matrices, we have the matrix for the period around one point. The energy found at the point is taken as an average for the period. The Twiss parameters are derived for each period, printed and plotted. In a second iteration,the choice of the 3 currents results from the maximum available aperture. If, as for LIL, this aperture is determined by the iris of the structures, it is quasi – constant. The ratio β / *energy* will be kept constant, as it represents the product $\beta * \varepsilon$, i.e. the square of the maximum transverse beam size. We have finally a set of currents which are optimized, and the corresponding values of $\beta_i\varepsilon_i$, $\gamma_i\varepsilon_i$ along the machine. Given the beam matrix at the system entry, we use the program standard matching procedure to trim the currents of the matching quadrupoles, forcing the elements of the beam matrix at a series of points to be equal to the preset values: $\sigma_{ii} = \beta_i * \varepsilon_i$, $\sigma_{jj} = \gamma_i * \varepsilon_i$, where i stands for x or y, and j for x′ or y′.
The number of points where matching is required gives the number of constraints while the number of varying currents is limited (to 5 in our application). Because of the lack of periodicity, we require matching at more points than is allowed by the number of variables, and therefore, the fitting accuracy has to be relaxed. We repeat the process with points chosen differently, for verification of the stability of the solution and his best achievement at all points. The required beam characteristics at the linac end are supplementary conditions for the matching. The automatic computation of the machine pseudo – periodic parameters ,as shown on Fig.1, is of a great help , the plotting facility also, for easy verification of the solution at each step. The LIL e+ beam envelopes are shown on Fig.2.

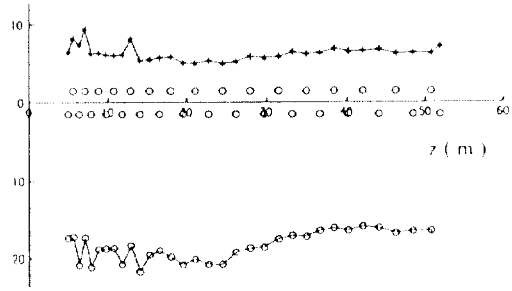

Fig.1 Variation of β_x / energy , β_y / energy at successive QF and QD of the linac

A rapidly cycled air core solenoid provides the focusing for the positrons leaving the target. This field does not penetrate the bulk of the target. At the contrary, the target lays in the field of the solenoid placed around the following accelerating section, because it is operated with dc power supply. On Fig.3, we compare the envelopes calculated when supposing no field on the target to those obtained when including the target in the constant stray field.

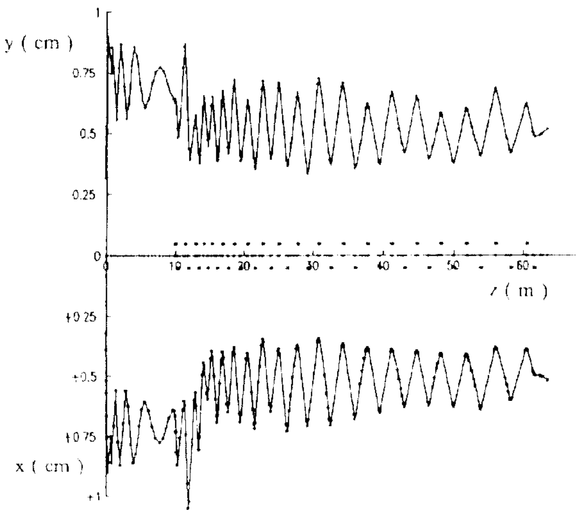

Fig.2 x and y envelopes for positron beam from the target to the end of the linac W

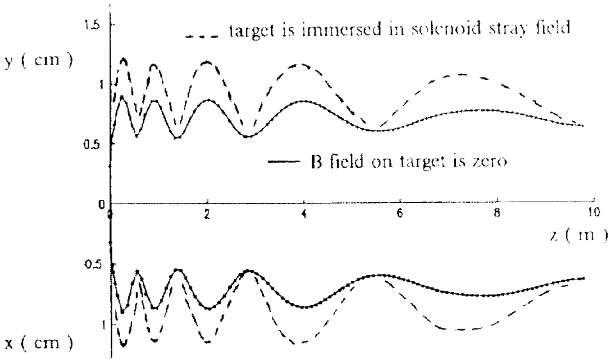

Fig.3 x and y envelopes for positron beam from the target to the end of accelerating sections with associated solenoids

Data cards.

Data cards for new elements.

TRANSPORT data cards hold parameters for describing the beam at entry or for program commands or for describing an action on the beam, which is further translated into transfer matrix. In order to take into account the variation of the electric field and the associated action of acceleration with magnetic fields, the number of the data of the corresponding cards had to be increased. A simple system allows the reading of the parameters in supplement, and even allows reading standard sets of cards as well as the new ones. 3 new parameters: $\frac{d\xi}{dz}$, L , ξ_2 are added after the standard ones for the description of acceleration. If the first card of a group describes a segment with pure acceleration, the sequence of its parameters is the above list. If the first card describes a segment in which a magnetic field is combined with acceleration, the list above follows the standard parameters for the magnetic fields. If the card is not the first one of the group, the first new parameter ($\frac{d\xi}{dz}$ for pure acceleration, or the energy gain for mixed action) , is replaced by a dummy number flagging the segment. The absence of this flag or the occurrence of a card describing another type of operation ends the domain of application of the acceleration as it has been described previously.

Input is also simplified by the program because it defines automatically the point where the constant field ξ_2 replaces (after L) the rising linear field, and splits any element which would lie across these 2 parts. in order to describe the acceleration correctly, as shown on Fig.4.

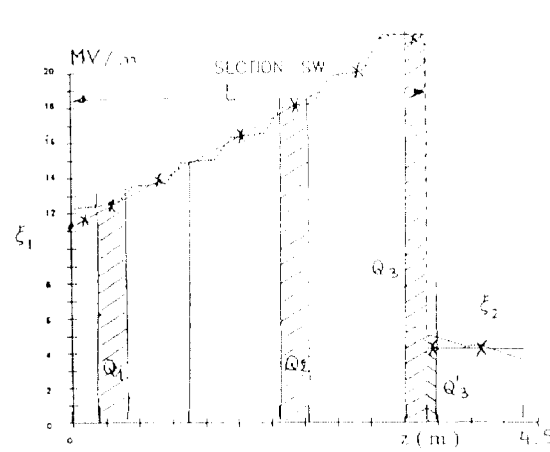

Fig.4 variation of the longitudinal component of the electric field along a LIL accelerating structure indication of the superposed quadrupoles and the partition for the data cards
interrupted line represents the exact field

Calculation of the pseudo periodic parameters.

A flag triggers the process in the program. For convenience, this flag consists in changing the sign of the value of $\sqrt{\sigma_i}$ in the card describing the beam, at the head of the data set. Then, the couple of cards
13. 4. 3. (beam matrix print out)
6. 0. 1. (update of the transfer matrix)
is systematically placed at each mid−quadrupole point,in case of a FODO structure. at the mid points of each second quadrupole, if triplets were used.

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