A Simulation Program for Low Energy Injection Study*

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Abstract

Due to the long damping time at low energy, the simulations usually involve with tracking tens of million turns of several thousands particles. It would require about one CPU year of VAX 2000 to do the simulation. By imposing some physics knowledge and a special tracking technique, we can do the simulation in several CPU hours by the same computer. The simulation results agrees with the measurement within the accuracy of the measurement.

Introduction

Due to the coupling between horizontal motion and vertical motion as well as the non-zero dispersion at the injection point (septum), the study of the injection kinematic always involves six dimensional phase space tracking. In order to get good statistics, we need to simulate at least several thousand particles. The tracking time should be equal to or greater than the damping time. Under these conditions, the injection simulation for low energy machine (*e.g.* 100 MeV) will require unreasonable CPU time, because the damping time of low energy machine is in the order of million turns. Therefore, it is necessary to develop a simulation program which can properly simulate the injection process in a reasonable CPU time. In this paper we use Aladdin as a study example. However, the program is not machine parameters restricted. Aladdin is a 1 GeV electron storage ring located at the Synchrotron Radiation Center of the University of Wisconsin, Madison. It has been in operation as a synchrotron radiation source since 1985 with average accelerated currents of about 200 mA.[1] The injector of Aladdin is a 100 MeV racetrack microtron[2].

We divide the injection process into three stages for the purpose of the simulation. The first stage is during the beam injection and the kickers vary with time. We treat the first 100 turns of the storage ring as a long transport line and track the particles in the full six dimensional phase space. The second stage is after the kickers have died out and before the next time we fire the kickers. During this stage the transverse oscillation is unimportant, except we should recognize that after one million turns the betatron phase of each particle is randomly distributed. Thus, in this stage, we can decouple the simulation of the longitudinal motion and the transverse motion. To track 30 million turns of 9,000 particles in longitudinal phase space still needs a lot of CPU time using the normal tracking method. The computing time can be greatly reduced by using L_TRA[3]. L_TRA is a code used to do longitudinal phase space tracking by using the amplitude-dependent synchrotron oscillation period. We will discrib L_TRA in detail later. The third stage begins right before the next time we fire the kickers, and lasts until the kickers reach their maximum amplitude when the next injected beam comes in. Before we start this stage, we should recouple the longitudinal oscillation and the transverse oscillation, then treat the storage ring as a long transport line again. After the third stage, the process goes back to the first stage. Figure 1 shows the block diagram of what we described above. The detail of how we decouple the tracking of the transverse motion and the longitudinal motion and how we recouple them will be discussed in the next section.

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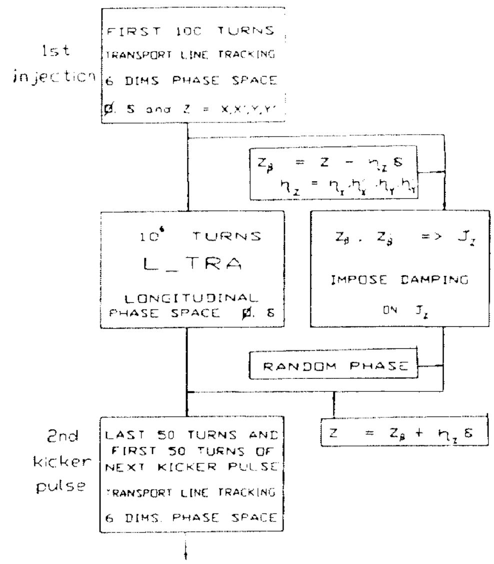

Figure 1: A block diagram of the simulation of the injection

The Simulation by the Transport Line Method

Single Particle Tracking

In this study, the lattice is assumed to be linear and have no imperfection. A 7×7 matrix is adequate for the six dimensional tracking of a linear lattice. We need one more dimension in the the matrix to produce the effect of kickers. The one turn transformation matrix is broken into several transformation matrices which represent segments of the whole ring. Between the two consecutive matrices, we can put the diagnostic command and/or the physical apertures. Because the closed orbit changes as the kicker excitation decreases, the first hundred turns in the storage ring are best tracked by a long transport line which consists of one hundred small transport lines. Each one of the small transport lines is one turn of the storage ring. About 25 of them have different kicker strength. After the basic structure is built, we put the control knobs into the program. Those are :

- the time delay between the injected particle and the inflector trigger.
- the time delay between the inflector trigger and the master trigger of the two kickers.
- the time delay between the kicker #1 trigger and kicker #2 trigger.
- the kicker strength of kicker #1 and that of kicker #2.

Figure 2 is an output of the program. Fig. 2(a) is a horizontal phase space plot. Fig. 2(b) is the horizontal position vs. time (in units of revolution time). Fig. 2(c) is the kicker strength seen by the tracked particle. The horizontal axis is the kicker strength as a percent of the full strength. The vertical axis is time in units of revolution time. On the right top quarter, the value of the parameters of this run is printed. In this run we only plot out 23 turns.

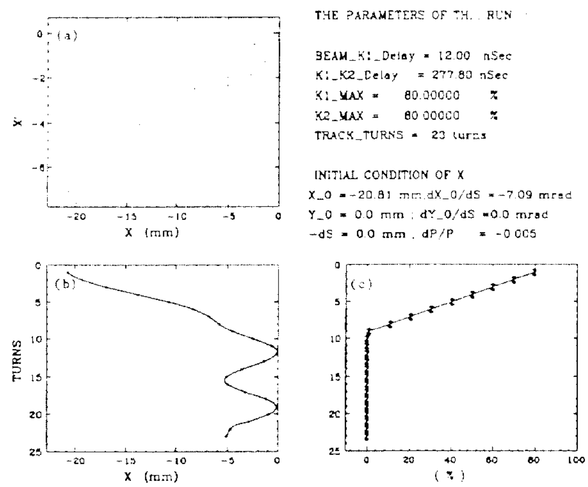

Figure 2: An output of the single particle transport line tracking. (the detailed description is in the text).

Pulsed Beam Simulation

After the completing of the single particle tracking program, the pulse beam transport line tracking can be done very easily. What needs to be done is to update the program to become able to accept the injected beam initial condition description, then run the particles sequentially. The output format is changed, too. Now we only plot those particles which have survived after the run. Any combination of two dimensions out of six dimensional phase space information can be plotted out.

Since the injected beam pulse length is about 4 times the ring circumference, when we plot the final position, the information as to exactly what portion of the injected beam pulse has survived, is hidden. This information can be unfolded by requesting the program to plot the original position of the surviving particles. This information is useful for us to adjust the injection parameters to get better injection efficiency. Figure 3 shows an output of this program. The horizontal axis is the time t_0 in nsec. for particles passing through an observation point at the storage ring and the vertical axis is the final horizontal position X in mm. Figure 4 shows another output of this program. Now, the horizontal axis is the distance S in meters along the beam pulse before the beam is injected into the storage ring and the vertical axis is the initial horizontal position X in mm. We can see there is a big portion of the front part of the beam pulse that is lost. In this case, we may want try to inject the beam pulse later (with respect to the inflector trigger) or change some other parameter to save particles which were lost in the front part of the beam pulse.

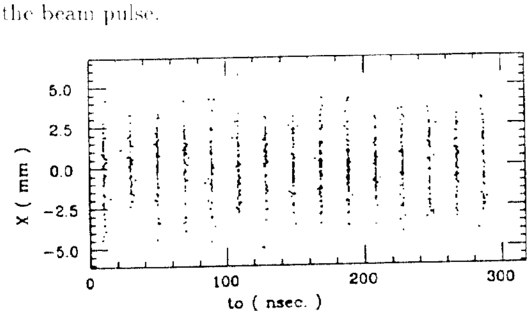

Figure 3: An output of the pulse beam transport line tracking. Final horizontal position X vs the time t_0 for particles passing through an observation point in the storage ring.

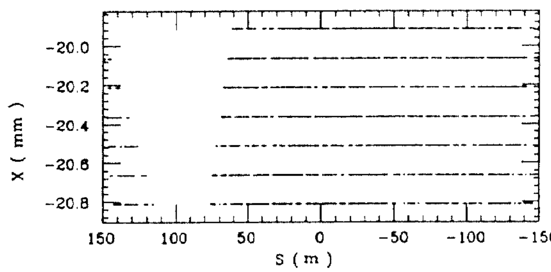

Figure 4: An output of the pulse beam transport line tracking. Initial horizontal position X vs distance S along the beam pulse before injecting into the storage ring.

Longitudinal Phase Space, Long Time Tracking

To track particles in longitudinal phase space for a long time period can be done in a much less CPU time by using the amplitude dependent synchrotron oscillation period. Figure 5 shows the block diagram of the tracking scheme. As soon as the initial conditions in the phase space ϕ_0 and ΔE_0 of the particle are given, the extreme phase ϕ_1 and ϕ_2 can be calculated from the equation[4] :

$$\frac{\dot{\phi}^2}{2} - \frac{\Omega^2}{cos\phi_s}(cos\phi + \phi sin\phi_s) = -\frac{\Omega^2}{cos\phi_s}(cos\phi_1 + \phi_1 sin\phi_s)$$
$$= -\frac{\Omega^2}{cos\phi_s}(cos\phi_2 + \phi_2 sin\phi_s) \quad (1)$$

where Ω is the small amplitude angular synchrotron oscillation frequency and ϕ_s is the synchronous phase. After we know ϕ_1 and ϕ_2 the synchrotron oscillation period $T_s(\phi_0, \Delta E_0)$ can be calculated from the following equation[4] :

$$T_s = \frac{2}{\Omega}\int_{\phi_1}^{\phi_2}\frac{d\phi}{\sqrt{f(\phi,\phi_s) - f(\phi_1,\phi_s)}} \quad (2)$$
$$f(\phi,\phi_s) = \frac{\sqrt{2}}{cos\phi_s}(cos\phi - \phi sin\phi_s).$$

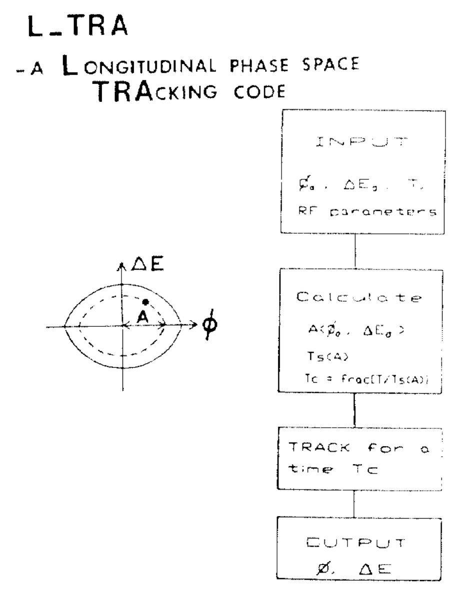

Figure 5: The block diagram of the the longitudinal phase space tracking program

Therefore, if we want to track a particle for a time T, what we need to track is only the fractional part of $\frac{T}{T_s(\phi_0,\Delta E_0)}$. For example, if T = 1 sec. and $T_s(\phi_0, \Delta E_0) = 0.14$ msec., $\frac{1000.0}{0.14} = 7142.857$ the time for which we need to track is only 0.857×0.14 *msec* = 0.120 *msec*. To track 1000 particles with different initial conditions for 1 second only takes 3 CPU minutes on a VAX 2000; it will take 36 CPU hours of the same computer using the normal tracking method.

If the tracking time T is much smaller than the radiation damping time, the radiation damping effect is included by reducing the oscillation amplitude at the end of tracking according to the tracking time and the radiation damping time. If the tracking time T is not much smaller than the radiation damping time, we will break T into smaller pieces, each of them much smaller than the radiation damping time, and then do the tracking. How small we will break T depends on how accurate the result we want and how much CPU time we can spend. The default value in the code L_TRA is one tenth of the radiation damping time. We can change that by inputting another value.

Simulation of Multipulse Multiturn Injection

To do multipulse multiturn injection simulation, the remaining problem is the connection between the six dimensional phase space transport line tracking and the long time longitudinal phase space tracking. The connection is done in the following way. We choose an arbitrary point in the ring; say the position where the inflector is located. Here we only describe how to decouple and then recouple the horizontal coordinate and the longitudinal coordinate. The same method applies to the vertical coordinate. From both a direct measurement and the output of SYNCH, the lattice functions β, α, η, and η' at the inflector position can be known. From the following relations,

$$X = X_\beta + \eta \frac{\Delta E}{E}, \tag{3}$$

$$X' = X'_\beta + \eta' \frac{\Delta E}{E}, \tag{4}$$

by knowing X, X', and ΔE from the end of the six dimensional tracking, X_β and X'_β can be calculated. Therefore, we know the horizontal betatron oscillation amplitude J_x. If there is any damping during the longitudinal tracking, that can be applied to J_x analytically using the values of the tracking time and the damping time.

At the end of the long time tracking we have J_x, J_y and $\phi, \Delta E$. According to Eq. 3 and Eq. 4, if we know X_β, X'_β and ΔE, we know X and X'. By the same argument we can get Y and Y'. This will provide us the initial conditions for the next stage tracking. Now the problem is how we get X_β and X'_β. To get X_β and X'_β, we need to know the betatron oscillation amplitude J_x and the betatron phase ϕ_x. After a long time, in our case 0.8 sec or 2.7×10^6 turns, the betatron phase ϕ_x of the injected particles is distributed randomly. Therefore, we can get X_β and X'_β by knowing J_x and using a randomly distributed betatron phase.

The argument of the randomly distributed phase is very important. Because of that we can decouple the tracking and the simulation of injection can be done in a reasonable CPU time. Also only if we take this random effect into account, can we describe the physical system correctly. Figure 6 is the result of the simulation of injection with the emittance coupling $\kappa = 10\%$ for different ΔE and kicker strength S_k. In this simulation, to save CPU time, we use a smaller beam size, so, we only need to track 2,140 particles. This leads to a higher injection efficiency.

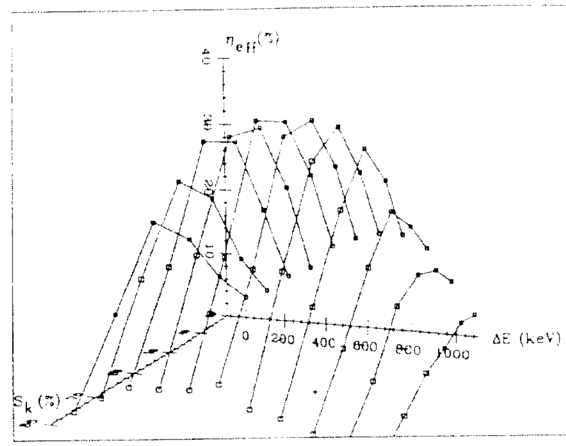

Figure 6: The result of the simulation of injection with the emittance coupling $\kappa = 10\%$. η_{eff} vs ΔE and kicker strength S_k. + is measured value of Aladdin parameters.

Conclusions

These simulation results and the particle loss rate during injection agree well with the measurement within the measurement accuracy.[5] It also help us to understand the anomalous fast stacking rate which let Aladdin be able to accumulate more than 200 mA.

Ackonwledgment

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