SURVEY OF TECHNIQUES FOR LONGITUDINAL EMITTANCE BLOW-UP IN THE KAON FACTORY COLLECTOR RING

S.R. Koscielniak
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3
G.H. Rees
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon. U.K.

Abstract

A common method to increase beam stability is to enlarge the longitudinal momentum spread. This paper discusses the efficacy, problems and merits of several blow-up schemes involving modulation of either the main rf system or AM/PM modulations of a dedicated high harmonic cavity. The relative efficiency of each scheme when applied to the KAON Factory Collector storage ring is demonstrated using a particle simulation code.

Manipulation of the RF Fundamental

Studies of emittance dilution using high harmonic cavities have been reported by Boussard[1], Katz[2], Koscielniak[3], Balandin[4] and Rees[5]. It is worthwhile asking why not use modulation of the rf fundamental to achieve some emittance growth. There are several schemes one might consider:

(1a) deliberate phase mismatch at injection to give a dipole-mode oscillation. The beam phase-loop is turned off.

(1b) deliberate rf-bucket height mismatch to give a quadrupole mode oscillation. The bunch-length loop is turned off.

(2) dipole and quadrupole mode ($n = 0, m = 1, 2$) loops with gains reversed to produce anti-damping. For instance:

$$\ddot{\phi} + 2K_p\dot{\phi} + \omega_s^2\phi = 0 \quad . \ K_p < 0 \ .$$

(3a) open loop phase modulation:

$$\ddot{\phi} + \omega_s^2[\phi\cos\Phi(t) - \sin\Phi(t)] = 0 \quad , \ \Phi(t) = \hat{\phi}\cos\omega_d t \ .$$

(3b) open loop amplitude modulation:

$$\ddot{\phi} + \omega_s^2[1 + \rho\cos(\omega_d t)]\phi = 0 \ .$$

In each case ϕ is the particle rf-phase, and approximate forms have been presented.

Mismatching

The ratio of emittances before and after filamentation is $(1+\mu)^2$, where for a dipole oscillation μ = (phase-offset)/(bunch half-length), and for the quadrupole mode $(1 + \mu)$ = ratio of bucket heights. Large emittance increase is favoured by large values of the mismatch parameter μ. However, this is also the condition for excessive filamentation, which invariably leads to large voids in the phase-space distribution, and hence multi-peaked bunch shapes. Such bunch shapes, for energy mismatching, have been observed at Rutherford ISIS[6]. There is also a strong tendency to form bunches with a dense core and long tails, which is undesirable from the point of view of beam loss.

Anti-damping

Phase-loop and bunch-length loops with positive feedback produce anti-damping. For small values of the feedback gain, the emittance growth soon comes to a halt since the error signals are lost because filamentation causes decoherence. For large values of gain, the phase-space distributions develop voids, cores-and-tails and the bunches become multi-peaked. It is unwise to base an emittance blow-up scheme upon delicate balance between these two extremes.

AM/PM modulation of fundamental

Both these schemes can be quite successful. Amplitude modulation gives Mathieu's equation. There are parametric resonances when $\omega_d = 2\omega_s/n$ with n an integer. Setting $n = 1$ gives the stongest resonance: the amplification of oscillations is roughly $1 + 2\rho$. However, for the Collector it takes 15 ms for filamentation to fill the voids created during the first 5 ms.

For phase modulation with $|\hat{\phi}| < 1$ the bunch centre approximates to a forced oscillator with drive term $\omega_s^2\hat{\phi}\cos\omega_d t$. The resonance condition is $\omega_d = \omega_s$. Large voids develop for $\hat{\phi} > 0.1$, and smaller values give very poor emittance growth.

In the absence of other means, amplitude modulation is an efficient use of resources; and is the preferred scheme.

High Harmonic Cavity

The basic problem with all the previous schemes is that a large-scale structure is imposed upon the bunch. High Harmonic Cavities (HHC) offer a valuable alternative whereby the wavelength of the perturbation is small compared to the bunch length. Extra RF cavities, operating at many times the fundamental, are inserted in the ring. For equal dilutions, the higher frequency has to be an integer multiple (k) of the main rf ($h\omega_0$) when the harmonic number is odd.

Phase-modulation

The scheme reported in Ref.[2] employs phase modulation. The single particle equation of motion becomes:

$$\ddot{\phi}/\omega_s^2 + \sin\phi = \rho\sin(k\phi - \hat{\phi}\cos\omega_d t) \ .$$

If the PM function is expressed as a Fourier-Bessel series, it is seen that the bunch is driven at a spectrum of frequencies $n \times \omega_d$ with $n = 1, 2, 3\ldots$ Which frequencies predominate depends on the values of $J_n(\hat{\phi})$. For instance, if $\hat{\phi} = \pi$ (the maximum of the J_2 Bessel function) the strongest component occurs at frequency $2\omega_d$ and there is an effective frequency doubling[5].

The emittance growth is understood[4] to be a resonant process, occuring for the condition $m \times \omega_s = n \times \omega_d$ with n, m integers. Since growth is due to resonant excitation, it is imperative to maintain the resonance condition by sweeping the modulation frequency along with the average synchrotron frequency : $n\omega_d = m\langle\omega_s\rangle$. In this way a $\sqrt{\text{time}}$ growth[1] law is converted to linear, but at the expense of enhancing the multipolar components of the phase-space distribution.

With many excitation frequencies present in the FM signal, the growth rate can be large. As a general rule[3], fast growth is associated with high modulation frequency. However, large amplitude particles are driven more strongly by the higher frequency components and so it is inevitable that fast growth rates are associated with formation of tails so that a parabolic bunch tends to become gaussian.

Another draw back is that wide-band amplifiers and cavities are required to deliver a good approximation to the true FM signal. For instance dilution in the Collector ($h = 225$) with $k = 14$, $\rho = 0.2$, $\hat{\phi} = \pi$ and $\omega_d = 4\omega_s$ requires a bandwidth $> 32\omega_s$ or 256 kHz. Since the revolution frequency is

$\omega_0/2\pi = 270$ kHz, this implies a quality factor $Q \leq 3300$. A further problem arises when the cavity resonance frequency lies on a strong beam harmonic: there is reactive beam-loading. For the Collector circulating current $I_b^0 = 3$ A and assuming $R/Q \sim 100$ this amounts to 50 kV per cavity.

Double Sideband Amplitude Modulation

Rees and Koscielniak[5] have suggested an alternative scheme using amplitude modulation of the HHC. The single particle equation of motion is:

$$\ddot{\phi}/\omega_s^2 + \sin\phi = \rho(1 + \hat{a}\sin\omega_d t)\sin k\phi \ .$$

Figure 1 compares the growth of root mean square (rms) emittance for excitation at $\omega_d = (4,8) \times \omega_s$ ($\rho = 0.1 \times \sqrt{2/3}$ and $\hat{a} = 1$) with FM at $4\omega_s$ (and $\rho = 0.1$). The simulations were made with the particle tracking code LONG1D[7].

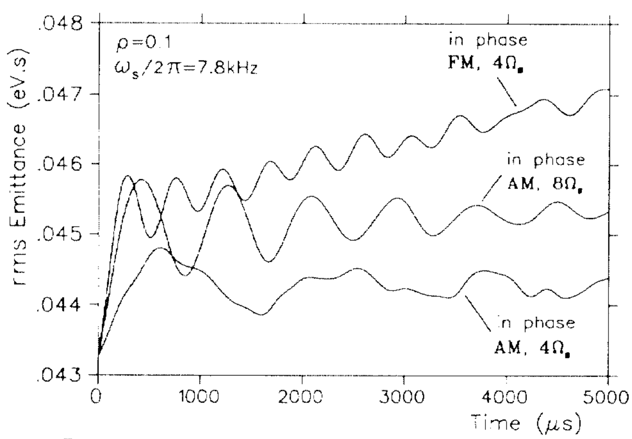

Fig.1 : Comparison of DS-AM with in-phase FM.

The scheme is conceptually simpler, since each multi-polar resonance (index m) can be selected by corresponding choice of $\omega_d = m \times \omega_s$. In phase space it is seen that $m = 4$ stimulates motion at the bunch centre, while $m = 8$ affects the periphery. Hence, one can choose to dilute the centre or periphery of the bunch; and thereby tailor the bunch-shape as desired.

The modulation signal contains a carrier and both upper and lower side-bands since

$$2\sin k\phi \sin\omega t = \cos(k\phi - \omega t) - \cos(k\phi + \omega t)\ , \quad \phi = h\omega_0 t\ .$$

Hence this is *double sideband* (DS) modulation. Figure 2 shows the frequency components superimposed on the cavity impedance. If the modulation frequency is high, then the cavity is of necessity wide-band and the power dissipation is large. For instance, the Collector dilution cavities would have a bandwidth of $16\omega_s$ or 128 kHz, and quality factor $Q \leq 6640$. The power can be reduced by dispensing with the carrier.

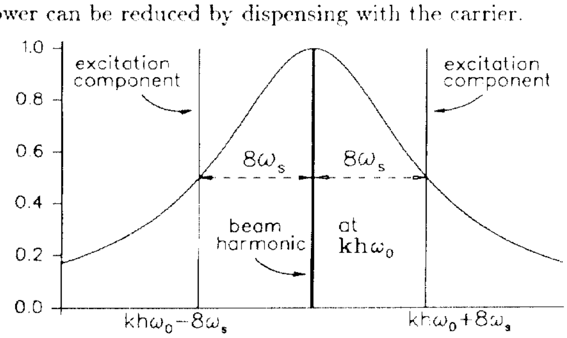

Fig.2 : Power spectra and cavity impedance for DS-AM.

Single Sideband Excitation

A variant is single-sideband (SS) modulation with the carrier suppressed. Since the carrier produces a steady state deformation of the rf bucket, it does not contribute much to the blow-up. The single particle equation of motion becomes:

$$\ddot{\phi}/\omega_s^2 + \sin\phi = \rho[\cos\omega_d t\cos k\phi \pm \sin\omega_d t\sin k\phi]$$

The plus sign (+) corresponds to use of the lower sideband at $kh\omega_0 - \omega_d$, and the minus sign (−) to use of the upper sideband at frequency $kh\omega_0 + \omega_d$. The growth rates for upper and lower sideband excitation are, of course, identical. There are three advantages to this scheme:

1. The cavity can be made high Q (narrow band) with much lower power dissipation for a given peak gap voltage.
2. The cavity resonance frequency does not lie on a beam harmonic. Therefore the beam-induced signal is much smaller.
3. The growth rate for the same signal power is in excess of that for the corresponding FM scheme.

Figure 3 shows the rms emittance growth for several excitation frequencies $\omega_d = (4,6,8) \times \omega_s$. Because of the frequency doubling, one should compare FM at $4\omega_s$ with SS-AM at $8\omega_s$.

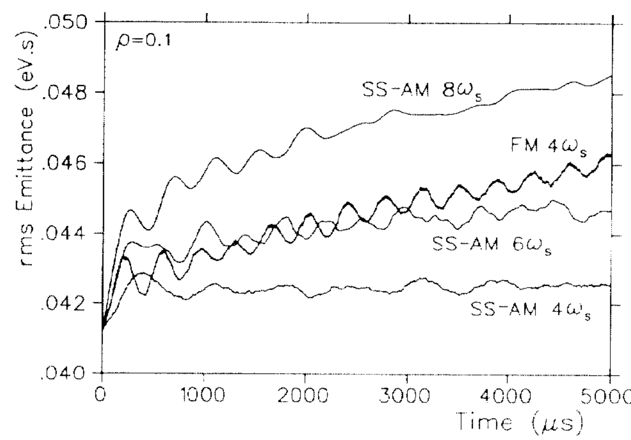

Fig.3 : Comparison of SS-AM with in-phase FM.

Nevertheless, one should be cautious that the HHC does not cause a longitudinal instability, and this possibility remains to be studied.

Dual-frequency excitation

The ability to selectively excite chosen m-polar moments is of course retained. Consequently one is led to consider simultaneous excitation at several frequencies. For instance a driving field of the form :

$$\rho\,[\sin(kh\omega_0 - 4\omega_s)t + \sin(kh\omega_0 - 8\omega_s)t]/\sqrt{2}\ .$$

As was observed for the case of phase modulation, it is imperative for the modulation frequency to track with the ensemble average synchrotron frequency. Figure 4 compares cases with and without such tracking. The frequency sweep rate is found by first running a case where the simulation code calculates the ensemble average synchrotron frequency at each time step and uses (a multiple of) this value for the modulation frequency. $\langle\omega_s\rangle$ is then fitted by a linear ramp $\omega_s(t)/\omega_s^0 = 1 - K_s t$ and this is used as a frequency law for a second run of LONG1D.

The KAON Factory Collector ring requires a four-fold emittance blow-up to be achieved in 20 *ms*. It is clear that $\rho = 0.1$ is adequate to the task, particularly as the bounding emittance grows more quickly than the rms. With two frequency components the peak voltage is $\rho\sqrt{2}$ times that of the main rf-system. The value is 256 kV to be divided between two cavity cells fed from a single klystron.

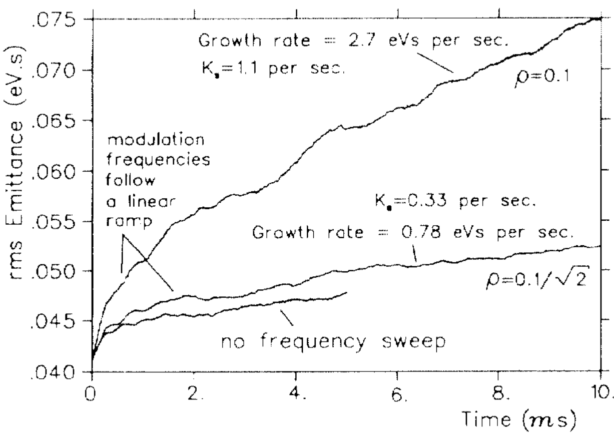

Fig.4 : Dual-frequency excitation for various signal power.

Examination of the phase space shows a remnant octapole component and a small core of particles unaffected by the blow-up. Both can be removed by the addition of a third frequency.

Triple-frequency excitation

The previous scheme does not make use of the cavity resonance at $kh\omega_0 - 6\omega_s$. Hence we consider:

$$\rho[\sin(kh\omega_0 - 4\omega_s)t + \sin(kh\omega_0 - 6\omega_s)t + \sin(kh\omega_0 - 8\omega_s)t]/\sqrt{3} .$$

Figure 5 shows the frequency components superimposed on the cavity impedance for the case of upper sideband excitation. The bandwidth is $4\omega_s$ or 32 kHz, and the $Q \leq 2.6 \times 10^4$. Assuming $R/Q \sim 100$ the beam induced voltage is found to be 125 kV per cavity, but this will fall as the bunch length increases.

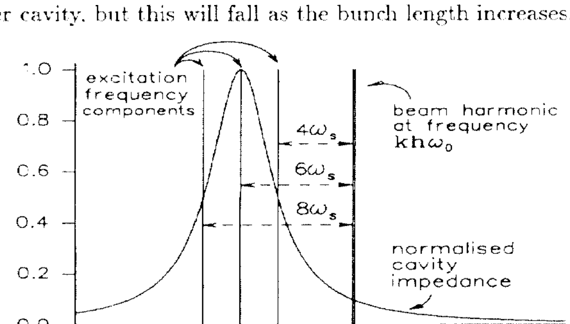

Fig.5 : Power spectra for triple-frequency SS–AM.

Figure 6 shows the blow-up to be smaller than for dual-frequency excitation of the same power, which suggests that the $6\omega_s$ excitation component should be reduced relative to the $(4,8)\omega_s$ components. It is anticipated that a total perturbing voltage of 300 kV should suffice. Figures 4 and 6 suggest that the blow-up scales roughly as the power in the driving field, that is as ρ^2 ; so that a small increase in the HHC gap voltage can give a substantial gain in growth rate.

Conclusion

Single sideband excitation is the most effective emittance blow-up scheme yet devised for the KAON Collector ring. Frequency sweeping is essential to maintain the resonance condition.

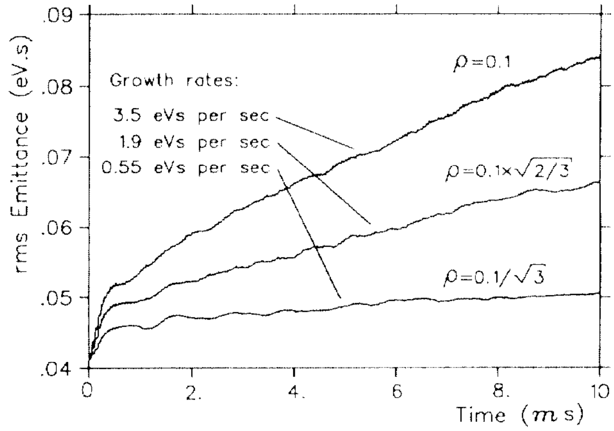

Fig.6 : Triple-frequency excitation for various signal power.

A frequency variation $\Delta\omega_s/\omega_s^0 = 0.02$ should be allowed. Multiple modulation frequencies are useful in tailoring the bunch shape, and a cavity bandwidth $\geq 4\omega_s$ is advised. Definitive optimisation, which could take many months of computer simulations, can best be done experimentally with the beam and high harmonic cavity in a few hours. Nevertheless, simulations have shown the important parameters. To allow full scope for this optimization, it is recommended that independent excitation at 4, 6 and 8 times the synchrotron frequency be allowed, with independent amplitude laws $\rho(t)$ and individual frequency ramps $\omega_d(t)$ for each frequency component.

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