Simplified Calculations for Beam Optics and Dynamics of the Saclay Superconducting Linac Booster

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Abstract

This paper will review the simplified programs used for on line simulmations of the radial and longitudinal behaviour of the heavy ion beams inside the helix loaded resonators. These programs are simplified versions of more accurate and general programsmade by P.M Lapostolle and S.Valéro {1}

1 Introduction

The booster design and the handling of the beam through the different sections of the Booster Linac have been described in detail in other papers {2},{3},{4}. .We will only recall the basic tools of the machine.The Saclay Booster is a superconducting Heavy ions Linac, using 50 helix loaded RF resonators made of bulk Niobium (4),(5). The injector is a 9MV FN Tandem The goals of the Booster is to increase the mass and energy ranges of heavy ion beams conserving the excellent beam qualities delivered by the Tandem Accelerator. (Small radial emittance and energy spread), and to reach the same energies as those achieved by a 25 MV Tandem; namely 7 MV/m for A=10 to 7MV/m for A=80. The intensity is, of course,strongly dependant on the ion source, charge selection at the strippers and may vary from 10 to 200nA. The main interest of such a machine is, as mentionned the excellent beam qualities, but also that the beam energy is continuously variable.

Now the RF booster which is the first superconducting heavy ion accelerator in Europe has already delivered several hundred hours of beams. Five beam extensions are used for different physics experiments (Irradiation for resistivity analysis or semi-conductor damage, γ ray spectroscopy, fast fission...)

2-Beam optics

2.1 Description of the booster

The design of the linac beam lines can be divided in 5 sections.

1- An achromatic S bend section which is the injection line between the Tandem and the first accelerating resonator
2- A first accelerating leg composed of 3 successive 8 resonators cryostats
3- An achromatis U bend section
4- The second accelerating leg (equivalent to the first one)
5- A L bend section which is the "analysing" line towards the experimental areas

2.2 Transverse motion inside a resonator

These calculations were performed, by Valero et al.{6},improved and generalized later {1},{7}. We give here a simplified model of beam optics inside a resonator; assuming that the resonator is running in a $TM_{0,1,n}$ mode which implies cylindrical symetry along the optical axis . In cylindrical coordinates the electric and magnetic fields solutions of Maxwell equations can be writen:

$$E_z(r,z)=\cos(\omega t+\varphi)\sum_1^n A_i I_0(k_i r)\cos(\lambda_i z)$$

$$E_r(r,z)=\cos(\omega t+\varphi)\sum_1^n \lambda_i\frac{A_i}{k_i}I_1(k_i r)\cos(\lambda_i z)$$

$$B_\theta(r,z)=\sin(\omega t+\varphi)\sum_1^n \frac{A_i}{k_i}I_1(k_i r)\sin(\lambda_i z)$$

$$\text{With } \lambda_i=\frac{\pi i}{L} \quad\text{and}\quad k_i=\sqrt{(\lambda_i^2-(\tfrac{\omega}{c})^2)}$$

L is the electric length, ω the RF pulsation, and ϕ the relative phase.

Since the geometry of our resonators is very complex, the electric field distribution along the optical axis (r=0), can only be determined experimentally by perturbation measurements. A least square fit of this distribution gives the coefficients A_i of the Fourier expansion.

The equations of motion are given by:

$$\frac{d^2r}{dt^2}=\frac{q}{m}(E_r(r,z)-v_z B_\theta(r,z))$$

$$\text{and}\quad \frac{dv_z}{dz}=\frac{1}{mv_z}E_z(r,z)$$

This differential system is solved by numerical integration, dividing the electric length in small steps of length h, where we assume that the longitudinal velocity v_z remains constant.

When $k_i r$ is small, $I_1(k_i r)$ can be approximated by:

$I_1(k_i r) \approx k_i r/2$

We then can write;

$$E_r(r,z)=rE^*_r(z) \qquad B_\Theta(r,z)=rB^*_\Theta(z)$$

Where $E^*_r(z)$ and $B^*_\Theta(z)$ are independent of r.

Then using Euler method, we have:

$r_{i+1}-r_i=hr_i'$

We can then write for the radial motion:

$$r'_{i+1}-r'_i=\frac{qr_i}{v_i^2}\int_{ih}^{(i+1)h}(E^*_r(z)-v_iB^*_\theta(z))dz$$

Which gives the expression of the transfert matrix from step i to step i+1 {6};

$$\begin{vmatrix} r_{i+1} \\ r'_{i+1} \end{vmatrix} = \begin{vmatrix} 1 & h \\ \frac{-1}{f} & 1 \end{vmatrix} \times \begin{vmatrix} r_i \\ r'_i \end{vmatrix}$$

$$\text{With}\quad -1/f=\frac{q}{mv_i^2}\int_{ih}^{(i+1)h}(E^*_r(z)-v_iB^*_\theta(z))dz$$

The matrix of the whole resonator is obtained by multiplication of all the step matrices among the electric length L. The coefficients of the matrix resonator can be approximated by;

$R_{11} \approx 1 \quad R_{12} \approx L \quad R_{22} \approx 1 \quad R_{21} \approx -1/F$

F is a function of the ion velocity and of the relative RF phase. Each cavity has a defocussing effect in the radial phase planes (cf.fig1)

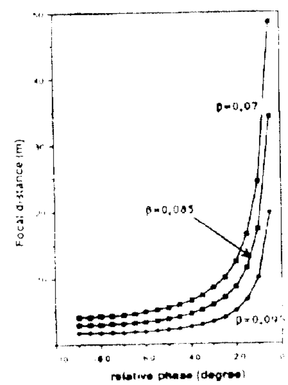

Fig.1 Defocusing effect of the resonator versus the relative RF phase $\Delta\phi$ for various ions velocities. ($\Delta\phi = \phi - \phi_0$) where ϕ_0 corresponds to the maximum energy gain

A"soft" refocussing with the solenoids is used to reduce the radial oscillations of the beam envelope inside the cryostat (cf fig2).

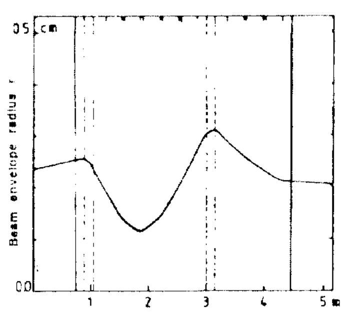

Fig.2 Beam envelope (including 95% of the beam) through a cryostat housing 8 resonators and 2 solenoids (dotted lines)

2.3 Control of the radial phase space along the booster lines.

From Liouville theorem, we know that the beam emittance $\varepsilon = \beta\gamma\varepsilon_t$ remains constant as far as the electromagnetic field derives from an .Hamiltonian ($\beta = v/c$, $\gamma = 1/\sqrt{(1-(v/c)^2)}$, ε_t is the surface of the beam ellipse in the radial phase space). For non relativistic particles ($\gamma = 1$), it means $\varepsilon_t\sqrt{E}$ is constant (E is the beam energy) In fact, because of the angular straggling due to the strippers, mounted in the Tandem terminal and in the S bend section, ε_t increases with the mass of the heavy ions beams(2). Thus the beam emittance ε_t will decrease inside the booster linac. Using the "3 gradients method"; we have measured the radial emittance of heavy ions beams at the entrance of the booster, after the first accelerating leg, and after acceleration with all the resonators (two legs). These measured emittances decrease as the square root of the beam energy.

To control beam optics along these beam lines, we dispose of triplets of quadrupoles used to realize the waists along the booster lines, and the achromaticity of the S and U bend sections Several beam profile monitors , faraday cups and gold targets are also necessary to optimize the beam transport, the transmission and to control, by elastic scattering of the beam on the gold target, the time and energy spread of the beam.

3. Longitudinal motion in a resonator

The complete expression of the energy gain ΔW and of the phase shift $\Delta\Phi$ due to the electric accelerating field in the resonator has been calculated (1). The coupling of the longitudinal and transverse phase spaces comes from second order terms in r^2 and rr'.that we neglect in our calculations.

The energy gain for a resonator can then be written

$$\Delta W = q(T\cos\phi - S\sin\phi)$$

and the phase shift :

$$\Delta\varphi = -q(T'\sin\phi + S'\cos\varphi)$$

With

$$T(k) = \int_0^L E(z)\cos(kz)\,dz$$

$$S(k) = \int_0^L E(z)\sin(kz)\,dz$$

$$T'(k) = \frac{\delta T}{\delta W} \qquad S'(k) = \frac{\delta S}{\delta W} \qquad k = \frac{\omega}{v}$$

f is the phase v the velocity at the entance of the resonator These expressions derive from an Hamiltonian :

$$H = -q(T\sin\phi + S\cos\phi)$$

$$\frac{\delta H}{\delta W} = \Delta\phi \qquad \frac{\delta H}{\delta\phi} = -\Delta W$$

Then the complete matrix of a resonator in the $(\Delta t, \Delta E)$ phase space is the product of matrices corresponding to the accelerating field inside the resonator and to the drift space equivalent when there is no field :

$$\begin{pmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{pmatrix} = \begin{pmatrix} 1 & L(E) \\ 0 & 1 \end{pmatrix} \times \begin{pmatrix} 1+\frac{\delta\Delta t}{\delta t} & \frac{\delta t}{\delta W} \\ \frac{\delta W}{\delta t} & 1+\frac{\delta W}{\delta W} \end{pmatrix}$$

Where $L(E) = \frac{10{,}828 \times L}{c} \times \sqrt{\frac{A}{E^3}}$ (A is the mass in a.m.u, E the beam energy in MeV, c is the light velocity in m/s)

The matrix has a determinant equal to :

$$\Delta = 1 + \frac{1}{\omega^2}\left(\frac{\delta^2 H\,\delta^2 H}{\delta^2 W\,\delta^2 t} - \left(\frac{\delta^2 H}{\delta t \delta W}\right)^2\right)$$

Valero et al (1) has shown that in accelerating conditions

$\Delta \approx 1$ .Then the beam ellipse surface in the longitudinal phase space remains constant.

This first order simulation of the longitudinal motion inside a resonator does not include the coupling between radial and longitudinal phase spaces which can be neglected as far as the beam size in the resonator is less than 1cm. This "on line" simulation should allow us to increase the transmission of the beam in the linac. Thus the phase of each resonator has to be optimized to

-reduce the spread in time of the beam to the phase acceptance of the resonators all along the accelerating legs

-have a better control of the energy spread of the accelerated beam at the end of each leg.

Conclusions

For light ions (up to mass A = 40 amu), the calculations in the radial and longitudinal phase spaces are in good agreement with the set up values, and many ions have already been accelerated {4}; for heaviest ions, the simplified "on line" simulations described above should help us to increase the transmission through the booster.

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