Stability of Channel Guided Laser Propagation *

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Abstract

The stability of short laser pulses propagating through plasma channels is investigated theoretically. Perturbations to the laser pulse are shown to perturb the ponderomotive pressure, which modifies the dielectric properties of the plasma channel. The channel perturbation then further distorts the laser pulse. Mechanisms for suppressing the instability are discussed.

1 INTRODUCTION

The stable propagation of laser pulses in underdense plasmas is fundamental to the development of laser wake-field accelerators. The laser pulses must be focused to a small spot size in order to generate a large amplitude plasma wave, and, thereby, a high accelerating gradient [1]. The laser will, in free space, be focused only over a diffraction length $Z_r = \omega\sigma^2/2c$, where ω is the laser frequency, c the speed of light, and σ the laser waist at the focus. A homogeneous plasma, which has a dielectric constant $\epsilon = 1 - \omega_{p0}^2/\omega^2$, where $\omega_{p0} = 4\pi e^2 n_0/m$ is the electron plasma frequency, $-e$ the electron charge, m the electron rest mass, and n_0 the plasma density, will only enhance the tendency of the light to diffract. To achieve a net acceleration of, say, 10 GeV, will require, with present terawatt lasers, propagation lengths of order 10-20 Rayleigh ranges, and TeV accelerators using a single laser would require hundreds of Rayleigh lengths. For overall efficiency reasons, the propagation lengths must be long enough for a substantial fraction of the laser energy to be converted into plasma oscillation. This will require propagation over many diffraction lengths.

Several schemes have been proposed to overcome diffraction. Relativistic guiding [2] relies on the energy dependence of the plasma frequency, $\omega_p^2 = \omega_{p0}^2/\gamma$, where $\gamma = \sqrt{1+\vec{p}\cdot\vec{p}/m^2c^2}$. The electron momentum $|\vec{p}\,|$ will be largest where the laser pulse is most intense, and therefore the plasma frequency will be lower there, and the pulse will generate a nonlinear index of refraction which is larger at the center of the pulse than at the pulse edges. Analysis has shown that, in steady-state, relativistic guiding can focus the pulse whenever the total power is greater than $P_c = 16.2(\omega/\omega_p)^2$ GW.

For pulses of order a plasma wavelength, however, relativistic guiding is substantially reduced [3]. An alternate scheme envisions guiding the laser pulse in a plasma density channel. The channel has a higher density on the outside than on the inside, resulting in an index of refraction of the plasma which decreases from the channel axis (due to the increase in plasma density). A fixed plasma channel is analogous to an optical fiber, and its guiding properties can be similarly analyzed. The plasma channel can be used to guide short pulses, and has been studied using axisymmetric models for parabolic density variation [3] and for hollow channels [4].

This paper considers the dynamic stability of channel guided pulses in the presence of plasma wakes. A perturbation to the guided equilibrium leads, through the ponderomotive force, to a plasma density perturbation which, in turn, couples back to the perturbed field. Thus, the plasma couples different longitudinal slices of the laser pulse. For example, a transverse instability of channel guided pulses occurs when the laser pulse is initially not centered on the channel axis. The underlying physics is straightforward: the off-centered laser produces a ponderomotive force with a dipole component; this causes the surrounding plasma electrons to try to follow the laser pulse. Thus the shape of the channel is distorted and its guiding properties are perturbed. There will be a coupling between higher order multipoles, so that the back of the laser pulse will widen.

2 THEORETICAL MODEL

With a quadratic density variation, the problem can be solved exactly. The physical model consists of a preformed neutral plasma channel with an unperturbed density given by

$$n_0(\vec{x}_\perp) = \bar{n}_0(1 + \frac{r^2}{W^2}), \tag{1}$$

where $r^2 = x^2 + y^2$. Since the duration of the laser pulse is assumed to be short compared to $2\pi/\omega_{pi}$, where $\omega_{pi}^2 = 4\pi e^2\bar{n}_o/m_i$, the ions can be considered immobile. Furthermore, the laser frequency is much larger than the plasma frequency, so that the evolution of the laser pulse, caused by the electron density wake, occurs on a time-scale much longer than the laser period. Thus, we consider an averaged (over a laser period), slow time-scale, weakly relativistic equation of motion for plasma electrons under the influence of the ponderomotive force of the laser field.

A fluid model, which is applicable before wave-breaking has occurred, is adequate to describe the plasma evolution for the short pulse duration of interest here. In particular, a plasma electron must have a thermal velocity in the

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plasma must be much less than its oscillatory velocity in the laser.

The channel density is taken to vary over a distance much larger than collisionless plasma skin-depth, so that $K = c/\bar{\omega}_p W \ll 1$. Then, as shown below, the unperturbed laser pulse has a spotsize $2w = 2\sqrt{Wc/\bar{\omega}_p} \ll W$, so that the density does not vary appreciably in the region where the ponderomotive force is nonzero.

We consider a circularly polarized radiation field $\vec{A} = mc^2\vec{a}/e$, where

$$\vec{a} = \frac{1}{2}(a(x_\perp, z, t))(\hat{e}_x + i\hat{e}_y)\exp\left(i(k_0 z - \omega_0 t)\right) + cc. \quad (2)$$

Introducing the variables $s = t - z/v_{g0}$, $z = z$, and using the eikonal approximation, the weakly relativistic limit ($|\vec{a}|^2 < 1$), and $\omega_p^2 \ll \omega^2$ results in

$$\left(\frac{\omega_0^2}{c^2} - k_0^2 + \nabla_\perp^2 - \frac{\omega_p^2}{c^2}\left(1 - \frac{|a|^2}{2}\right) + 2ik\frac{\partial}{\partial z}\right)a = 0, \quad (3)$$

where $v_{g0}/c = k_0 c/\omega_0$. The amplitude is expanded as the sum of the unperturbed guided laser pulse and a perturbation driven by the generation of plasma density modulations: $a(\vec{x}_\perp, z, t) = a_0(\vec{x}_\perp, s) + a_1(\vec{x}_\perp, s, z)$. We use the dimensionless transverse coordinates $\bar{x} = x/w, \bar{y} = y/w$, and $\bar{\nabla} = w\nabla$. Retaining leading order terms in Eq. 3 then results in

$$(-\bar{\nabla}_\perp^2 + \bar{r}^2)a_0(s, \vec{x}_\perp) = w^2\left(\frac{\omega_0^2}{c^2} - k_0^2 - \frac{\bar{\omega}_p^2}{c^2}\right)a_0(s, \vec{x}_\perp). \quad (4)$$

Since the primary concern in this paper is guiding by a channel and not relativistic self-focusing, we will assume that the laser power is below the self-focusing threshold P_c. This allowed us to neglect the nonlinear terms in Eq.(4).

The calculation proceeds by linearizing Eq.(3) and expanding the perturbation a_1 in a complete set of transverse eigenfunctions ψ_n^m which can be expressed, in cylindrical coordinates (r, θ), as

$$\psi_n^m(r, \theta) = \exp\left(-\bar{r}^2/2\right)\bar{r}^m L_n^m(\bar{r}^2)\exp\left(im\theta\right), \quad (5)$$

where L_n^m are the modified Laguerre polynomials [5].

The unperturbed equilibrium profile a_0 can be taken as

$$a_0(s, z, \vec{x}_\perp) = \bar{a}_0(s)\exp\left(-\bar{r}^2/2\right), \quad (6)$$

which corresponds to the lowest ($m = 0, n = 0$) eigenfunction.

The equation for the perturbed field is

$$[(2 + \bar{\nabla}_\perp^2 - \bar{r}^2) + 2ikw^2\frac{\partial}{\partial z}]a_1 = \frac{\omega_{p0}^2}{c^2}w^2\left(\frac{\delta n}{n_0} - b\right)a_0. \quad (7)$$

where $b = (a_0^* a_1 - a_0 a_1^*)/2$

The equation for the density modulation δn is given by [6]:

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{p0}^2\right)\frac{\delta n}{n_0} = c^2\nabla^2 b. \quad (8)$$

Equation (8) for $\delta n/n_0$ can be broken into a longitudinal and a transverse piece by use of the quasistatic approximation, $c^2\nabla_\parallel^2 \approx \partial^2/\partial s^2$.

Changing Eq. (8) into an integral equation, integrating twice by parts, and inserting the resulting expression for $(\delta n/n_0)$ into Eq.(7), yields

$$\left(2 + \bar{\nabla}_\perp^2 - \bar{r}^2 + 2ikw^2\frac{\partial}{\partial z}\right)a_1(s) =$$
$$a_0(s)\int_{-\infty}^{s}\omega_{p0}ds'\sin\omega_{p0}(s - s')\left(\bar{\nabla}_\perp^2 - \frac{1}{K}\right)b(s', z). \quad (9)$$

Here both a_0 and a_1 are implicitly assumed to depend on $\vec{x}_\perp$.

The fundamental has $m = 0$, so that perturbed modes with differing azimuthal numbers are decoupled. Thus, we can concentrate on the evolution of a particular azimuthal mode with an arbitrary radial profile. This profile can be decomposed as a weighted sum of radial eigenmodes with the same azimuthal mode number:

$$a_{(1)}^m(\vec{x}_\perp, z, s) = \sum_{n=0}^{\infty}\bar{a}_n^m(z, s)\psi_n^m(\bar{r}, \theta). \quad (10)$$

It is convenient to introduce dimensionless time and space coordinates, normalizing them to a plasma period and Rayleigh length, respectively: $\bar{s} = \omega_{p0}s$, $\bar{z} = z/kw^2$ and to set $b_n^m = \bar{a}_0\bar{a}_n^{m*} + \bar{a}_0^*\bar{a}_n^m$. Then multiplying Eq.(9) and its complex conjugate by $\psi_{n_1}^m(r, \theta)$, integrating over the transverse dimensions and making use of the orthogonality condition for Laguerre polynomials [5] results in

$$\left(2 - \lambda_{n_1}^m + 2i\frac{\partial}{\partial \bar{z}}\right)\bar{a}_{n_1}^m(\bar{s}, \bar{z})\frac{(n_1 + m)!}{n_1!} =$$
$$= \frac{\bar{a}_0(s)}{2}\sum_{n_2}G_{n_1 n_2}^m\int_{-\infty}^{\bar{s}}d\bar{s}'\sin\left(\bar{s} - \bar{s}'\right)(b(\bar{s}', \bar{z}) \quad (11)$$

and its complex conjugate. Here $\lambda_n^m = 2 + 2m + 4n$. The coupling coefficients can be calculated explicity using relations between Laguerre polynomials [5]:

$$G_{n_1 n_2}^m = -\frac{L!}{2^{L+1}n_1!n_2!}\left(\frac{1}{K} + \frac{L+1}{2}\right), \quad (12)$$

where $L = n_1 + n_2 + m$. Eq.(11) can be used to find the evolution of the instability for a finite duration pulse. If $\bar{a}_0$ has an arbitrary longitudinal profile, these equations need to be solved numerically.

3 MATRIX DISPERSION RELATION

Further analytical progress may be achieved by assuming that either (i) $\bar{a}_0$ varies slowly on a time-scale of a plasma oscillation, or (ii) that $\bar{a}_0$ has a flat-top profile. We speculate that for pulses where relativistic guiding effects play some role, that this approximation will give an over estimate of the growth for a short pulse. (See the discussion of BNS-like damping below). Equation (11) can then be solved by Fourier transform in s.

The result is a set of coupled differential equations

$$\left(\frac{\partial^2}{\partial \bar{z}^2} + \tilde{m}^2\right) b^m_{n_1} = \frac{|\,\bar{a}_0^2\,|}{\tilde{m} 2^{m+2}(1-\bar{\omega}^2)} \sum_{n_2} A^m_{n_1 n_2} b^m_{n_2}, \quad (13)$$

where $\tilde{m} = m + 2n_1$, $\bar{\omega}$ is normalized to ω_{p0}, and $A^m_{n_1 n_2} = (L!/(((m+n_1)!n_2!)2^{n_1+n_2}))(1/K + (L+1)/2)$. Eq.(13) describes the evolution of any initial perturbation of a flat-top laser pulse. As it is obvious from Eq.(13), all the radial modes are coupled to each other. This is a consequence of both the nonlinear nature of the laser-plasma interaction and the finite transverse size of the unperturbed equilibrium.

4 LASER-HOSE INSTABILITY

It is instructive to examine the evolution of a laser pulse which is initially displaced from the center of the channel as a "rigid body", which corresponds to the $(m,n) = (1,0)$ mode. The coupling of this mode to other modes decreases rapidly with the radial mode number n, so that we can approximate the evolution of the instability by keeping only the diagonal element A^1_{00}. A more accurate treatment would involve keeping a finite number of modes and diagonalizing the resulting matrix.

The dispersion relation for the dipole mode, obtained by keeping only the diagonal element, is:

$$k^2 = \left(1 - \frac{\mu}{1-\omega^2}\right), \quad (14)$$

where $\mu = |\,\bar{a}_0^2\,| /8 \left(1 + \frac{1}{K}\right) \approx |\,\bar{a}_0^2\,| /(8K)$.

Asymptotic behavior of the solution for $\bar{z} \geq 1, \bar{s} \geq 1$ can be obtained from Eq.(14) by an inverse Laplace transform and a steepest descent integration (see, for example, [7]) in regimes that are delineated by relations between the length of the pulse s, the interaction length z and the coupling parameter μ. For the analysis presented above to be valid, $\mu \ll 1$. With $z_R = k_0 w^2$ and returning to dimensional variables, the asymptotic amplitude in the short pulse regime is:

$$\omega_{p0} s \ll \mu(z/z_R)$$

$$\bar{a}_1 \sim \bar{a}_{10} \exp\left[\frac{3\sqrt{3}}{4}\,|\,a_0^2/(8K)\,|^{1/3} (z/z_R)^{\frac{1}{3}} (\omega_{p0} s)^{\frac{2}{3}}\right] \quad (15)$$

Expressions valid in other regimes and details of the calculation will be published elsewhere [8].

Some of the conclusions drawn in this paper rely on the quadratic radial variation of the plasma density. Another simple model [4], which also has better accelerating properties, is an inverted step-function radial density dependence, which, due to finite variation in the plasma density, will have only a finite number of discrete eigenmodes. By a careful choice of parameters, the unstable modes can perhaps be pushed into the continuum, which may reduce the instability. Furthermore, since each mode has a distinct phase velocity the coupling between modes may also provide a natural mechanism for the instability to damp.

The perturbation has, in the various regimes, an exponential spatio-temporal growth rate proportional to $(z/z_R)^p(\omega_{p0}s)^q$, where $p + q = 1$. It is not surprising that this behavior is similar to that seen in beam break-up instabilities encountered in linacs–here the laser pulse is somewhat analogous to an electron beam and the plasma channel to the metallic structure in which the electron beam propagates. While the analogy proves useful in understanding the physical picture, and in developing suppression techniques similar to BNS damping, the details of the interaction in the two cases are different. The similarity between the laser-hose and electron-hose [7] instability can also be seen by choosing the perturbed field a_1 to be of a dipole type and interpreting the expression $(a_0^* a_1 + a_0 a_1^*)$ as the transverse displacement of the beam. By analogy with BNS damping, a longitudinal variation in the "betatron frequency," given here by $1/Z_r$, should reduce the growth rate. We have analyzed [8] two mechanisms for achieving this, either by introducing a frequency chirp on the pulse or using the natural variation that occurs from the nonlinear relativistic guiding. Future investigations need to address the critical issue of how these plasma instabilities, even if substantially suppressed, will affect the final energy and emittance of an accelerated bunch.

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