Excitation of Non-linear Plasma Waves for Electron Acceleration

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Abstract

The excitation of travelling non-linear Langmuir plasma waves by a relativistic electron bunch is considered. Large amplitude longitudinal electric fields specific for non-linear excitation regime provide a high acceleration rate for the trailing electrons. An analytical theory of the process is developed. The non-linear excitation is described as relativistic oscillations of the momentum of the plasma electrons in an effective potential well produced by the bunch and by the perturbation of the electron density of the plasma. A condition for the optimal excitation of the wave is found which yields a relation between the density of the plasma, the density of the bunch and the length of the bunch. Numerical results support the theoretical analysis and demonstrate a deep, non-linear modulation of the electron plasma density and the excitation of a large amplitude longitudinal field at realistic plasma and electron bunch parameters. The back action of the wake field on the bunch is also considered. Numerical results show a formation of sharp peaks in the bunch before its break-up.

1 INTRODUCTION

During the last decade plasma-based accelerator concepts have attracted much attention due to the possibility to reach ultra-high acceleration gradients, up to 1–10 GeV/m. A few wakefield schemes have been studied (see, e.g., review [1]). The most elaborated one is the beat-wave acceleration (PBWA) concept [2]. Recent experiments with CO_2 lasers at UCLA have demonstrated acceleration gradient of 0.7 GeV/m [3]. This concept relies on a linear, resonant process in a homogeneous plasma. This means that the density modulation remains small, and that the acceleration gradient is limited to $d\epsilon/dz < m\omega_p$ [1] where ϵ is the particle energy and ω_p is the plasma density.

The excitation of a longitudinal plasma wave by a single laser or electron pulse is a non-resonant process and does not require a strictly homogeneous plasma density. The electric field of a linear plasma wave in the wake of an electron bunch is restricted to the same range as the case of the PBWA concept [4–5]. Much higher electric fields, however, can exist inside plasma in the case of relativistic non-linear longitudinal oscillations of plasma electrons [6]. The process of the excitation of a non-linear Langmuir plasma wave (NLPW) by a relativistic bunch was considered in [7–8]. It has been shown that the profile of the density of plasma electrons and accompanying electric field sharply steepens when the density of the bunch approaches half the density of plasma electrons.

At the same time, the description formalizm used in [7–8] is based on an introduction of different dependent variables that does not allow to link all important parameters of the problem selfconsistently. In the present publication we consider a strightforward approach based on the behaviour of the momentum of the plasma electrons. Within this approach a direct connection between the density of plasma, the density of the bunch and the length of the bunch can be obtained. This method gives a clear physical picture and allows to maximize the field behind the bunch and, therefore, the acceleration gradient.

2 PLASMA WAVE EXCITATION

Let an electron pulse with density n_b travel with velocity v_b through a plasma with average electron density n_p. We are interested in plasma waves propagating at a relativistic phase velocity; hence, the plasma ions can be considered as immobile. The evolution of the pulse shape due to the back influence of the excited wake-field is neglected. Let our system be one-dimensional without magnetic fields. The Maxwell equations are in this case

$$\frac{\partial E}{\partial z} = 4\pi\rho, \qquad \frac{\partial E}{\partial t} + 4\pi j = 0 \tag{1}$$

where $\rho = e(\delta n_p + n_b)$ is the charge density and $j = e(\delta n_p + n_p)v + e n_b v_b$ is the current density, with δn_p being the perturbed electron density and v the fluid velocity of the electrons. The equation of motion of a plasma electron under the action of the field in the wake of the electron bunch is

$$\frac{dp}{dt} = eE \tag{2}$$

where p is the momentum of the plasma electron.

The driving electron pulse has a relativistic velocity v_b, and we will look for solutions of Eqs. 1–2 in the form of travelling waves $E = E(z - v_b t)$. Introducing a new variable $\xi = z - v_b t$, one has

$$\frac{dE}{d\xi} = 4\pi e(\delta n_p + n_b),$$
$$v_b\frac{dE}{d\xi} = 4\pi e[(\delta n_p + n_p)v(p) + n_b v_b], \tag{3}$$
$$(v(p) - v_b)\frac{dp}{d\xi} = eE(\xi),$$

where $v(p) = p/(p^2 + m^2)^{1/2}$.

[1] Relativistic units $h = c = 1$ are used throughout the paper.

One may note from here that for the type of perturbations under consideration the current density and the charge density are related by $j = \rho v_b$, which yields the relationship

$$\delta n_p = -\frac{n_p}{1 - v_b/v}. \tag{4}$$

To solve the above set of equations, we assume the profile of the density of the electron bunch to have a rectangular form

$$n_b(z,t) = \begin{cases} n_b & \text{for } -\tau < \xi < 0 \\ 0 & \text{for } \xi < -\tau, \xi > 0 \end{cases}$$

where τ is the length of the beam pulse. The plasma is initially unperturbed:

$$\delta n_p(\xi = 0) = 0, p(\xi = 0) = 0, \left(\frac{dp}{d\xi}\right)_{\xi=0} = 0, E(\xi = 0) = 0.$$

Let us introduce a new variable α through

$$d\alpha = \frac{d\xi}{v_b - v} \tag{5}$$

In terms of this variable, Eqs. 3 can be written as a single equation for the momentum p of the plasma electrons

$$\frac{d^2p}{d\alpha^2} = -\frac{\partial U}{\partial p}, \tag{6}$$

where the potential field is

$$U(p) = 4\pi e^2(n_p - n_b)(p^2 + m^2)^{1/2} + 4\pi e^2 n_b v_b p \tag{7}$$

Equations 6–7 describe two different regimes of the motion of plasma electrons, depending on the relation between the plasma density and the beam density. In the present paper we shall only discuss the case when the beam density is below the threshold of NLPW breakdown: $n_b < n_p/(1 + v_b)$. In this case the potential field (7) has a shape of an asymmetric well in which the motion of plasma electrons is finite. This means that there are oscillatory solutions for the motion of plasma electrons under the action of the electron beam pulse. These solutions move with the phase velocity equal to the beam velocity, $v_{ph} = v_b$. The oscillatory solutions exist even when the velocity of the plasma electrons becomes relativistic, $p \gg m$.

The process of the NLPW excitation starts from the point $\xi = 0, p = 0$ corresponding to the leading edge of the electron pulse. To get the highest amplitude of the oscillations behind the bunch, the pulse duration should last until plasma electrons reach the opposite turning point in the potential $U(p)$. It means the existence of some optimal bunch duration τ_0 for effective NLPW excitation. At this point plasma electrons have acquired the momentum p_m. The value of p_m can be found from the conservation law

$$\frac{1}{2}\left(\frac{dp}{d\alpha}\right)^2 + U(p) = U(0). \tag{8}$$

The derivative $dp/d\alpha$ vanishes at the point p_m, therefore

$$p_m \approx -2mv_b \frac{n_b(n_p - n_b)}{n_p[n_p - n_b(1 + v_b)]}. \tag{9}$$

The optimal bunch duration τ_0 can be found as

$$\tau_0 = \int_{p_m}^{0} \frac{v_b - p(p^2 + m^2)^{-1/2}}{[2(U(0) - U(p))]^{1/2}} dp \tag{10}$$

It is seen that p_m can be much larger then m if n_b tends to $n_p/(1+v_b) \approx n_p/2$. Therefore, if the density of the driving beam is close to but smaller than half of the plasma density, a non-linear Langmuir plasma wave can be excited. This wave is much more effective for electron acceleration then the linear LPW.

Behind the bunch $n_b = 0$, therefore the potential well $U(p)$ becomes symmetric but still anharmonic. Plasma electrons perform free nonlinear oscillations with amplitude p_m in this potential well . These oscillations correspond to nonlinear relativistic Langmuir plasma waves [6]. It follows from Eq. 4 that when the velocity of the plasma electrons v approaches the beam velocity v_b, the plasma density perturbation $\delta n_p/n_p$ can become larger than unity (one should note that the wave breaks down when the plasma fluid velocity reaches the bunch velocity, i.e. when $|p_m| \approx m/(1 - v_b^2)^{1/2} = m\gamma_b$). The longitudinal electric field of the plasma wave with such a density distribution is much larger than that of a linear Langmuir wave. The same is valid for the acceleration gradient, $d\epsilon/dz = eE$.

Since $eE = -dp/d\alpha$, the maximum field strength behind the bunch occures at $p = 0$. For $|p_m| \gg m$ one obtains

$$eE_{max} \approx (2m|p_m|\omega_p^2)^{1/2} \tag{11}$$

The plasma electrons momenta were shown to be limited with the quantity $m\gamma_b$. One can thus see from the above equation that the acceleration rate in NLPW is restricted to the value

$$eE_{max} < m\omega_p(2\gamma_p)^{1/2}$$

which is much larger than that for linear LPW.

3 NUMERICAL RESULTS

To confirm our analytical considerations, we have made numerical simulations of the process of NLPW excitation. Our 1D code is fully non-linear and includes the back influence of the excited plasma wave on the electron bunch. Numerical results demonstrate that an electron bunch with a density below the wave-breaking limit and with the optimized length can excite the NLPW effectively. Such nonlinear behaviour has also been reported in [8].

Fig.1 shows the density of plasma electrons and the longitudinal electric field behind a bunch. The energy of the electron beam is 50 MeV and the beam current density is 5 kA/cm^2. This corresponds to an electron density in the bunch of 10^{12} cm^{-3}. The bunch length corresponds to the optimal duration defined by Eq. 10 and equals to 3.6 cm. The plasma density is $2.5 \cdot 10^{12}$ cm^{-3}, so that $n_b = 0.8(n_p/2)$. It is seen that the excited plasma density modulation is much larger than unity and that the amplitude of longitudinal electric field exceeds more than two times the theoretical limit for the amplitude of a linear Langmuir wave.

The simulations confirm that when the bunch is either shorter or longer than the optimal length determined by Eq. 10, or when the beam density n_b deviates too much from $n_p/2$, the acceleration gradient reduces significantly. The dependence of the acceleration rate on the plasma density for fixed beam current density of 5 kA/cm^2 is shown in Fig.2.

Another important result concerns the evolution of the driving electron bunch during its travel through the plasma. Simulations show practically no significant change of the bunch shape until it has lost almost all its energy in the excitation of the NLPW. This is due to the fact that the velocity of a relativistic particle weakly depends on its energy.

Fig. 3 shows the shape of the bunch after a pathlength of 72.5 cm in the plasma of the density $3.3 \cdot 10^{12}$ cm^{-3}. During the first 60 cm the bunch keeps its initial rectangular shape. Hence, an electron moving in the wake of the bunch can be accelerated along a relatively long distance. After that distance under the action of the wake field sharp irregular peaks appear in the density distribution of the bunch. The excitation of the NLPW deteriorates and the amplitude of the wake field and the plasma density modulation decrease.

The results shown in the Fig. 3 contain also a promissing possibility of a new kind of bunch compression technique. After travelling a certain distance through plasma, very dense and short microstructures (microbunches) emerge on the background of the broadened initial density distribution. The electron density of these microbunches exceeds the initial bunch density more than an order of magnitude. At this stage the velocity spread in the microbunch is of order unity. But if the microbunches are quickly accelerated again, the final energy spread can be small, because all electrons acquire the same energy and an initial spread is related now to a much larger energy. This kind of beam-plasma interaction can thus be used for the production of short and dense electron bunches.

4 REFERENCES

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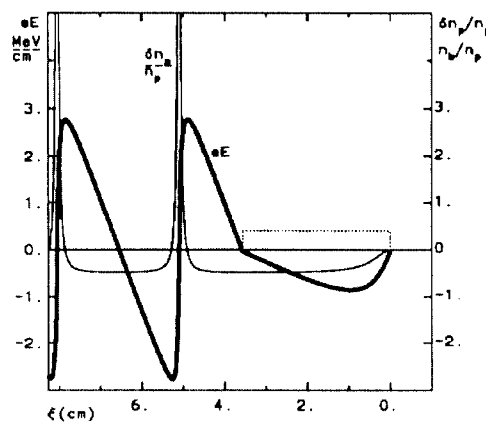

Fig.1. The distributions of the perturbed plasma density, $\delta n_p/n_p$, and the longitudinal electric field, eE, behind electron bunch (the bunch itself is shown with the dotted line); beam and plasma parameters see in the text.

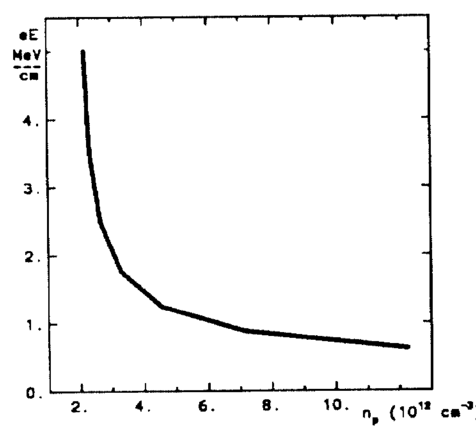

Fig.2. The acceleration rate as a function of the plasma density; beam current density is fixed at 5 kA/cm^2

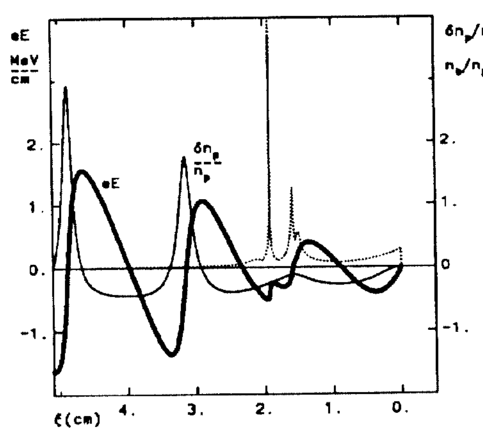

Fig. 3. Same as in Fig.1 but after a pathlength of 72.5 cm in the plasma of the density of $3.3 \cdot 10^{12}$ cm^{-3}; initial length of the bunch is 1.7 cm, initial electron density is $1 \cdot 10^{12}$ cm^{-3}.