

Acceleration of Two Ion Beams in Undulator Linear Accelerator *

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Abstract.

The simplest variants of undulator linear accelerator (UNDULAC) is discussed. Shielded pair of longitudinal electrodes are used as RF-system. Uniform electrodes are placed symmetrically in periodical magnetostatic fields (in undulator-M) parallel to the axis. Undulators of two types are under consideration: (i) plane undulator and (ii) axially symmetric undulator. The focusing and acceleration of one or two beams in the same undulator are considered. The special conditions correlating amplitude values of magnetostatic and RF fields were found.

1. INTRODUCTION

Earlier an idea to apply a combination of electrostatic undulator field (undulator-E) and radio-frequency field for acceleration of intense ion beams with low injection energy was suggested [1,2]. General expressions correlating the amplitude value of electrostatic undulator and RF fields were deduced. It was shown, how to choose the field configurations to provide effective focusing and acceleration of ribbon beam. The current in the electrostatic version of UNDULAC (UNDULAC-E) was shown to be increased using large width of the ribbon beam. UNDULAC-E is preferable at small values of initial ion energy ($W \geq 30$ keV for proton beam). The magnetostatic undulator might be substituted by electrostatic one in case of high injection energy [3]. It is impossible to obtain the large cross-section area because of technical problems of strong magnetic fields formation in large volumes occupied by the beam. However there is the opportunity to accelerate more than one beam in a magnetic channel. Since in undulator linear accelerator (UNDULAC-M) drift tubes (DT) are absent, the task is to choose higher symmetry of the transverse radio-frequency and magnetostatic fields. The RF-system must have small transverse size to be located inside an undulator. Therefore it is preferable to use shielded multielectrodes line, where transverse electromagnetic (TEM) waves are traveling.

Two simple examples of UNDULAC-M with shielded-pair electrodes will be considered in this work.

2. PARTICLE MOTION EQUATIONS

Let us see the beam dynamics in the TEM waves and in the alternating-sign magnetostatic field. The two inner electrodes of the line have radii a and the shield has radius b , but their axes and axis of the shield are assumed to be coplanar and parallel (fig. 1).

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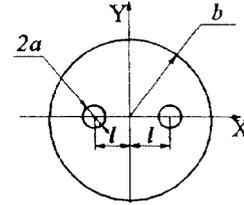


Figure 1. Cross-section of RF-system.

In Cartesian system wave components E_x and E_y can be expressed as:

$$E_{x,y} = E_{v,x,y}(x,y)\sin\varphi, \quad \varphi = \frac{2\pi z}{\lambda} - \omega t \quad (1)$$

The components of magnetic field are connected with E_x and E_y , as:

$$E_{v,x} = cB_{v,y}, \quad E_{v,y} = -cB_{v,x} \quad (2)$$

Only fundamental space harmonics of the undulator field will be taken into account.

$$B_x = B_u(x,y) \sin \int_0^z k_0 dz, \quad \vec{B}^{\perp} = \vec{B}_{u,1} \cos \int_0^z k_0 dz, \quad (3)$$

$$k_0 = \frac{2\pi}{\lambda_0(z)}$$

It is assumed, that quasiperiod of undulator increase slowly with coordinate. As it was shown [3], nonrelativistic motion equation averaged over quick oscillation can be written in the form:

$$\frac{d\vec{q}}{dt} = -\frac{\lambda}{8\pi} \vec{\nabla} U, \quad (4)$$

where the potential function

$$U = \vec{b}_x^2 + \vec{b}_y^2 - 2\vec{b}_x \vec{b}_y \sin\psi \quad (5)$$

Here

$$\vec{b}_{v,0} = \frac{e\vec{B}_{v,0}\lambda_{v,0}}{2\pi mc}$$

dimensionless amplitudes of transverse components of the wave magnetic field \vec{B}_x^{\perp} and undulator field \vec{B}_0^{\perp} , $\tau = \omega t$,

$$\psi = \omega \int \frac{dz}{v_x} - \tau + \psi_0$$

the particle phase in a combined wave field; ψ_0 - the initial phase. The reduced velocity of a synchronized particle is

$$\beta_x = v_x/c \approx \lambda_0/\lambda_v,$$

when $\lambda_v \gg \lambda_0$.

The acceleration gradient is proportional to the amplitudes of RF and undulator fields

$$\frac{dW}{dz} \sim \vec{b}_x \vec{b}_y \cos\psi.$$

The energy increase ΔW on the length λ_0 will be maximum, when $\vec{B}_x^{\perp} \parallel \vec{B}_0^{\perp}$. Therefore the choosing of magnetic undulator type and its orientation depends on the choosing of RF-structure. For the shielded-pair line (fig.1) two types of undulator can be used: (i) plane undulator (PU), where $B_x = 0$

(fig.2) and (ii) axially symmetric undulator (ASU) - a periodical sequence of axial magnetic lenses (fig.3). In absence of undulator the transverse RF field is focusing on x, defocusing on y and do not accelerate on z. Undulator without RF field is focusing on x and/or on y. The combined field of wave and undulator undulator is accelerating on z and defocusing on transverse direction. The total effect of transverse focusing by RF and undulator fields (RFU) is possible under definite conditions. The equilibrium trajectory exists, if

$$\nabla_1(\vec{b}^2 + \vec{b}_0^2) = 0, \quad \nabla_1(\vec{b}, \vec{b}_0) = 0, \quad (6)$$

The motion about equilibrium trajectory will be stable, if potential function U has the absolute minimum.

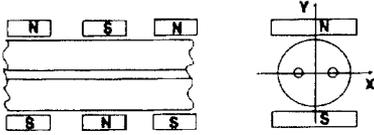


Figure 2. RF-system with plane undulator

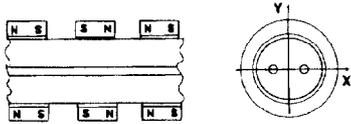


Figure 3. RF-system with axially symmetrical undulator.

3. RF SYSTEM

Let us consider the field, created by two longitudinal round electrodes with radius a . Each electrode is supplied by RF potential $\pm U \cos(\varphi)$. Main transverse geometrical sizes of RF system are shown on fig.1. For the first approximation consider screen radius $b \gg a$. For description of fields it is possible to use the method of image line charges, where electrodes are replaced with infinitely thin threads. Equipotential surfaces of charged threads coincided with real surfaces of electrodes in this case. The distances between threads $2h$ is defined through l and a by expression

$$2h = 2\sqrt{l^2 - a^2} \quad (7)$$

In Cartesian coordinates E_x, E_y components can be written as

$$E_{x,x}(x,y) = \frac{E_0 h}{2} \left[\frac{x+h}{(x+h)^2 + y^2} - \frac{x-h}{(x-h)^2 + y^2} \right] \cos \varphi, \quad (8)$$

$$E_{x,y}(x,y) = \frac{E_0 h y}{2} \left[\frac{1}{(x+h)^2 + y^2} - \frac{1}{(x-h)^2 + y^2} \right] \cos \varphi, \quad (9)$$

where E_0 is amplitude of RF field at $x=0, y=0$.

The method of image line charges enables to take into account the influence of screen under zero potential. For this purpose we need to add two charged threads in parallel axis on distances $x = \pm b^2/h$ from center. As a result new expressions for E_x and E_y containing four items instead of two occurs. The proposed method of simple analytical account of field distribution is in good agreement with results of exact numerical simulation.

The geometry of RF structure must be found. The main pur-

pose is to choose the optimum size of electrodes. It is necessary to provide the maximal field amplitude in beam area and minimal field amplitude beside the electrodes in order to reduce losses of RF power in walls. As simple analysis shows for trajectory of particles, passing close to the beginnings of coordinates, losses of RF power will be minimum at

$$0,5 \leq a/h \leq 0,9$$

and will slightly change in this interval. The optimum radius a will differ for the equilibrium trajectory passing outside the axis.

4. MAGNETOSTATIC UNDULATORS

For RF structure discussed two types of magnetic undulators PU and ASU having large transverse components B_x might be used. As it was mentioned above, arrangement of magnetic system should be so that the transverse component of magnetic undulator field \vec{b}_0 was parallel \vec{b} , near the equilibrium trajectory.

For PU

$$B_{0,y} = -B_0 \sum_{n=1}^{\infty} C_n \cosh nk_0 y \cos n \int_0^z k_0 dz_1, \quad (10)$$

$$B_{0,x} = B_0 \sum_{n=1}^{\infty} C_n \sinh nk_0 y \sin n \int_0^z k_0 dz_1$$

For ASU

$$B_{0,x} = -2B_0 \sum_{n=1}^{\infty} D_n I_1(nk_0 r) \cos n \int_0^z k_0 dz_1, \quad (11)$$

$$B_{0,z} = B_0 \sum_{n=1}^{\infty} D_n I_0(nk_0 r) \sin n \int_0^z k_0 dz_1$$

In the first case $B_{0,y} \neq 0$ on the axis of system, in the second one $B_{0,y} = 0$ and is increased with removal from the axis z reaching the maximum near poles.

Amplitudes of the highest harmonics of undulator field C_n, D_n ($n > 1$) are determined by form of poles and should be taken into account when the stability of transverse motion is studied.

The large magnetic fields might be obtained by reducing the transverse sizes of aperture or by bringing nearer equilibrium trajectory to magnetic poles when the beam is accelerated far from axis.

5. EQUILIBRIUM TRAJECTORIES

Consider now the realization of conditions (6) for PU. The conditions might be satisfied only if $x=y=0$. Hence, one trajectory, passing along the axis, exists. Focusing takes place for all ψ , if

$$k_0^2 h^2 b_0 (b_0 + b, \sin \psi) > 2b_0 (b_0 + b, \sin \psi) > 0 \quad (12)$$

To provide capture of particles in acceleration regime for all phases, two conditions should be satisfied.

$$b_0 = b, \quad \frac{2\pi h}{\lambda_0} > \sqrt{2} \quad (13)$$

Under these conditions the amplitude of accelerating combinational wave has one absolute minimum at point $x=0, y=0$. The first condition connects B_x and B_0

$$B_x = B_0 \lambda_0 / \lambda$$

The second condition restricts the distance between two electrodes

$$h > \frac{\lambda_0}{\sqrt{2\pi}} \equiv \frac{\beta\lambda}{\sqrt{2\pi}} \quad (14)$$

More interesting case corresponds to ASU. The potential function $U(x,y)$ has two minima, i.e. two equilibrium trajectories are exist: $x=0, y=y_0; x=0, y=-y_0$. The trajectories lie in (y,z) plane and coordinates $\pm y_0$ might be found from

$$\frac{d}{dy} \left(\frac{I_1(k_0 y)}{h^2 + y^2} \right) = 0 \quad (15)$$

The amplitude of wave field and undulator are correlated by condition

$$b_1^2/b_0^2 = \frac{2(h^2 + y_0^2)}{h^2 y_0^2} [I_1(k_0 y_0) I_0(k_0 y_0) - I_1(k_0 y_0)] \quad (16)$$

In elementary case, when $kh \ll 1, y_0 = h$. Thus the condition (16) might be rewritten as

$$b_1/b_0 = 2kh \quad (17)$$

While kh increases, the trajectories move away from the axis. For example, if $kh=0.5, y_0=1.07h$. Thus the equality (17) varies slightly. The analysis of potential function $U(x,y)$ shows that U has two absolute minima for all phases so that focusing conditions take place for two beams. This conclusion is great significance for problem of increasing of beam current in UNDULAC.

The choosing of optimum parameters for bunching and acceleration of two ion beams might be satisfied similarly to case of one beam [3].

6. CONCLUSION

The acceleration of one (PU) or two (ASU) beams is possible in UNDULAC, where RF structure is shielded pair of longitudinal electrodes. The absence of drift tubes and uniformity of RF-structure makes UNDULAC quite attractive.

The choice of functions $b_1(z), b_0(z)$ and $\lambda_0(z)$ could be made independently. The efficient acceleration and focusing of two identical beams in one channel may be realized. Beam bunching will be successive by means of geometrical size variation of system. The variation must keep the conditions (13) for PU and (15), (16) for ASU.

The acceleration of beams far from axis allows to approach the beams to the magnetic poles and to increase acceleration gradient. The field in resonator (standing wave) could be considered like the field in waveguide calculated above (travelling wave). In this case the longitudinal electrodes will act as vibrators of $\frac{1}{2}\lambda$ or $\frac{1}{4}\lambda$. The acceleration of more than two beams makes necessary to use more than two electrodes. For n electrodes in PU and ASU it allows to accelerate $(n-1)$ and n beams correspondingly.

7. REFERENCES

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