FEL microbunching of electron beam at half-wavelength of an energy modulating laser.

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Abstract

The bunching of a FEL energy-modulated electron beam at optical wavelength is studied assuming a drift space composed of a magnetic isochronous section. The effect of the second order term on the longitudinal distribution is considered to provide periodic modulation at the second harmonic of the laser wavelength. The beam quality requirements to obtain 80 nm bunching from a modulating excimer laser at 160 nm are defined.

1 INTRODUCTION.

In a FEL interaction the energy modulation impressed by the e.m. field laser of wavelength λ to an electron beam causes a density modulation at the same wavelength λ after drift through a dispersive section. In a standard Optical Klystron this happens in the magnetic field of the undulator itself. If the beam emittance is low, so that negligible longitudinal straggling is caused by transverse motion, more complex dispersive transport lines can be conceived [1], as shown in fig.1. Provided that at the end of the line $\eta, \eta' = 0$, the dispersive effect depends on the value of the first order TRANSPORT coefficient R_{56} given by the integral along the magnets

$$R_{56} = \int_M \frac{\eta}{\rho} ds \quad \begin{cases} = 0 & \rightarrow \text{isochronous line} \\ \neq 0 & \rightarrow \text{dispersive line} \end{cases} \tag{1}$$

Since the integral is positive in the first and last magnet, the minimal configuration for an isochronous line includes a third magnet where $\eta < 0$; this inclusion is shown in dashed line in fig. 1. At first order the displacement with respect to the unperturbed motion is

$$\delta l = a_1 \frac{\delta\gamma_{max}}{\gamma} \sin\psi_o \tag{2}$$

where $\delta\gamma_{max}/\gamma$ is the energy modulation depth and $\psi_o = (k + k_u)z - \omega t + \phi$ is the electron phase in the FEL pendulum-like ponderomotive potential resulting from the coupling of the laser field $\mathcal{E}cos(kz - \omega t + \phi)$ with the transverse velocity $-(K/\gamma)sin(k_u z)$ in the undulator. The coefficient a_1 used in eq. (2) is given by

$$a_1 = R_{56} - \frac{z}{\gamma^2} \tag{3}$$

The term R_{56} accounts for the longitudinal displacement caused by the trajectory dispersion in the bending magnets, and the last term accounts for the dispersive effect of velocity modulation $\delta\beta/\beta$ after a drift space of length z

$$\delta l_{drift} = z\frac{\delta\beta}{\beta} = z\frac{\delta\gamma}{\gamma^3} \tag{4}$$

The conditions to neglect the last term so that $a_1 = 0$ will be discussed later. Assuming $a_1 = 0$ the next most significant term of a series development in $\delta\gamma/\gamma$ is of course

$$\delta l = a_2 \left(\frac{\delta\gamma_{max}}{\gamma}\right)^2 \sin^2\psi_o \tag{5}$$

giving a density modulation at $\lambda/2$ as shown in fig. 2.

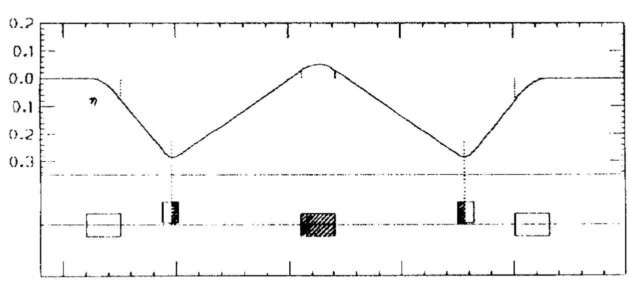

Figure 1: Schematic of magnetic transport line and dispersion function η for bunching. Dashed elements are to be included for the isochronous transport line which requires $\eta < 0$ in the middle magnet.

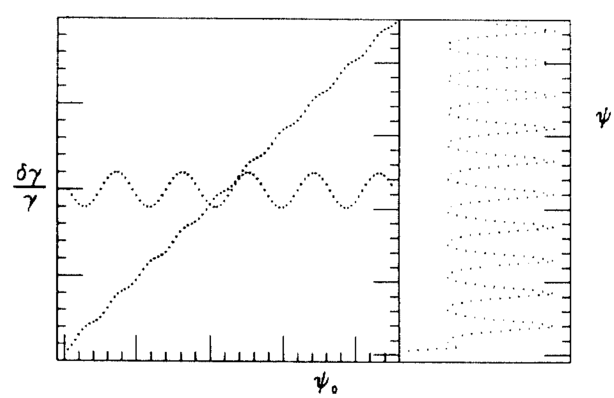

Figure 2: Energy modulation at ψ_o and longitudinal position ψ after an isochronous tranport line. The resulting longitudinal density modulation of an initially monoenergetic beam is shown in the vertical projection on the right side.

2 TRANSPORT LINE SPECIFICATIONS.

Eq. (5) can be valid, up to second order terms, only for an ultrarelativistic and zero emittance, monochromatic beam. In a real beam both the emittance ϵ_x, ϵ_y and the initial energy spread $\Delta\gamma/\gamma$ cause different path length, changing the amplitude of the Fourier components of the density modulation. The ideal beam description is appropriate if the effects of ϵ and $\Delta\gamma/\gamma$ are found negligible. This estimate implies the use of full second-order expansion of the dispersive effect of the transport line; eq. (5) is therefore substituted, using the TRANSPORT notation, by

$$\delta l = \sum_{ij} T_{5ij} x_i x_j \quad (i, j = 1, .., 6) \tag{5'}$$

where $T_{566} x_6 x_6$ is just the rhs of eq. (5). Limits to either the tranport matrix elements T_{5ij} or to the initial conditions $x_i \quad (i, j = 1, .., 5)$ must be set so that their overall contribution is $\delta l \ll \lambda/4$. The transverse phase space variables $x_i \quad (i, j = 1, .., 4)$ are easily related to beam emittance and optical funtion of the transport line, assuming for simplicity that the beam has a waist at $z = 0$, by

$$\begin{cases} x_1 = (\epsilon_x \beta_x)^{\frac{1}{2}} & x_3 = (\epsilon_y \beta_y)^{\frac{1}{2}} \\ x_2 = (\epsilon_x / \beta_x)^{\frac{1}{2}} & x_4 = (\epsilon_y / \beta_y)^{\frac{1}{2}} \end{cases}$$

No simple analytical expression is available for the matrix T of an isochronous transfer line, therefore the performances of the minimal configuration of fig. 1 have to be extimated from a specific design. A more complex line is described extensively in literature [2]; it is composed by 4 cells of a periodic structure ensuring by symmetry the cancellation of the transverse second order chromatic terms of single cell, so that $T_{hij} = 0 \quad (h = 1, ..., 5$ and i or $j = 5)$ The term T_{566} can still be tailored after these cancellations have been obtained. Thereafter it is assumed that all second order term linearly depending on energy can be neglected.

3 BEAM QUALITY REQUIREMENTS.

Bunching at short wavelength $\lambda \approx 80\,\text{nm}$ can be exploited in high power UV FEL design required in heavy-ion inertial fusion scheme [3], so the parameters are defined for this wavelength; for comparison both the beam parameters of the linac in [3] and those of the TESLA Test Facility (TTF) [4] are considered. The most relevant ones are listed in table 1. The TTF linac is conceived as a feasibility test bench in view of a future very large superconducting linear collider, but it could provide in a few years the best beam for UV-FEL experiment.

Table 1: Electron beam and undulator parameters for 80 nm bunching.

Beam parameters	Heavy ions fusion	TESLA Test Facility
Energy [MeV]	215	500
Duty cycle [%]	≤ 10	1
$<$ Current $>_{macro}$ [mA]	2	8
Inv. emitt. $\gamma\epsilon_x[\pi m \cdot rad]$	1.7×10^{-5}	1.0×10^{-5}
Inv. emitt. $\gamma\epsilon_z \pi m \cdot rad$	1.7×10^{-5}	1.0×10^{-5}
Energy spread	2×10^{-3}	1×10^{-3}
Undulator @ 160 nm		
period λ_u [mm]	28.3	153.2
length [m]	0.283	1.532

The longitudinal spread contribution by $\Delta\gamma/\gamma$ must be $\ll \lambda/4$, to avoid blurring of the modulation at $\lambda/2$. The energy modulation depth must be comparable with the energy spread, so $\delta\gamma_{max}/\gamma \approx \Delta\gamma/\gamma$. Assuming a realistic length $z \approx 10\,m$ for the isochronous transfer line eq. (4) sets the upper limit to $\Delta\gamma/\gamma$ vs γ at different value of the ratio $\lambda/\delta l_{drift}$ as shown in fig. 3. Assuming that the

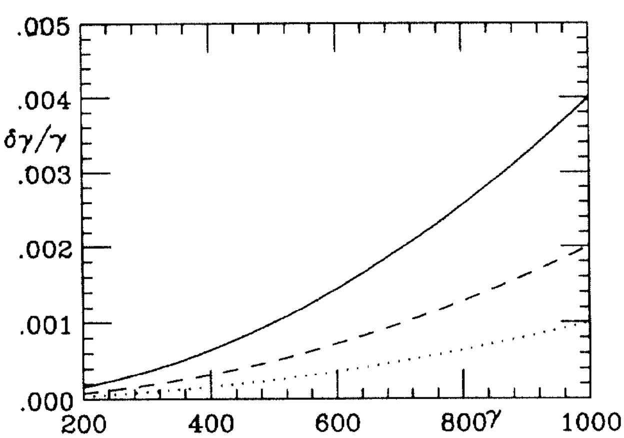

Figure 3: Upper limit to $\Delta\gamma/\gamma$ to have $\delta l_{drift}/\lambda < 1/4$ (continuous line), $\delta l_{drift}/\lambda < 1/8$ (dashed line), $\delta l_{drift}/\lambda < 1/16$ (dotted line)

optimal value of the undulator parameter $K_{rms} = 1.0$ the undulator period is given by the basic FEL relationship

$$\lambda = \frac{\lambda_u}{2\gamma^2}(1 + K_{rms}^2) \tag{6}$$

The modulator itself acts as a dispersive element for the being-modulated beam. When $\delta\gamma \neq 0$ in an undulator of length L_{mod} the dispersive effect is

$$\frac{ds}{d\gamma} = \frac{1 + K_{rms}^2}{\gamma^3} L_{mod} \tag{7}$$

Assuming that the energy modulation is linearly increasing along the modulator the maximum displacement is

$$\Delta s = \int_0^{L_{mod}} \frac{1 + K_{rms}^2}{\gamma^3} \frac{\delta\gamma_{max} s}{L_{mod}} ds = \frac{1 + K_{rms}^2}{2\gamma^3} \delta\gamma_{max} L_{mod} \tag{8}$$

and with substitution of (6)

$$\frac{\Delta s}{\lambda} < 2N \frac{\delta\gamma_{max}}{\gamma}$$

Since $\delta\gamma_{max}/\gamma < 0.01$ the condition $\Delta s/\lambda \ll 1$ is satisfied assuming $N = 10$.

4 MODULATING LASER.

The laser field required for the modulation increases with energy

$$\mathcal{E} = \frac{2mc_2}{eK_{rms}L_{mod}}\gamma^2\frac{\delta\gamma_{max}}{\gamma} \tag{9}$$

Assuming $K_{rms} = constant$ the γ^2 dependance of eq. (9) is cancelled replacing $L_{mod} = N\lambda_u = \lambda\gamma^2$ from eq. (6), so that

$$\mathcal{E} = \frac{2mc_2}{eK_{rms}\lambda}\frac{\delta\gamma_{max}}{\gamma} \tag{9'}$$

and the laser power density $\mathcal{P}$ is constant when $\delta\gamma_{max}/\gamma$ is constant . The laser power P_L depends on the electron beam section

$$\Sigma = \sigma_x\sigma_y = (\epsilon_x\beta_x)^{\frac{1}{2}}(\epsilon_x y\beta_y)^{\frac{1}{2}} = \frac{\epsilon_n}{\gamma}\frac{L_{mod}}{2}$$

where it has been assumed equal emittance in both planes, i.e. $\epsilon_x = \epsilon_y = \epsilon_n/\gamma$, and matched optical functions $\beta_x = \beta_y = L_u/2$ in the undulator. The condition $\delta\gamma_{max}/\gamma = constant$ must be reconsidered, since fig. 3 shows that a trade off is possible between electron beam quality and laser power. The curves in fig. 4 show the laser power density $\mathcal{P}\, vs\, \gamma$ and the peak power $P_L\, vs\, \gamma$ required to get $\delta\gamma_{max}/\gamma = \Delta\gamma/\gamma$ where $\Delta\gamma/\gamma$ has been defined by the condition that the longitudinal straggling $\delta l_{drift} = \lambda/4$ according to eq. (4) assuming a normalized beam emittance $\epsilon_n = 1. \times 10^{-5}$. Since the energy spread

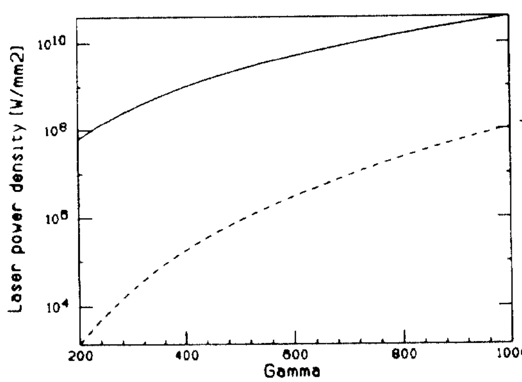

Figure 4: Laser power density $\mathcal{P}\, vs\, \gamma$ and laser power $P_L\, vs\, \gamma$.

requirement is very tight for low energy linacs, it would be easier to realize this FEL with higher energy machines. This also favours the reduction of emittance depending blurring. Of course, a trade-off must be obtained with the increase of cost of both the linac and the laser. However, the advantages of prebunching deserve a depeer study from the economical point of view. It is worthwhile noting that clean prebunching provides a localized distribution of electrons in the FEL pendulum-like phase space; subsequent evolution does not cause filamentation and the exausted beam can be easily driven to an energy recovery system.

5 BUNCHING IN AN ISOCHRONOUS TRANSFER LINE.

The density distribution $\rho(\psi, z)$ after the drift space of length z is determined by the time delay at z caused by the impressed energy variation $\delta\gamma = \delta\gamma_{max}\sin\psi$ according to eq. (5). Assuming that the spreading given by eq. (4) is negligible, the phase displacement on the scale of the laser wavelength λ is

$$\psi = \psi_o + \frac{2\pi\delta l}{\lambda} = \psi_o + \frac{2\pi}{\lambda}a_2(\frac{\delta\gamma_{max}}{\gamma})^2\sin^2\psi_o \tag{10}$$

Assuming an initially flat distribution ρ_o the final distribution is

$$\rho(\psi; z) = \rho_o\frac{d\psi_o}{d\psi} = \rho_o\delta(\psi,\psi_o)(1 + 2c_2\sin\psi_o\cos\psi_o)^{-1} \tag{11}$$

where $c_2 = a_2(2\pi/\lambda)(\delta\gamma_{max}/\gamma)^2$ and $\delta(\psi, \psi_o)$ is the Dirac function indicating that only the value of ψ_o satisfying the relation (10) must be considered. The Fourier series expansion of $\rho(\psi, z)$ gives fon the n^{th} harmonic component

$$\rho_n = \frac{1}{2\pi}\int_{-\pi}^{\pi}\cos\{n(\psi_o + c_2\sin^2\psi_o)\}d\psi_o \tag{13}$$

The shape of ρ_2 as a function of c_2 from eq. (13) is plotted in fig. 5 (continuous line) for comparison with the shape of $J_2(2c_2)$ (dashed line), which would have been obtained in a dispersive transport line.

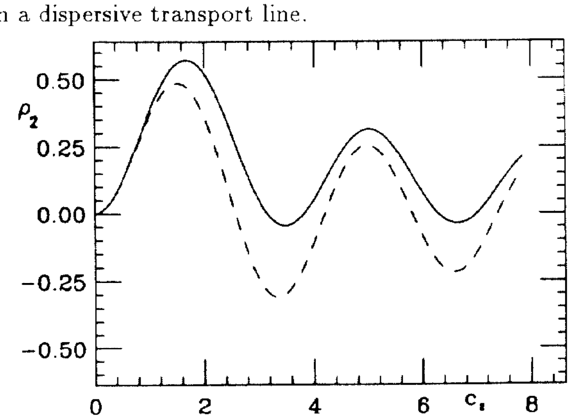

Figure 5: Amplitude oscillation of the harmonic component ρ_2 in an isochronous transport line (continuous line) and in a dispersive one (dashed line) .

6 REFERENCES

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