GIOTTO: an interactive program for the analysis of 2D area preserving mappings

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Abstract

A description of an interactive program for the visualization of the orbits of an area preserving mapping is given. The code includes zooming and coloring facilities, and advanced analytical and numerical tools for the analysis of the map dynamics such as the computation of normal forms, interpolating hamiltonians, fixed points, tune shifts, Fourier spectra and invariant manifolds emanating from the hyperbolic fixed points. Implemented models are both simple maps and complicated polynomial maps which can be defined by the user, representing more realistic lattice models. The program is based on the X-Window environment and on the OSF/Motif graphic interface, allowing to work with several windows and a friendly interaction with the user.

1 INTRODUCTION

Betatronic oscillations of a single particle in a magnetic lattice can be conveniently described by a series of symplectic mappings which represent the behaviour of the nonlinear elements [1]. Several analytical and numerical tools have been worked out in the past decades in order to analyse the dynamical behaviour of such models [2]. A first step towards the understanding of these dynamical systems is the analysis of symplectic 2D mappings (i.e. area-preserving mappings).

In this paper we briefly outline the basic features of GIOTTO, which is a graphic interactive program for the analysis of two dimensional area-preserving maps. This code consists of three parts: a tracking module which computes the iterates of the map (complicated lattices given as a MAD input file [3] can be analysed), a module which computes the indicators of the nonlinear motion such as tunes, width of resonances and perturbative series, and a graphic module for the visualization of the phase space portrait and of the nonlinear indicators. The X-window environment of the code provides a friendly interface with the user.

The aim of the authors is twofold: on the one hand, to create a tutorial code that allows any user without a specific mathematical background to familiarize oneself with the basic concepts of 2D nonlinear hamiltonian discrete dynamics. On the other hand, GIOTTO can be used to test both analytical and numerical techniques on simple two-dimensional models of a magnetic lattice. A detailed description of the code is given in Ref. [4].

2 GENERAL FEATURES

2.1 Technical Survey

GIOTTO is based on the X-Window environment (Release X11R4 or higher); and on the OSF/ Motif graphic interface (Release Motif1.1 or higher); it has been developed on the Hewlett-Packard workstations of the 9000/700 series running the hp-ux 9.0 operating system, and it uses the low level graphic library STARBASE to draw pictures on the workstation screen. This is the only machine dependency of GIOTTO but a version which uses the standard X library is in preparation.

2.2 Installation & starting

GIOTTO's tar file can be asked to the e-mail address servizi@bologna.infn.it. During the installation, depending on the workstation performances, one can choose between versions which manage simultaneously at most 10 or 20 windows, and have a resolution for the pictures of 512x512 or 1024x1024 pixels. A public domain installation of GIOTTO is available at CERN in the directory /users/missy/giotto of the node hpariel.

Assuming that the directory /usr/giotto has been added to the PATH environment variable, the command which starts GIOTTO simply is giotto.

2.3 General appearance

GIOTTO has an X-windows environment: when the program starts three windows are displayed (see Fig. 1):

- the 'menu bar' control window which consists of eight buttons: all the actions that can be taken by the user are grouped under these buttons in the associated pulldown menu.

- a graphic window, where pictures are drawn. Any graphic window has a set of data attached to it (i.e. the given model, the region of the phase space displayed by the window, etc.), in such a way that, when many windows are displayed together on the screen, each one can be modified independently.

- a semaphore-shaped information window that describes the state of the program: green (idling), yellow (waiting for data input), red (processing data) and green, yellow and red (error message).

3 MAP INPUT

Map input is driven by the START/INPUT button in the menu bar. Some maps are already implemented in the program, such as the conservative Hènon mapping which modelizes 2D betatronic oscillations of a linear machine with a sextupole kick [2]:

$$\begin{pmatrix} x' \\ p' \end{pmatrix} = \begin{pmatrix} \cos\omega & \sin\omega \\ -\sin\omega & \cos\omega \end{pmatrix} \begin{pmatrix} x \\ p + x^2 \end{pmatrix} \qquad (1)$$

Other relevant input options are the following:

- User programmed map: the user is asked to supply a FORTRAN or a C code which contains the map;

- Polynomial (read from file) map: the input map is a polynomial whose coefficients are read from a file provided by the user. Also maps generated through SIX-TRACK (see Ref. [5]) can be read;

- MAD input file: one can analyse a lattice given as a file generated by the STRUCTURE command of MAD [3].

These options provide a wide spectrum of models, of different complexity, which can be implemented in GIOTTO.

4 MAKING PHASE PLOTS

4.1 Drawing and erasing

Once a map is chosen for a graphic window, trajectories are drawn for any initial condition which is clicked on the window itself; trajectories can be erased by clicking the Erase cascade button on the main menu.

GIOTTO does not store the data relative to all the trajectories, but only keeps in memory the array of 512×512 pixels (low resolution installation) or 1024×1024 pixels (high resolution installation) relative to the graphic window.

4.2 Coloring and zooming

Colors can be very useful to better analyse the geometry of the orbits: using the Color option one can draw trajectories with up to 16 million of different colors.

Using the Zoom button, the user can select a small region of a graphic window and enlarge it to see in more detail the topology of trajectories inside. A series of zoomings of the phase space of the Hènon map for $\omega/2\pi = 0.255$ are shown in Fig. 2.

4.3 Printing pictures

GIOTTO can produce printable PostScript file of each graphic window both in black and white and in color simply by activating the PostScript file or PostScript mono push button in the Actions pulldown menu.

5 NONLINEAR ANALYSIS

Here we give a list of the different analytical and numerical tools which are implemented in GIOTTO; they are all located in the Dynamics button in the menu bar.

- Fixed points of arbitrary order can be computed starting from an initial guess, using a numerical method due to Prof. M. N. Vrahatis (see Ref. [6]). The position, the period, the stability, and the eigenvalues of the fixed point are evaluated.

- The Fourier spectrum of the iterates of the mappings is evaluated through an FFT and displayed.

- The nonlinear tunes in their dependence on the initial condition are evaluated using the numerical method of the average phase advance or the perturbative approach of normal forms (see Ref. [2]).

- The normal forms series [2] relative to the implemented map are evaluated; approximation of the dynamics provided by the perturbative series can be displayed and compared with the original phase space. Both resonant and nonresonant normal forms are implemented; in Fig. 1 the resonant normal form relative to the Hènon map close to resonance four is shown.

- The homoclinic tangle option allows one to draw the invariant manifolds which emanate from the hyperbolic fixed points computed through the fixed point option, and to analyse the relation with the stability boundary (see Ref. [7]).

- The Lyapunov exponent can be evaluated and displayed as a function of the number of iterates using the method of the nearby trajectories or the tangent map method [8].

6 REFERENCES

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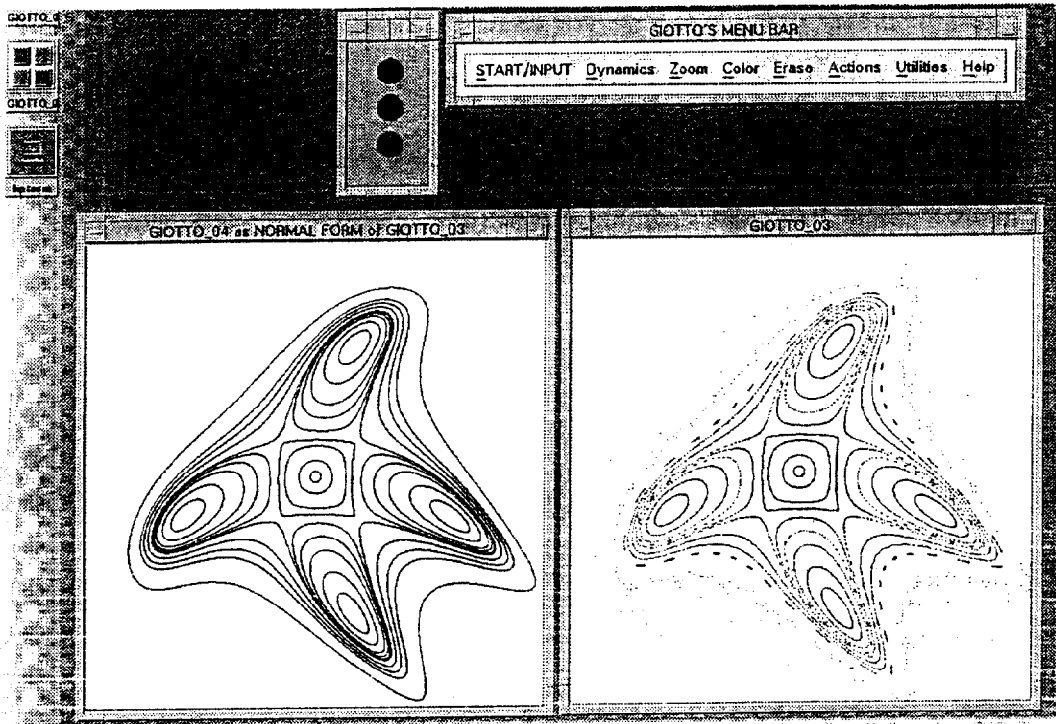
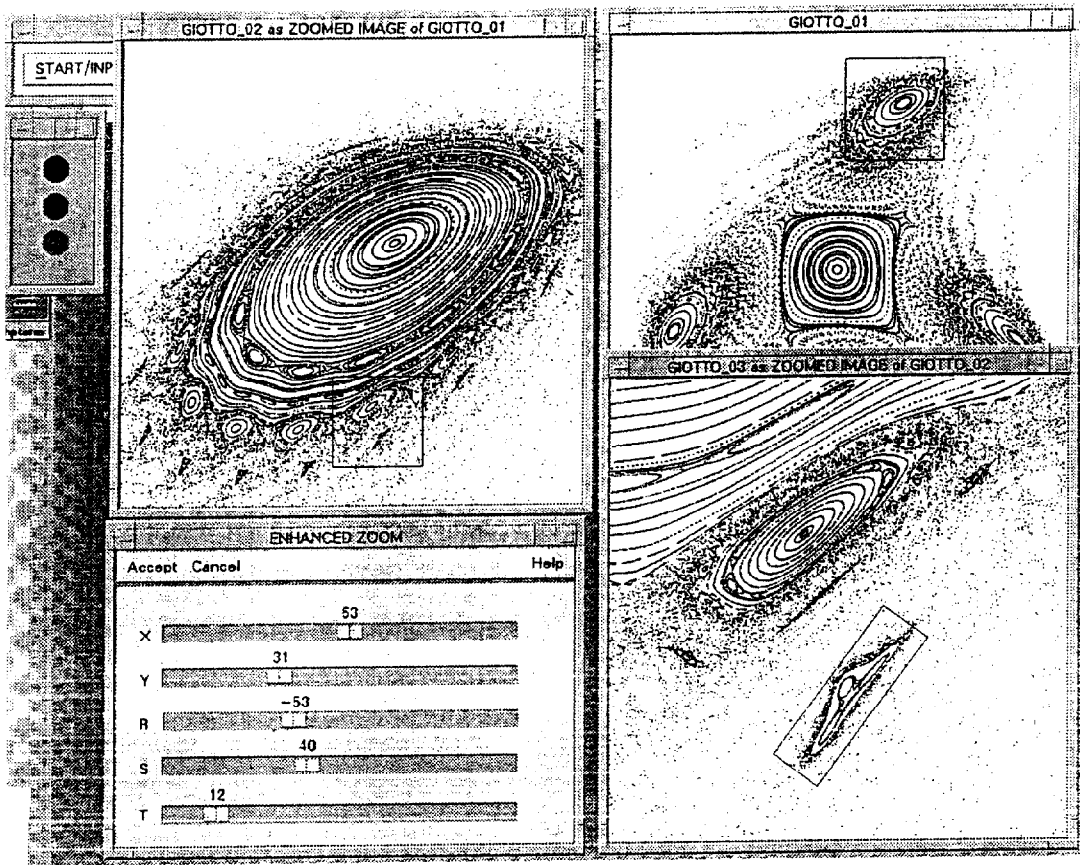Fig. 1: Lay-out of the screen of GIOTTO: menu bar, semaphore, graphic window and normal form approximation.

Fig. 2: Successive zoomings of the Hénon map close to resonance four.