Longitudinal Phase Space Tracking of Particles in a Multiple Harmonic RF-System

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Abstract

The cooler synchrotron COSY [1], now being under operation, is intended to cover the large energy range from 40 MeV up to 2.5 GeV. Beam cooling will be done by means of electron as well as stochastic cooling. Depending on the operational mode of the machine, transition crossing may become unavoidable. To study the various effects arising at transition, the longitudinal tracking code TIBETANC, originally written by J. Wei [2] had been enhanced so as to include acceleration in higher harmonic electrical fields up to arbitrary order. It now allows to simulate transition crossing by conventional methods as well as by the more promising technic of using a higher harmonic rf-system [3]. Also, acceleration in a higher harmonic rf-system suitable for stochastic bunched beam cooling [4] is implemented.

1. INTRODUCTION

Conventional phase switching methods suffer from the fact that due to chromatic effects particles with different momenta will cross transition at different times and in some circumstances will never cross transition which leads to an increase of particle losses during transition crossing. This makes the timing of phase switching a tagging procedure. Various methods, such as the method of transition energy jump or the "duck under" procedure [3,5] has been developed to circumvent this lack and have been studied in detail in [3]. The most challenging one is the use of a higher harmonic rf-system. In a neighborhood of transition one or more harmonics of the fundamental accelerating voltage are switched on for a period of time around transition. The relative phases and amplitudes are chosen in such a way that the resulting voltage will exhibit a broad and flat maximum around the accelerating phase, additionally to the required energy gain per turn. Substantial beam blow up can be drastically reduced.

As is pointed out in [6] a variety of technical efforts had been done to develop higher harmonic cavities. However conventional technics generally require a number of separate cavities which have to be precisely phased in order to achieve the required voltage pattern. To circumvent, a new type of broad band cavity is developed in [6]. In addition precise phasing becomes available by employing digital technics at COSY in synthesising the higher harmonic voltages [7]. A tracking code as TIBETANC becomes then a meaningful tool to investigate higher harmonic rf-waves for applying to stochastic cooling of bunched beams, transition crossing, bunch lengthening procedures, e.g. to stabilise the beam against space charge effects.

The most common options of TIBETANC have been discussed in detail in [2]. We thus restrict our contribution to the new features concerning higher harmonic voltage patterns in application to transition crossing. For clarity, all calculations shown below neglect space charge effects.

2. FEATURES OF TIBETANC

The former version of TIBETAN [2] written in Fortran is now available as standard ANSI-C version, TIBETANC, running on HP computer of COSY or on-line on a VAX workstation, DEC 5000/240, under Ultrix V4.2 or VMS 5.5 supported by GKS graphic packages. Up to six different windows can be chosen to display bunch and bucket as well as projections. Furthermore, moments up to 3rd order, longitudinal emittance and particle losses versus time can be displayed. The original version has been updated to include a momentum ramping function following the scheme F1Q1LQ2F2 where F1, F2 stands for flat bottom and flat top of the ramp, respectively. We assume that the magnetic field and thus momentum will be varied linearly (L) or quadraticly (Q1,Q2) from flat bottom to flat top with momenta p_i and p_f, respectively. We denote the momenta at the end of Q1 by p_1, at the end of L by p_2. Then, the complete ramping function can be determined by the program input parameters p_i, p_1, p_2, p_f and δE_S, the constant energy gain in the linear part of the ramp. Particles are usually accelerated by a single *sinusoidal voltage*. The voltage amplitudes are given at user specified times. In between, the program automatically interpolates. The necessary energy gain is calculated from the ramp data which in turn gives the desired synchronous phase. The higher harmonic voltage can be switched on at user specified times with arbitrarily chosen parameters. To study bunch dilution phase and/or amplitude noise can be switched on.

3. BEAM DYNAMICS

3.1. General Equations

The particle motion in longitudinal phase space is derived as usual from a Hamiltonian in which the higher harmonic voltage wave form of order M can be written as

$$g(\phi) = \sin\phi + \sum_{i=1}^{M} \frac{V_i}{V_0} \sin(\frac{h_i}{h_0}\phi + \phi_{i,0}) \qquad (1)$$

in which ϕ is the particle phase, $\phi_{i,0}$, V_i and h_i the phase voltage and the harmonic number of the ith harmonic, respectively. Fundamental quantities are labelled by 0. The energy gain per turn is calculated with (1) from

$$\delta E = qV_0 g(\phi) \tag{2}$$

The motion from turn number n to n+1 is then described for each particle of charge q in a given initial longitudinal phase space distribution by a set of coupled equations for the conjugate variables ϕ and $W = \Delta E/(h\omega_s)$

$$\begin{aligned} W_{n+1} &= W_n + \frac{qV_0}{h_0\omega_S}(\sin\phi_n - \sin\phi_{s,n}) \\ &+ \sum_{i=1}^{M} \frac{qV_i}{h_0\omega_S}\left\{\sin(\frac{h_i}{h_0}\phi_n + \phi_{i,0}) - \sin(\frac{h_i}{h_0}\phi_{s,n} + \phi_{i,0})\right\} \end{aligned} \tag{3a}$$

and

$$\phi_{n+1} = \phi_n - \frac{2\pi h_0^2 \omega_s \eta(W_{n+1})}{\beta_s^2 E_s} + (\phi_{s,n+1} - \phi_{s,n}) \tag{3b}$$

where β_s and ω_S are the normalised velocity and angular revolution frequency of the synchronous particle ($\phi_{s,n}$)with total energy $E_s = \gamma_s A m_0 c^2$. ΔE denotes the energy deviation with respect to the synchronous particle. For clarity we left out contributions - actually incorporated in the code - to eqs. (3) due to space charge and noise. As indicated in eq. (3b) the frequency slip factor η generally depends on momentum spread $\delta = \Delta p/p$, $\eta(W) = \eta_0 + \eta_1\delta + ...$, leading to chromatic single particle effects [2]. Calculations below include non-linearities up to first order with η_0 and η_1 given by [2]

$$\eta_0 = \frac{1}{\gamma^2} - \frac{1}{\gamma_{tr}^2} \text{ and } \eta_1/2 = -\frac{1}{\gamma_{tr}^2}(\alpha_1 + \eta_0) - \frac{3\beta_s^2}{2\gamma_s^2} \tag{4}$$

The quantity α_1 is determined by lattice functions [2]. In our calculations the computer code MAD was used [5] to find for COSY $\alpha_1 = 0.16$. The transition gamma is assumed to be tuned to $\gamma_{tr} = 2.222$. Due to non-linearities particles with different momenta will cross transition at different times so that the bunch shape will be sheared non-uniformly in phase by the amount

$$\Delta\phi(\tau,\Delta t,\delta) \approx h_0\omega_S\delta\left\{\frac{\dot\gamma\,(\Delta t^2-\tau^2)}{\gamma_{tr}^3} - \frac{(\Delta t+\tau)\,\delta\,(2\alpha_1+3\beta_S^2)}{\gamma_{tr}^2}\right\} \tag{5}$$

where $2\Delta t$ is the switching-on time of the higher harmonic system and τ, $|\tau| < \Delta t$, measures the time with respect to crossing point where the acceleration rate is $\dot\gamma = d\gamma/dt$.

3.2. Parameters for Transition Crossing

Transition crossing is studied using a three octave higher harmonic system, that is $h_1 = 2$ and $h_2 = 4$ in eq. (1). For the fundamental we have $h_0 = 1$ at COSY. Since phase focusing at transition is lost [3] we use a voltage wave form which ensures the necessary energy gain per turn and exhibits a broad flattened maximum at the synchronous phase ϕ_s. For maximum ramping speed the energy gain per turn will be $\delta E_s = 1.316$ keV. If we choose the stable phase to be $\phi_S = 90^0$, maximum flatness will be achieved by setting the first four derivatives of the wave form to zero. From these conditions the relative phases and amplitudes of the wave forms can be derived together with the necessary energy gain δE_s. We thus end up with the wave form

$$g(\phi) = \sin\phi + \frac{5}{16}\cos(2\phi) + \frac{1}{64}\cos(4\phi) \tag{6}$$

The fundamental peak voltage V_0 is determined by $qV_0 = (64/45)\,\delta E_s$. With values given above we achieve a maximum deviation from the centre value of < 0.2% for $\Delta\phi = \pm 30^0$. In this range all particles will receive the same energy gain, provided bunch shearing according to (5) does not appreciably exceed this value. However, the effect of any deviation can be estimated by tracking using TIBETANC. Note that the corresponding potential in the Hamiltonian is chair like and will only be focusing for values of $\phi < 60^0$. The potential is de-focusing for values above 120^0.

Similarly, the relative phases and amplitudes of a three octave rf-system suitable for stochastic cooling may be found for given peak voltage V_0 and energy gain δE_s. The potential will then exhibit a broad flattened minimum if the 5th derivative of (1) does not vanish [6].

4. TRACKING RESULTS

With a three octave rf-system (HHS) we tracked a bunch of particles with initial parabolic distribution and bunch area A = 1.09 eVs. The initial voltage was set to 2.5 kV with the corresponding phase $\phi_s = 31.8^0$ (= 0.555 rad). The bunch with relative momentum spread ± 0.2% is shown in figure 1.

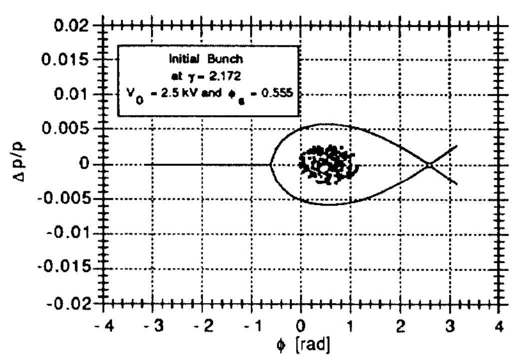

Figure 1. Bunch and bucket before transition. HHS off.

The bunch is then tracked through transition with the higher harmonic system switched on for different time intervals, ±5 ms, ± 10 ms and ± 20 ms with respect to the transition crossing time. According to eq. (2) the voltage V_0 is set to 1.87 kV, thus $d\gamma/dt = 2/s$. Bunch and voltage wave form are shown close to transition in figures 2, 3 and 4. It is clearly seen that an optimum crossing is found for $\Delta t = \pm 10$ ms (fig. 3). As compared to fig. 2 the relative momentum spread becomes now confined in the interval [-0.4%,0.4%]. The crescent bunch shape due to the chromatic effects is visible and is well described by the analytical formula (5) with $\tau = 0$.

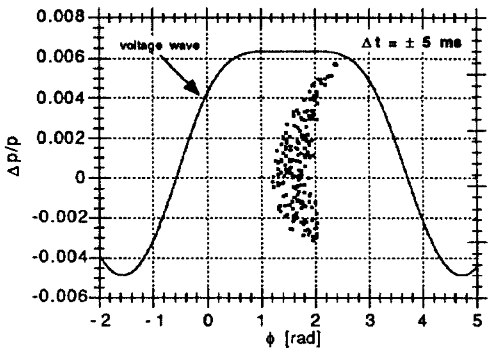

Figure 2. At transition. The HHS is switched on for 10 ms.

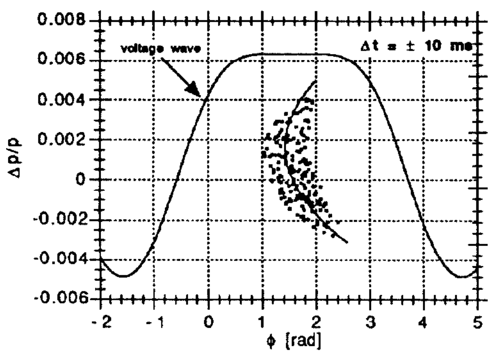

Figure 3. At transition. The HHS is switched on for 20 ms. Phase shearing formula (5) is plotted for $\tau = 0$.

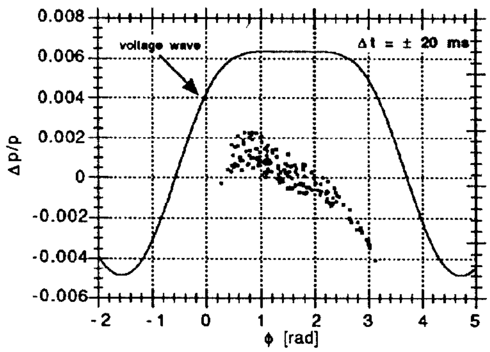

Figure 4. At transition. The HHS is switched on for 40 ms.

Also, figure 3 shows that the lower momentum tail of the bunch just enters the decreasing area of the voltage wave form where the voltage is about 2% off the centre value. Particles entering this area are slowly decelerated leading to a small tail in the final distribution after transition crossing, fig. 5. Some of the particles still remain outside the bucket and will eventually lost. The final relative momentum spread is comparable to that before transition crossing (fig. 1). The effect of a too long on-time is clearly demonstrated in figure 4. In this case the long low-momentum tail deeply enters the decreasing part of the voltage and particle losses are considerably enlarged.

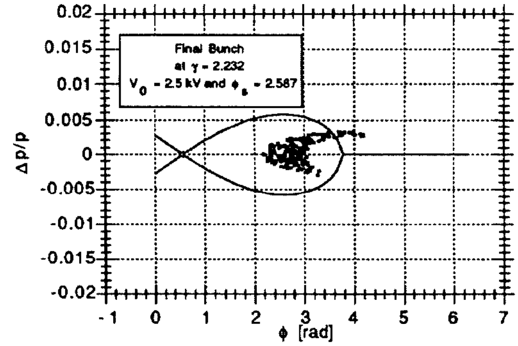

Figure 5. Bunch after transition. The HHS is switched off and the voltage is reset to 2.5 kV.

5. CONCLUSIONS

To conclude, the tracking calculations using the updated version of TIBETANC have shown that transition crossing at COSY by applying a three octave harmonic rf-system would be a challenging procedure. Although the bunch slightly leaves the flat area of the voltage pattern at transition, particle losses are less then 10%. This is to be compared with the duck-under method in which particles losses are about 30% [5]. The necessary time interval around transition for the higher harmonic system to work properly is found to be 20 ms. This value is consistent with those derived in [3]. Further studies to optimise voltage and chromatic mis-match including space charge can now be done. Additionally, formula (5) describing bunch shearing during transition crossing is shown to be a handy equation allowing a quick estimate of the minimum required flat-topped portion of the higher harmonic wave pattern. Application to technical design of rf-cavities however should be accompanied by tracking simulations.

6. REFERENCES

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