Transition-Energy Crossing with a γ_t-Jump *

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Abstract

Expressions for the minimum size and speed of a transition-energy (γ_t-) jump needed to diminish the chromatic non-linear effect, the self-field mismatch, and the microwave instabilities in the Relativistic Heavy Ion Collider (RHIC) are obtained. A γ_t-jump of 0.8 units is needed to be performed within 60 ms in order to achieve a "clean" transition crossing.

1 INTRODUCTION

During the past several decades, the γ_t-jump method[1] has been extensively used to minimize the beam loss and growth during transition crossing. In the RHIC, intensive particle beams are accelerated through transition at a relatively slow rate due to the slow ramping rate of the superconducting magnets. Effects of chromatic non-linearities (so-called Jøhnsen Effect) and self-field mismatch both become significant. Crossing transition using a γ_t-jump provides an effective method in minimizing these undesired effects.

With a γ_t-jump, the amount of distortion in the machine lattice is proportional to the size of jump necessary to eliminate the undesired effects, while the speed of jump is restrained by the pulsing of the superconducting quadrupole correctors. This paper first studies the longitudinal particle motion during the γ_t-jump process. In section 2, the particle motion is discussed in terms of longitudinal amplitude functions. Section 3 reviews the growths in longitudinal bunch area due to various effects in the absence of a γ_t-jump. Section 4 presents the minimum size and speed of γ_t-jump required to diminish the bunch growth.

Ion beams in the RHIC of all species except proton will cross transition at about $\gamma_t = 23$. Due to the slow ramping rate of the superconducting magnets, the acceleration rate $\dot{\gamma}$ for the Au^{79+} beam is about 1.6 per second. With the nominal intensity of 1×10^9 ions per bunch in an area 0.3 eV·s/u and the space-charge coupling impedance $|Z/n|$ of about 1.2 Ohms, a γ_t-jump is needed to diminish both chromatic non-linear and self-field effects. The various γ_t-jump schemes for the RHIC using different rf system are compared in Section 5. The conclusion is given in section 6.

2 PARTICLE MOTION DURING γ_T-JUMP

It is convenient to measure time relative to the instant that the synchronous particle crosses the reference transition energy γ_{t0} and to assume that particles are accelerated through transition with a constant rate $\dot{\gamma}$,

$$\gamma_s(t) = \gamma_{t0} + \dot{\gamma}\, t. \tag{1}$$

For simplicity but without losing generality, we approximate the process of γ_t-jump in the time period of interest by the expression

$$\gamma_t(t) = \gamma_{t0} - \dot{\gamma}_t\, t, \quad -T_J/2 \le t \le -T_J/2, \tag{2}$$

where T_J is the duration of the jump, and $\Delta\gamma_t = \dot{\gamma}_t T_J \ll \gamma_{t0}$ is the size of jump.

In the absence of the γ_t-jump, the characteristic transition time T_c during which the particle motion is non-adiabatic is

$$T_C = \left(\frac{\pi E \beta_s^2 \gamma_t^3}{qeV|\cos\phi_s|\dot{\gamma} h \omega_s^2}\right)^{\frac{1}{3}}, \tag{3}$$

where the subscript s represents the synchronous value, and,

qe = electric charge, $E = Am_0c^2\gamma_s$ synchronous energy,
V = accelerating rf voltage, h = rf harmonic number,
ω_s = revolution frequency, $\beta_s c$ = velocity,
ϕ_s = synchronous phase.

In the presence of the γ_t-jump, the characteristic transition time $\tilde{T}_C$ is reduced to be

$$\tilde{T}_C = \left(\frac{\dot{\gamma}}{\dot{\gamma} + \dot{\gamma}_t}\right)^{1/3} T_C. \tag{4}$$

In this paper, we denote the corresponding values during the γ_t-jump by the sign $\tilde{}$, and describe the motion of the particle by the deviation of rf phase ϕ and energy $W \equiv \Delta E/h\omega_s$. The trajectory of the particle of action J before the start of the jump can be described as

$$\beta_L(x)\phi^2 + 2\alpha_L(x)\phi W + \gamma_L(x) W^2 = 2J, \tag{5}$$

where x is the time normalized by T_C. The longitudinal amplitude functions β_L, α_L, and γ_L are[2],[3]

$$\beta_L = k^{-1} T_C \hat{\beta}, \; \alpha_L = \hat{\alpha}, \; \gamma_L = k T_C^{-1} \hat{\gamma}, \tag{6}$$

where $k = 2\pi h/qeV|\cos\phi_s|$, and the normalized amplitude functions are

$$\hat{\beta}(x) = \frac{\pi}{3} x \left\{ \left[J_{-\frac{1}{3}}(y)\right]^2 + \left[N_{-\frac{1}{3}}(y)\right]^2 \right\} \approx \begin{cases} \dfrac{4\pi}{3^{4/3}\Gamma^2(2/3)}\left[1 - \dfrac{3^{5/6}\Gamma^2(2/3)}{2\pi} x\right], & x \ll 1, \\ \dfrac{1}{x^{1/2}}\left(1 - \dfrac{5}{32}\dfrac{1}{x^3}\right), & x \gg 1, \end{cases} \tag{7}$$

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$$\begin{aligned}\hat{\alpha}(x) &= -\frac{\pi}{3}x^{3/2}\left\{J_{\frac{2}{3}}(y)J_{-\frac{1}{3}}(y)+N_{\frac{2}{3}}(y)N_{-\frac{1}{3}}(y)\right\}\\ &\approx \begin{cases} -\frac{1}{\sqrt{3}}\left[1-\frac{3^{5/6}\Gamma^2(2/3)}{\pi}x\right], & x\ll 1,\\ -\frac{1}{4x^{3/2}}, & x\gg 1,\end{cases}\end{aligned} \tag{8}$$

with $\hat{\beta}\hat{\gamma}=1+\hat{\alpha}^2$. Here, J and N are the Bessel functions, $y=2x^{2/3}/3$, and $\Gamma(2/3)\approx 1.354$. It can be shown that the instantaneous synchrotron-oscillation period is $\Omega_s=\hat{\beta}T_C$.

At time $t\le -T_J/2$ before the start of the γ_t-jump, the particle motion is given by Eq. 5 with

$$x=\frac{\dot{\gamma}_t}{\dot{\gamma}}\frac{T_J}{2T_C}+\frac{|t|}{T_C},\quad t\le -T_J/2. \tag{9}$$

Immediately after the start of the γ_t-jump ($-T_J/2<t\le T_J/2$), the particle motion is given by Eqs. 5 and 6 with T_C replaced by $\tilde{T}_C$, and with

$$x=\frac{|t|}{\tilde{T}_C},\quad -T_J/2<t\le T_J/2. \tag{10}$$

The γ_t-jump typically starts well before the particle motion becomes non-adiabatic, i.e. $x_J\gg 1$ and $\tilde{x}_J\gg 1$, or

$$\Delta\gamma_t\gg 2\dot{\gamma}_t\tilde{T}_C. \tag{11}$$

According to Eqs. 7, 9, and 10, the particle trajectories before and after the start of the jump are matched. At the time that the reference particle crosses the transition energy, the maximum energy deviation and the minimum phase deviation are given by the expressions

$$\begin{aligned}\tilde{W}(0) &= \sqrt{2k^{-1}\tilde{T}_C\hat{\beta}(0)}=\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{1/6}W(0),\\ \tilde{\phi}(0) &= \sqrt{2k\tilde{T}_C^{-1}\hat{\gamma}(0)}=\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{-1/6}\phi(0).\end{aligned} \tag{12}$$

Compared with the case without the γ_t-jump, the energy and phase deviations are scaled by the factor $\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{1/6}$. On the other hand, the synchrotron period is reduced by a factor $\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{1/3}$,

$$\tilde{\Omega}_s=\hat{\beta}\tilde{T}_C=\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{1/3}\Omega_s. \tag{13}$$

In section 4, we show that this change in beam configuration reduces the growth of bunch area due to both single and multi-particle effects.

3 BEAM GROWTH AT TRANSITION

In the absence of a γ_t-jump, both single- and multi-particle effects can produce bunch mismatch and growth. In Ref. [3], scaling laws are obtained for the effects of chromatic non-linearity, self-field mismatch, and microwave instabilities at transition.

Chromatic non-linearities cause particles of different momenta to cross transition at different time. The so-called non-linear time is defined as

$$T_{nl}=\frac{\left|(\alpha_1+\frac{3}{2}\beta_s^2)\right|\hat{\delta}(0)\,\gamma_{t0}}{\dot{\gamma}}, \tag{14}$$

where $\hat{\delta}(0)=h\omega_s W(0)/E\beta^2$ is the maximum momentum spread at transition, and the non-linear momentum-compaction factor describes the dependence of the circumference on the momentum

$$\frac{L}{L_s}=1+\frac{\delta}{\gamma_{t0}^2}\left[1+\alpha_1\delta+O(\delta^2)\right]. \tag{15}$$

The mismatch in particle trajectories produces effective growth in the bunch area. The effective increase in the bunch area during the crossing depends on the ratio of T_{nl} to T_c,

$$\frac{\Delta J}{J}\approx\begin{cases}0.38\,\frac{T_{nl}}{T_c}, & \text{for } T_{nl}\ll T_c;\\ e^{\frac{4}{3}\left(\frac{T_{nl}}{2T_c}\right)^{3/2}}-1, & \text{for } T_{nl}\ge T_c.\end{cases} \tag{16}$$

Beam loss occurs if the effective bunch area $2\pi(J+\Delta J)$ after transition is larger than the bucket area.

Beam self fields cause mismatch in the nominal bunch shape at the time the synchronous phase is switched at transition. The effective increase in the bunch area is proportional to the ratio of the self-field to the rf accelerating field,

$$\frac{\Delta J}{J}=\frac{2h\hat{I}(0)\,|Z_L/n|}{V|\cos\phi_s|\,\hat{\phi}^2(0)}, \tag{17}$$

where 0 indicates the time $t=0$ of transition, $|Z_L/n|$ is the coupling impedance at bunch frequency, $\hat{\phi}$ is the maximum phase spread of the bunch, and the peak current $\hat{I}$ for a parabolic distribution is

$$\hat{I}=\frac{3hN_0 qe\omega_s}{4\hat{\phi}} \tag{18}$$

with N_0 the number of particle per bunch.

The threshold for microwave instability to occur at transition has been approximately obtained as

$$\frac{8h\hat{I}(0)\,|Z_H/n|}{3V|\cos\phi_s|\,\hat{\phi}^2(0)}\ge 1, \tag{19}$$

where $|Z_H/n|$ refers to the coupling impedance in the microwave frequency range.

In the absence of a γ_t-jump, the beam growth and intensity loss in the RHIC caused by chromatic non-linear effect and self-field mismatch, is significant.[3] γ_t-jump schemes are thus proposed[4],[5] to achieve a "clean" crossing.

4 REQUIREMENTS FOR γ_T-JUMP

An effective way to cure both the beam self field and the chromatic non-linear effect is to increase the transition-crossing rate by temporarily adjusting the lattice to achieve a γ_t-jump. The required size and speed of γ_t-jump is determined by the strength of the non-linearity and the self fields.

The γ_t-jump minimizes the effect of chromatic non-linearity by reducing the momentum spread and the duration of mismatch. With the γ_t-jump, the non-linear time T_{nl} can be significantly reduced,

$$\tilde{T}_{nl} = \left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{7/6} T_{nl}. \quad (20)$$

From Eqs. 4, 20, and 16, the growth in bunch area is reduced by a factor $\left(\frac{\dot{\gamma}}{\dot{\gamma}+\dot{\gamma}_t}\right)^{5/6}$. If Δ_g is the maximally allowed fractional growth in bunch area, it can be derived from Eq. 16 that the speed of γ_t-jump should satisfy

$$\frac{\dot{\gamma}_T}{\dot{\gamma}} > \left(\frac{1}{\Delta_g}\frac{T_{nl}}{T_c}\right)^{6/5} - 1. \quad (21)$$

The total size $\Delta\gamma_t$ of jump needed to prevent the non-linear growth is proportional to the time T_{nl},

$$\Delta\gamma_t > 2\dot{\gamma}T_{nl}. \quad (22)$$

The γ_t-jump also reduces the amount of self-field mismatch by preventing the bunch length from becoming small at transition (Eq. 12). If Δ_g is again the maximally allowed fractional growth in bunch area, the speed of jump should satisfy

$$\frac{\dot{\gamma}_T}{\dot{\gamma}} > \left(\frac{1}{\Delta_g}\frac{2h\hat{I}(0)\,|Z_L/n|}{V|\cos\phi_s|\,\hat{\phi}^2(0)}\right)^2 - 1, \quad (23)$$

where $\hat{I}(0)$ and $\hat{\phi}(0)$ are the peak current and the bunch length if no jump is applied. The size of jump should satisfy

$$\left.\frac{2h\hat{I}(\gamma)\,|Z_L/n|}{V|\cos\phi_s|\,\hat{\phi}^2(\gamma)}\right|_{\gamma=\gamma_t\pm\Delta\gamma_t/2} < \Delta_g, \quad (24)$$

where $\gamma_t \pm \Delta\gamma_t/2$ correspond to the instants just after and before the γ_t-jump. Using the expression for the bunch length ϕ, this condition can be explicitly written as

$$\Delta\gamma_t > \frac{3^{4/3}\pi^{1/3}E_s\beta_s^2 qeV|\cos\phi_s|\gamma_t^3}{8h^{1/3}\omega_s^2 J^2}\left(\frac{1}{\Delta_g}\frac{\bar{I}\,|Z_L/n|}{V|\cos\phi_s|}\right)^{4/3}, \quad (25)$$

where $\bar{I} = N_0 qe\omega_s/2\pi$.

In order to eliminate microwave instabilities caused by the coupling impedance Z_H/n, the needed speed of jump is derived from Eq. 19 as

$$\frac{\dot{\gamma}_T}{\dot{\gamma}} > \left(\frac{8h\hat{I}(0)\,|Z_H/n|}{3V|\cos\phi_s|\,\hat{\phi}^2(0)}\right)^2 - 1, \quad (26)$$

where $\hat{I}(0)$ and $\hat{\phi}(0)$ are again the peak current and the bunch length if no jump is applied. The needed size of jump is given by

$$\Delta\gamma_t > \frac{3^{4/3}\pi^{1/3}E_s\beta_s^2 qeV|\cos\phi_s|\gamma_t^3}{8h^{1/3}\omega_s^2 J^2}\left(\frac{4\bar{I}\,|Z_H/n|}{3V|\cos\phi_s|}\right)^{4/3}. \quad (27)$$

The condition on the minimum size (Eqs. 22, 25, and 27) and speed (Eqs. 21, 23, and 26) of γ_t-jump has been verified with the computer simulation program TIBETAN[3]. The agreement between the simulation and experiment has been partly achieved in a recent study in the AGS.[6]

5 CHOICE OF RF SYSTEM FOR γ_T-JUMP

The accelerating rf system in the RHIC operates at voltage $V = 300$ kV and frequency 27 MHz ($h = 342$). Assume the non-linear factor $|\alpha_1 + 1.5| \leq 3.5$ with the γ_t-jump lattice.[3],[5] In order for the effective growth in the Au^{79+} bunch area to be less than 20%, a γ_t-jump of 0.8 units (Eqs. 22) is needed to be performed in 60 ms (Eqs. 21). With the nominal intensity of $N_0 = 1\times10^9$ ions per bunch in an area $2\pi J = 0.3$ eV·s/u, the maximally allowed coupling impedance $|Z/n|$ is about 1.5 Ohms.

The storage rf system in the RHIC operates at a voltage up to $V = 6$ MV and frequency 196 MHz ($h = 2508$). In principle, this system could be used to capture the beam before transition and then accelerate it to the top energy. However, with this scheme a minimum voltage of 600 kV is needed to provide the adequate bucket area. With this voltage, both the bunch momentum spread and the synchrotron-oscillation frequency are significantly increased. In order to diminish the chromatic non-linear effect, a γ_t-jump of 1.6 units is required to be performed in 10 ms. Compared with the requirements on the previous γ_t-jump scheme with the 27 MHz rf system, this scheme appears less attractive.

6 CONCLUSION

From longitudinal beam-dynamics point of view, the minimum size and speed of jump needed to diminish the chromatic non-linear effect, the self-field mismatch, and the microwave instabilities are obtained in Eqs. 22, 25, 27, and Eqs. 21, 23, 26, respectively. A γ_t-jump of 0.8 units is needed to be performed within 60 ms in order to achieve a "clean" transition crossing in the RHIC.

7 REFERENCES

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