Interaction Region Quadrupole Roll and Solenoid Misalignment in RHIC *

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Abstract

The main sources of linear coupling in the interaction regions (IRs) of the Relativistic Heavy Ion Collider (RHIC) are IR quadrupole roll errors and coupling from the PHENIX experiment solenoid. The magnitudes of these effects are estimated. We show that coupling from the IR quadrupole roll errors can be locally corrected with skew quadrupole trims, using the local vertical dispersion in the IR as a convenient diagnostic. Coupling from the PHENIX solenoid is shown to be negligible. Solenoidal misalignment effects are small compared to the effects of a 3.85 milliradian closed orbit angle offset required when protons collide with gold ions. These effects are shown to modify local dipole corrector strengths to first order in the solenoid strength.

1 INTRODUCTION AND RHIC IR LAYOUT

Here we evaluate the impact of quadrupole roll and an experiment solenoid on local coupling in the RHIC IRs, and show how this coupling may also be corrected locally using measurements of vertical dispersion through the IR.

The layout of RHIC around the PHENIX interaction region for a low-beta $\beta^\star = 1$ m lattice is shown in Figure 1 and Table 1; this layout is also typical of the low-beta IR at the STAR experiment. The triplet quadrupoles, numbered from 1 to 3 outwards from the IR, cluster near the beta function peaks, and skew quadrupole correctors are located in the spool corrector packages on the inner (IR) ends of each quadrupole 3. The triplets are also local peaks for horizontal dispersion — therefore the vertical dispersion through the IR is dominated by quadrupole roll errors and skew quadrupole components in the triplets. These sources, along with the PHENIX solenoid, are also the primary local contributions to linear coupling in the IR region.

RHIC operational plans include colliding dissimilar ion species for the PHENIX and STAR experiments. To collide protons and gold nuclei, for example, the orbit must be skewed through the common DX splitting dipoles and the IR, resulting in an orbital pitch of 3.85 mrad about the vertical axis through each interaction point (IP). The effects of this orbit skew on coupling and orbit errors from the PHENIX solenoid are also addressed here.

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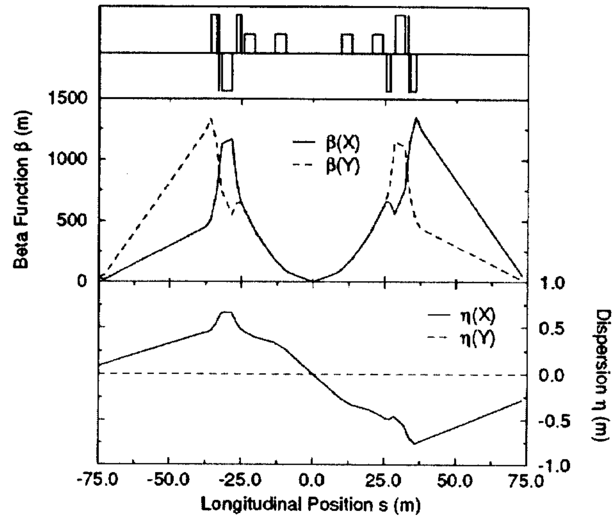

Figure 1: Optical functions near the 8 o'clock IR and the PHENIX experimental area for a $\beta^\star = 1$ m lattice. Longitudinal position is measured from the interaction point. The positions of IR quads 1–3, IR skew quad correctors (next to IR quad 3), and the DX and D0 dipoles are also shown.

2 VERTICAL DISPERSION AND LINEAR COUPLING

The differential equation for vertical dispersion η_y generated by a skew quadrupole of strength K_S, or a normal quadrupole of strength K_N rolled by a small angle ϕ, is given by

$$\eta_y'' + K_N\,\eta_y = (2\phi K_N + K_S)\,\eta_x\ , \qquad (1)$$

where K_N is positive for vertical focusing. The solution of Eq. 1 is straightforward for each source of vertical dispersion with focal length f:

$$\begin{aligned}\eta_y &= \frac{\eta_{x0}\sqrt{\beta_y\beta_{y0}}}{2\sin(\pi Q_y)}\cos(|\Delta\phi_y| - \pi Q_y)\left[\frac{2\phi}{f_N} + \frac{1}{f_S}\right] \\ &\equiv H_N\phi + \frac{H_S}{f_S}\ . \qquad (2)\end{aligned}$$

At the source point, where $\beta_y = \beta_{y0}$ and $\Delta\phi_y = 0$, the coefficients for vertical dispersion generation are:

$$H_N = \frac{\beta_{y0}\,\eta_{x0}}{f_N}\cot(\pi Q_y)\ ,\quad H_S = \frac{1}{2}\beta_{y0}\,\eta_{x0}\cot(\pi Q_y)\ . \quad (3)$$

Element Name	β_x [m]	β_y [m]	η_x [m]	μ_x $/2\pi$	μ_y $/2\pi$	L [m]	f [m]	H	G
IR quad 1	718	668	0.520	0.2448	0.2433	1.44	12.03	44 m	18.3
IR quad 2	1354	550	0.731	0.2455	0.2444	3.40	-5.25	-117 m	-52.3
IR quad 3	575	1313	0.490	0.2463	0.2453	2.10	8.46	116 m	32.7
Skew quad corrector	845	999	0.586	0.2459	0.2451			446 m^2	146.2 m
PHENIX solenoid	1.0	1.0	0.0	0.0	0.0	1.0			
Focusing arc quad	49.6	9.8	1.843			1.11	-11.02	-2.5 m	-0.6
Defocusing arc quad	10.4	48.6	0.940			1.11	10.66	6.5 m	0.7

Table 1: Optics of the RHIC IR triplet quadrupoles and local skew quadrupole corrector in the design low-beta storage lattice, with positive f implying vertical focusing and $Q_y = 29.185$. Phase advances are measured from the interaction point. The parameters for typical arc focusing and defocusing quadrupoles are included for comparison.

ϕ_1 [mr]	ϕ_2 [mr]	ϕ_3 [mr]	f_{corr}^{-1} [km^{-1}]	η_{y1} [m]	η_{y2} [m]	η_{y3} [m]	$\eta_{y,\mathrm{corr}}$ [m]	ΔQ_{min}
0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.0	0.0	0.0	0.0	0.044	0.040	0.063	0.055	0.018
0.0	1.0	0.0	0.0	-0.138	-0.125	-0.193	-0.168	0.052
0.0	0.0	1.0	0.0	0.085	0.077	0.118	0.104	0.033
0.0	0.0	0.0	-1.0	-0.718	-0.649	-1.001	-0.872	0.157
1.0	-1.0	1.0	0.0	0.339	0.308	0.474	0.414	0.106
1.0	-1.0	1.0	-0.7	-0.004	-0.002	-0.004	-0.003	0.000

Table 2: Dispersion and coupling from quadrupole roll errors in one low-beta triplet of the RHIC lattice, from simulation. Horizontal and vertical fractional tunes were adjusted to be exactly $Q = 0.185$, so that ΔQ_{min} is solely due to coupling.

These coefficients have their ring-wide maxima in the IR triplet quadrupoles at low-beta storage.

Another first-order effect of rolled normal quadrupoles and skew quadrupoles is linear coupling. A standard measure of coupling is the minimum fractional tune difference that is possible when coupling alone is present. It has been shown [1] that this minimum tune split, $\Delta Q_{\mathrm{min}} = |Q_x - Q_y|$, is

$$\Delta Q_{\mathrm{min}} = \left| G_N \phi + \frac{G_S}{f_S} \right| , \quad (4)$$

$$G_N = \frac{\sqrt{\beta_{x0}\beta_{y0}}}{f_N \pi}, \quad G_S = \frac{\sqrt{\beta_{x0}\beta_{y0}}}{2\pi} \quad (5)$$

for coupling induced by rolled normal quadrupoles and skew quadrupoles, respectively. These decoupling coefficients also have their ring-wide maxima at the IR quadrupoles at low-beta storage.

The coefficients listed in Table 1 show that linear coupling is a much more serious problem than vertical dispersion generation. It would be convenient to set the triplet quad roll angle tolerance such that the minimum tune split caused by a single triplet quad is much less than the nominal tune separation of $\Delta Q = 0.01$ in RHIC, and the vertical dispersion created is much less than 0.1 m. Using quad 2 as a worst case example, the coupling condition leads to a practically unattainable roll requirement of $\phi \ll 0.19$ mrad, while the dispersion constraint leads to a more reasonable constraint of $\phi \ll 0.85$ mrad. A direct conclusion is that local coupling correction is necessary in the RHIC triplets at low-beta operation; fortunately vertical dispersion may be used as a diagnostic for adjusting the strength of the skew quad correctors in each IR triplet.

3 LOCAL TRIPLET CORRECTION

Table 2 shows results of a numerical experiment to test the above predictions. Minimum tune split values are in good agreement with those predicted by the coupling coefficients, except in the fifth row where one of the eigentunes approaches the integer resonance. The measured vertical dispersions at the locations of each single source are also in good agreement with those predicted by their respective coefficients. It is worth noting that vertical dispersion is not affected by the fact that all these tests are in fully coupled machines, except again near the integer resonance.

The last two rows show that a single skew quad corrector in each triplet very effectively compensates the local coupling caused by 1 mrad roll angles in all three triplet quads — this makes a good deal of sense since there is little phase advance across the triplet. Vertical dispersion is also almost perfectly compensated by this setting.

Vertical dispersion is therefore a convenient diagnostic for local coupling in the IRs. There is a dispersive dual-plane BPM located in each triplet that is able to measure vertical dispersion to better than the 0.1 m criterion mentioned above. Variation in vertical dispersion through a low-beta squeeze will allow measurement and local correction of vertical dispersion and coupling without concern for other sources of vertical dispersion, such as arc quad roll errors.

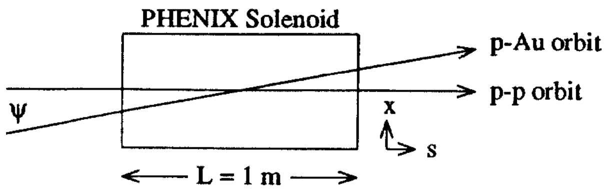

Figure 2: Orbit through the PHENIX solenoid, for proton-proton and proton-gold collisions. The orbit skew angle is $\psi = 3.85$ mrad.

4 SOLENOID COUPLING

For the PHENIX experiment solenoid with longitudinal field $B_s = 0.5$ T and length $L = 1$ m, a convenient dimensionless measure of strength is given by

$$\theta = \frac{B_s L}{2|B\rho|}, \tag{6}$$

where $|B\rho| = 97.5$ T − m and 839.5 T − m, and $\theta =$ 2.56 mrad and 0.30 mrad, for RHIC injection and storage respectively. The solenoid strength θ is the angle by which the transverse coordinate axes are rotated by the solenoid field. Solenoidal effects are stronger at injection rigidity, in contrast to coupling and dispersion from IR quad roll. The design horizontal dispersion over the solenoid length is near zero and the solenoid contributes little to vertical dispersion.

The minimum tune split from a solenoid is given by [1]:

$$\Delta Q_{\text{min}} = \frac{g\theta}{2\pi} = 8.2 \times 10^{-4} \tag{7}$$

at storage, where g, an optical factor, is 2 at an IR with equal beta functions in both planes, independent of hadron species and $\beta^\star$. This tune separation is an order of magnitude lower than the RHIC design fractional tune split and more than two orders of magnitude smaller than the coupling expected from triplet quad roll errors. This coupling therefore need not be considered in the local coupling correction scheme at low beta, as it can be handled by normal global corrections. The coupling at injection, while an order of magnitude larger, is still small compared to coupling from skew quad components of the RHIC arc dipoles which dominate dynamics at injection.

5 SKEW SOLENOID EFFECTS

The counter-rotating beams of RHIC pass through common splitting magnets (DX magnets) in each IR; since different ion species with the same relativistic velocity have different magnetic rigidities, collisions require an orbital tilt around the vertical axis when colliding different ion species. This orbital skew is 3.85 milliradians for proton-gold collisions and is shown in Figure 2. Here we examine the effect of this skew on PHENIX solenoid orbital effects through the IR.

A small-angle approximation of the transfer matrix for a thick solenoid with edge effects is [2]:

$$\begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_2 = \begin{pmatrix} 1 & L & \theta & L\theta \\ 0 & 1 & 0 & \theta \\ -\theta & -L\theta & 1 & L \\ 0 & -\theta & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_1$$

$$= \begin{pmatrix} \mathbf{D}_2 & \theta\mathbf{D}_2 \\ -\theta\mathbf{D}_2 & \mathbf{D}_2 \end{pmatrix} \begin{pmatrix} x \\ x' \\ y \\ y' \end{pmatrix}_1, \tag{8}$$

where $\mathbf{D}_2$ is the 2×2 transfer matrix for a drift of length L. The orbit skew has the effect of introducing a horizontal orbit offset of $x_1 = L\psi/2$ and $x_1' = \psi$ upon entry to the solenoid. Propogating this through Eq. 8 and comparing to a drift of length L gives an orbit error from the skew orbit in the solenoid of

$$\begin{pmatrix} \Delta y \\ \Delta y' \end{pmatrix}_2 = \begin{pmatrix} -\frac{3}{2}\theta\psi L \\ -\theta\psi \end{pmatrix} = \begin{pmatrix} -0.015\text{ mm} \\ -0.009\text{ mrad} \end{pmatrix} \tag{9}$$

at injection, with no horizontal orbit error to first order in θ. The change in linear coupling from this orbit skew is also of order $\theta\psi$, and so is a small perturbation on the already small coupling from the solenoid.

The general closed orbit offset created by a single orbit error $(\Delta y, \Delta y')$ at a point with beta function β_{y0}, and observed at phase ϕ_y and beta function β_y is

$$y_{\text{co}}(\phi_y) = \frac{\sqrt{\beta_y}}{2\sin(\pi Q_y)} \left[\frac{\Delta y}{\sqrt{\beta_{y0}}} \sin(\pi Q_y - \phi_y) + \Delta y' \sqrt{\beta_{y0}} \cos(\pi Q_y - \phi_y) \right]. \tag{10}$$

This orbit error is worst at injection, giving a closed orbit error in the IR triplets of 0.75 mm as compared to 0.28 mm at low-beta storage; this scales linearly with the solenoid strength and is locally correctable near the IR.

6 CONCLUSIONS

The dominant source of coupling in the RHIC interaction regions is quadrupole roll in the IR triplets at low beta, which may be corrected locally by correcting vertical dispersion in each triplet through the low-beta squeeze. The variation of vertical dispersion in the triplets with $\beta^\star$ is a convenient diagnostic for the presence of local coupling in the IR. Coupling from the PHENIX solenoid is negligible at all RHIC energies, as is vertical dispersion introduced by this coupling. The orbit skew through this solenoid when colliding protons and gold ions in RHIC creates an orbit error of 0.75 mm at injection in the triplet quads, and is correctable by standard techniques.

7 REFERENCES

[1] S. Peggs, "Coupling and Decoupling in Storage Rings", in IEEE Trans. Nucl. Sci., Vol. NS-30, No. 4, August 1983, pp. 2460-2464.

[2] R. Larsen, SPEAR-107 (March 1971).