RHIC Susceptibility to Variations in Systematic Magnetic Harmonic Errors *

G.F. Dell, S. Peggs, F. Pilat, T. Satogata, S. Tepikian, D. Trbojevic, and J. Wei
Brookhaven National Laboratory
Upton, New York 11973, USA

Abstract

Results of a study to determine the sensitivity of tune to uncertainties of the systematic magnetic harmonic errors in the 8 cm dipoles of RHIC are reported. Tolerances specified to the manufacturer for tooling and fabrication can result in systematic harmonics different from the expected values. Limits on the range of systematic harmonics have been established from magnet calculations, and the impact on tune from such harmonics has been established.

1 INTRODUCTION

Higher order magnetic fields from superconducting dipoles and quadrupoles depend on the profile of the cross section of the magnet iron as well as the location, orientation, and uniformity of the superconducting cable. The profiles for RHIC dipoles and quadrupoles have been carefully iterated to adjust allowed harmonics to acceptable values. In particular, the values of systematic b_2 (sextupole) and b_4 harmonics in the arc dipoles have been reduced to zero at excitation corresponding to the transition energy. The systematic multipoles corresponding to the magnet design are termed "expected" systematic multipoles and are denoted by $< b_n >$ and $< a_n >$ for the normal and skew components, respectively.

Production magnets will most probably have harmonic errors that differ from their expected values. These deviations are unknown but are divided into random and systematic components. The random component arises from sources that differ from magnet to magnet and include variations in coil size and positioning that are within specified manufacturing tolerances. These errors have the usual notation of σb_n and σa_n. The unknowm systematic components have sources, such as tolerances in tooling, that are common to all magnets. These errors are denoted by $d < b_n >$ and $d < a_n >$. The concept of $d < b_n >$ and $d < a_n >$ provides a useful tool during pre and early production to establish the possible limits of systematic errors. As production progresses and the number of magnets increases, systematic and random errors will be defined, and the $d < b_n >$ and $d < a_n >$ will become zero.

The tuneshift from random errors should be small. On the other hand, the $d < b_n >$ and $d < a_n >$ will produce nonzero tuneshifts. The sign of the $d < b_n >$ and $d < a_n >$ is not known apriori; hence the systematic harmonic errors are expected to be within the limits:

$$< b_n > -d < b_n > \ \leq b_n \leq \ < b_n > +d < b_n > \quad (1)$$

$$< a_n > -d < a_n > \ \leq a_n \leq \ < a_n > +d < a_n > \quad (2)$$

The present study was made to determine whether the tuneshifts produced by systematic harmonic errors whose values are at the center or extremes of the limits defined in Eq.1 and Eq.2 are acceptable.

2 MULTIPOLES

2.1 *Magnet body*

The profiles of RHIC dipoles and quadrupoles have been iterated to adjust allowed magnet harmonics to desired values. Small systematic deviations from the ideal design can produce significant changes, $d < b_n >$ and $d < a_n >$, in the allowed systematic harmonic errors. The multipoles for arc dipoles at injection are listed in Table 1.

2.2 *Magnet ends*

The fields from the magnet ends, denoted by lead and nonlead, have been parametrized with integral coefficients B_n, $d < B_n >$, σB_n, A_n, $d < A_n >$, and σA_n. The ends of the magnet coils have been tailored to minimize the sum of harmonic errors from the body and the ends.

2.3 *Magnet orientation*

Magnet multipoles are specified for a standard geometry in which the measuring probe is inserted from the nonlead end. When the probe is inserted from the lead end, certain order multipoles change sign. In dipoles odd order b_n's and even order a_n's change sign; in quadrupoles even order b_n's and odd order a_n's change sign.

Table 1: Expected systematic and random harmonic errors in the RHIC arc dipoles at injection (primed units)

n	$< b_n >$	$d < b_n >$	σb_n	$< a_n >$	$d < a_n >$	σa_n
1	.4	.0	.8	0.3	1.	1.3
2	-3.5	4.0	2.3	-.4	.0	.5
3	.2	.0	.3	.0	.3	1.0
4	-.2	1.0	.6	.0	.06	.2
5	.0	.03	.1	-.1	.0	.26
6	.15	.1	.2	.0	.03	.1
7	.0	.03	.1	.0	.03	.1
8	.3	.1	.1	.0	.03	.1
9	.0	.03	.1	.0	.03	.1

*Work performed under the auspices of the U.S. Department of Energy

All arc dipoles have the same physical orientation in both rings of RHIC. In the "Blue ring", where the beam traverses in a clockwise direction, the arc dipoles are positioned in the positive sense; in the "Yellow ring" they are oriented in the negative sense. Orientation of all quadrupoles and most insertion dipoles is mirror symmetric with respect to the crossing points; hence half have positive and half have negative orientations. The signs of multipoles in all magnets have been assigned to be consistent with the magnet orientation.

3 TUNE DETERMINATION

3.1 Baseline Lattice

The baseline lattice includes all linear effects plus contributions from expected systematic harmonic errors. It includes:

1). random displacements of all elements with $\sigma_x = \sigma_y = 0.5mm$,
2). random rolls of all elements, $\sigma_{theta} = 0.001 radian$,
3). σb_1 and σa_1 in all elements,
4). $< b_n >$ and $< a_n >$ for $2 \leq n \leq 9$,
5). decoupling for skew quadruole errors, and
6). chromaticity corrected to nominal values (-3 for injection or 2 for storage).

3.2 Procedure

Tuneshifts produced by magnetic harmonic errors are functions of the momentum error $\Delta p/p$ and the initial action, $\epsilon_{tot} = \epsilon_x + \epsilon_y$. Tunes have been determined by particle tracking for $\epsilon_{tot} = (n \times \sigma)^2/\beta^*$ when $n = 2, 4, 6$ and 7 with aspect ratios, $\epsilon_x/\epsilon_{tot} = 0.96, 0.75, 0.50, 0.25$, and 0.04 at $\Delta p/p = 0$ and $\pm 2.5(\sigma_p/p)$ where $\sigma_p/p = 0.044$ percent at injection. The average horizontal and vertical phase advance per turn are plotted in tune space. Points corresponding to the same initial action and points corresponding to the same initial aspect ratio are connected with lines. The results, a "*tuneleaf*" plot, are shown in Figure 1 for the baseline lattice when a particular seed is used to initiate the random number generator.

3.3 Baseline Lattice plus $d < b_n >$

Tuneshifts have been determined when a selected $d < b_n >$ is added as a perturbation to the baseline lattice. Inspection of Table 1 indicates the allowed harmonics are largest. Of these, b_2 is corrected to first order by sextupole correctors. The expected value of $< b_4 >= 0.2$ is small, but $d < b_4 >= 1$ is large and produces an appreciable tuneshift. This is shown in Figure 2.

The most pronounced effect occurs when $d < b_4 >$ is added in the arc dipoles; the resulting tuneleaf plot is shown in Figure 2. A simple model is adequate to explain the basic features as being the summation of tuneshifts from $< b_3 >$ and $< b_4 > +d < b_4 >$. The tuneshifts from these multipoles have the forms:

$$\Delta\nu_x(3) = b_3[A_3 \times \eta_x(\tfrac{dp}{p})^2 + B_3 \times f_3(\epsilon_x, \epsilon_y)] \quad (3)$$

$$\Delta\nu_x(4) = b_4[A_4 \times \eta_x(\tfrac{dp}{p})^3 + B_4(\tfrac{dp}{p}) \times f_4(\epsilon_x, \epsilon_y)] \quad (4)$$

where A_3, A_4, B_3, and B_4 are constants and $f_3(\epsilon_x, \epsilon_y)$ and $f_4(\epsilon_x, \epsilon_y)$ are functions of ϵ_x and ϵ_y.

The contribution to $\Delta\nu_x(3)$ from action is independent of $\Delta p/p$, while the the contribution from action to $\Delta\nu_x(4)$ is an odd function of $\Delta p/p$. In general, for multipole orders > 2, tuneshifts from odd ordered multipoles are symmetric functions of $\Delta p/p$, and tuneshifts from even order multipoles are antisymmetric functions of $\Delta p/p$.

Inspection of Table 1 indicates the only nonzero, unallowed harmonics are $< b_1 >$ and $< b_3 >$. The basic shape of each leaf in Figure 1 results from $< b_3 >= 0.2$ The superposition of $d\nu_x(3)$ and $d\nu_x(4)$ causes addition of tuneshifts at one extreme of $\Delta p/p$ and cancellation at the other extreme. Reversing the sign of either b_3 or b_4 reverses the sense of addition and cancellation.

3.4 Baseline Lattice with Grumman Dipoles

The magnet multipoles, measured at 660A for the first Grumman dipole DRG101, are listed in Table 2. The column labels b_n and a_n indicate multipole coefficients per unit length, while the labels B_n and A_n indicate integral values for the magnet ends.

A tuneleaf plot, generated when the multipoles from DRG101 were used as systematic multipoles in all 8 cm dipoles of the arcs and insertions, is shown in Figure 3. The shape suggests the tuneshift from b_4 is increasing the tunespread at $\Delta p/p = +0.11$ percent and decreasing it at $\Delta p/p = -0.11$ percent. Change of orientaion of all leaves results from the change of sign of both b_3 and b_4 used to generate Figure 3.

Table 2: Measured multipoles in RHIC arc dipole DRG101 at injection (primed units)

	Body		Lead		Nonlead	
n	b_n	a_n	B_n	A_n	B_n	A_n
1	-0.45	1.24	1.02	-3.25	0.57	-1.46
2	-1.20	-0.03	19.34	-9.97	2.64	0.33
3	-0.15	0.47	0.22	0.26	0.06	-0.31
4	-1.63	0.02	0.31	2.16	0.63	0.03
5	-0.01	0.02	0.06	0.06	0.00	0.04
6	-0.45	0.00	1.15	-0.91	0.06	-0.01
7	-0.01	-0.02	-0.01	0.06	-0.02	0.03
8	0.26	0.00	-0.06	0.22	-0.21	0.01
9	0.04	0.05	-0.03	0.00	-0.07	0.03

4 RESULTS

The investigation of impact on tunes from harmonic errors in arc dipoles being different than expected indicates the only multipoles of concern are b_2 and b_4. The large uncertainty represented by $d < b_2 >= 4$ in Table 1 is of little concern at injection where the magnetic rigidity of the charged particles is low. The adequacy of sextupole correctors when storing heavy ions at $100GeV/u$ is sufficient when as many as four insertions are operating at $\beta^* = 1m$ [1]. The importance of uncertainty in $d < b_4 >$ at storage is expected to be negligible, since the aperture there is limited in the high β quadrupoles and the corresponding betatron amplitudes in the arc dipoles will be small. The principal concern is the $d < b_4 >$ at injection. If b_4 is near the extremes in Eq.1, it may be necessary to use decapole correctors or make a slight dipole cross section modification [2].

5 REFERENCES

[1] G.F. Dell and S. Peggs, Parametrization of Sextupole Correctors, RHIC/AP/21, March 11, 1994

[2] G.F. Dell, F. Pilat, and S. Peggs, Modification of RHIC Dipole Cross Section, RHIC/AP/32, June 1994

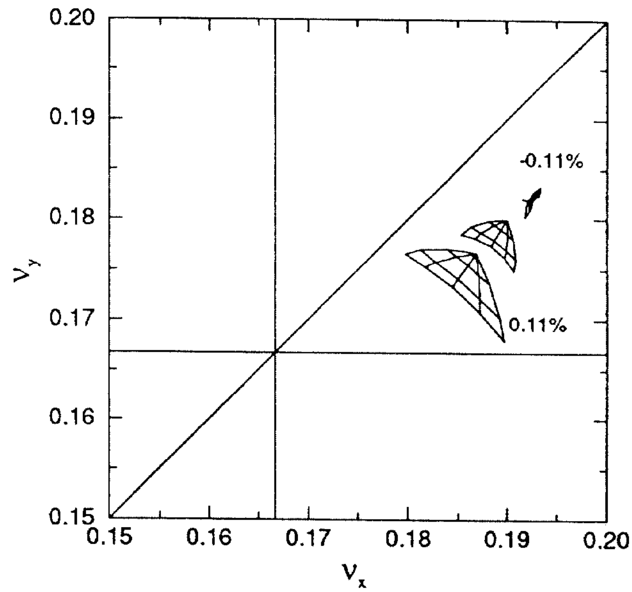

Figure 2: Tuneleaf when $d < b_4 >= 1$ in 8 cm dipoles is added as a perturbation to the baseline lattice: $b_3' = 0.2$ and $b_4' = +0.8$

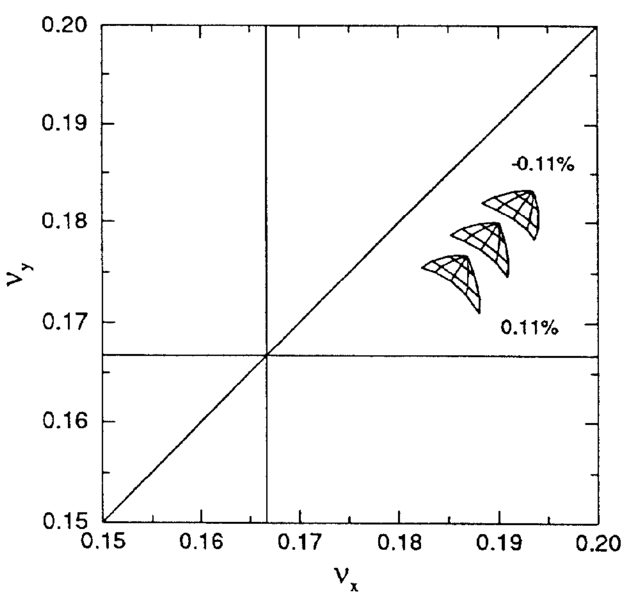

Figure 1: Tuneleaf for the baseline lattice. All $d < b_n >$ and $d < a_n >$ are zero in all elements. In 8 cm dipoles $b_3' = 0.2$ and $b_4' = -0.2$

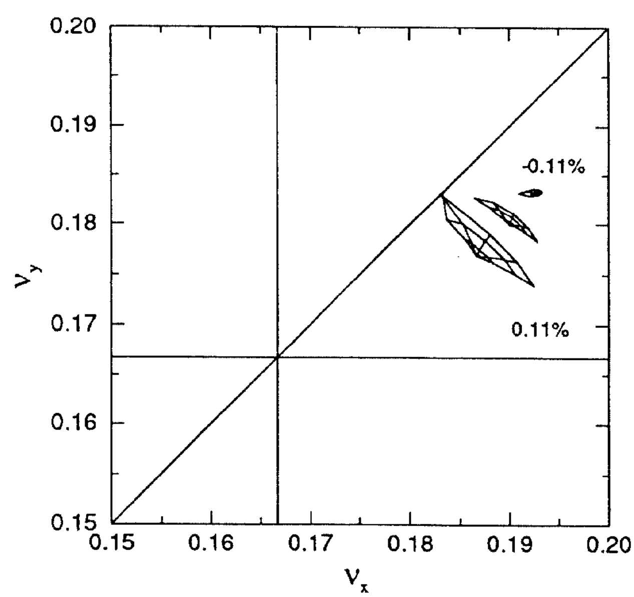

Figure 3: Tuneleaf when multipoles from Grumman dipole DRG101 are used in all 8 cm dipoles as a pertubation of the baseline lattice: $b_3' = -0.15$ and $b_4' = -1.63$