Transverse Mode Coupling Instability with a Double RF System*

Yong Ho Chin
Lawrence Berkeley Laboratory
University of California
Berkeley, CA 94720

Abstract

Effects on a transverse mode coupling instability in a storage ring by a strongly deformed rf bucket in a double rf system are studied. An eigenvalue equation, respectively the vanishing of the determinant of an infinite matrix, is derived from a Hamiltonian formalism. Truncation of this determinant permits solution of the problem on a computer, and a code MOSDRF has been written which finds the complex mode frequencies. The stability limits are determined by equating the imaginary part of the solution to the radiation damping rate. The theory was applied to LEP with third higher-harmonic cavities, and the results were compared with those obtained by the Multi-particle simulation program FEDBAK. They agree well, and show that the threshold current will be reduced rather than improved.

1 INTRODUCTION

A number of methods have been proposed to overcome the limitation of bunch current by the transverse mode-coupling (TMC) instability. One means to increase the threshold current is to lengthen the bunch, e.g., by wigglers or changing the damping partition numbers. However, this method is limited by the concurrent increase of the energy spread which cannot exceed the energy aperture of the machine. A way to increase the bunch length without increase of energy spread is the use of a second "higher harmonic" rf system [1]. The increase of the bunch length is largest when the phase and amplitude of the second rf system is adjusted such that the slope and the second derivative of the total rf voltage are zero at the bunch center (then, the synchrotron frequency vanishes there).

The effects on the TMC instability is then not so obvious. On one hand, a longer bunch and a larger spread in synchrotron frequency should increase the threshold current, but on the other hand, a smaller average synchrotron frequency tends to reduce it. The zero synchrotron tune at the bunch center may imply that the TMC instability can occur even at zero bunch current. In order to study this situation in detail, it was necessary to develop a theoretical description which permits study of the equations of motion in a strongly deformed rf bucket. This is done by describing the system in a Hamiltonian formalism [2]. The result can be expressed as an eigenvalue equation, respectively the vanishing of the determinant, of an infinite matrix. Such a problem can be solved numerically by truncating the matrix to finite dimensions, given by the product of the number of radial and azimuthal modes to be included. For not too long bunches, these dimensions can be kept to quite small values.

The eigenvalues yield the complex mode-frequencies - and hence the frequency shift and growth rates - as function of bunch current. Contrary to the problem for a single rf system, there is always a finite imaginary part, i.e. the growth rates are always non-zero. However, due to radiation damping (neglected in the Vlasov equation formulation), a finite threshold is found nevertheless. The computer code MOSDRF has been written and is available on the CERN-IBM system. It has been applied to a third harmonic system for LEP at injection. Simulations have been also carried out using Myers' multi-particle simulation program FEDBAK [3]. Both results show excellent agreement. It is found that the third harmonic rf system does not increase the TMC threshold at LEP.

2 DISPERSION RELATION

In this paper, we show only a final result of the Hamiltonian formalism. The derivation is given in ref. [2]. The complex coherent tune ν can be obtained by solving the following dispersion relation:

$$det(\delta_{mn}\delta_{kl} - N_{nl}^{mk}) = 0, \tag{1}$$

where the matrix N_{nl}^{mk} is given by

$$N_{nl}^{mk} = \sum_{j=0}^{\infty} F_{kj}^{(m)} M_{nl}^{mj}, \tag{2}$$

where

$$\begin{aligned} F_{kj}^{(m)} &= \sum_{q=-\infty}^{\infty} (\delta_{1q} - \delta_{-1q}) \\ &\times \int_0^{\infty} dz \frac{\exp(-\lambda z^4) z^{2|m|+2} e_k^{(|m|)}(z) e_j^{(|m|)}(z)}{\nu - m\nu_s(z) - q\nu_y} \end{aligned} \tag{3}$$

and

$$\begin{aligned} M_{nl}^{mj} &= -i \cdot 0.05104 \frac{I_b \beta_y}{E_0/e} \\ &\times i^{n-m} \sum_{p=-\infty}^{\infty} Z_T(p') I_{mj}(\frac{p'}{h}\sigma_\phi) I_{nl}(\frac{p'}{h}\sigma_\phi), \end{aligned} \tag{4}$$

*This work was supported by the Director, Office of Energy Research, Office of High Energy and Nuclear Physics, High Energy Physics Division, of the US Department of Energy under Contract No. DE-AC03-76SF00098.

where Z_T is the transverse impedance at harmonic $p' = p + \nu$. Other notations are defined as follows: h is the harmonic number, I_b is the bunch current, β_y is the beta function at impedance, E_0 is the energy of the synchronous particle, e is the elementary charge, ν_y is the betatron tune, and $\lambda = 0.11419$. The quantity σ_ϕ is the rms bunch length in units of rf phase angle in the double rf system, and can be calculated from the rms bunch length in the single rf system σ_s as

$$\sigma_\phi = \frac{1.28678}{(n^2-1)^{1/4}}(\sigma_s \frac{h}{R})^{1/2}, \quad (5)$$

where R is the average machine radius and n is the ratio of the harmonic number of the higher-harmonic rf system to that of the main rf system. In Eq. 3, the amplitude dependent synchrotron tune is given by

$$\nu_s(z) = 0.34587 \cdot \nu_{s0}\sqrt{n^2-1}\sigma_\phi z, \quad (6)$$

where ν_{s0} is the synchrotron tune in the absence of the higher-harmonic system. The function $e_k^{(|m|)}(z)$ is given by

$$e_k^{(|m|)}(z) = 2\lambda^{\frac{|m|}{4}+\frac{3}{8}}(\frac{k!}{\Gamma(\frac{|m|}{2}+k+\frac{3}{4})})^{1/2} L_k^{(\frac{|m|}{2}-\frac{1}{4})}(\lambda z^4), \quad (7)$$

where $L_k^{(\alpha)}$ are the generalized Laguerre polynomials, and Γ is the Gamma function. The function $I_{mj}(\frac{p'}{h}\sigma_\phi)$ is given by

$$I_{mj}(\frac{p'}{h}\sigma_\phi) = \int_0^\infty dz \mathrm{J}_m(\frac{p'}{h}\sigma_\phi z)\exp(-\lambda z^4) z^{|m|+2} e_j^{(|m|)}(z), \quad (8)$$

where J_m is the Bessel function.

3 CALCULATION RESULTS

Let us apply the present formalism to LEP to see how the mode-coupling instability will be affected by installation of third harmonic cavities into the current single rf system. The main LEP parameters used for calculations are summarized in Table 1. The impedance model is based on Zotter's estimate [5] (the peak value is inflated to accommodate the contribution from bellows in this single resonator model). Figures 1 and 2 show analytical results of the coherent tune shift and the growth (damping) rate of an unstable mode in the double rf system. Five azimuthal modes (m=-3,-2,-1,0,1) and four radial modes (k=0,1,2,3) were included in this calculation. Calculations with a larger number of modes confirmed that the results were converged with these dimensions. Contrary to the problem for a single rf system, the growth rate is always non-zero. By equating the growth rate to the radiation damping rate, one can determine the threshold current to be 0.306 mA. The corresponding tune shift is -0.0153.

Simulations have been also carried out using the program FEDBAK for the identical LEP parameters. The squares and diamonds in Figure 3 show the bunch length σ_z and the threshold current I_{th} as a function of the rf phase of the 3rd harmonic cavity, respectively. The bunch length is largest at the right phase which diminishes the slope and the second derivative of the total rf voltage. The threshold current is found to be 0.305 mA, in excellent agreement with the analytical result of 0.306 mA. Figure 4 shows the coherent vertical tune shift as a function of the rf phase of the 3rd harmonic cavity. Simulation results marked by squares agree well with the analytical result of -0.0153.

Next, let us examine effects of the bunch length on the threshold current. Figure 5 shows the threshold current as a function of the bunch length, when the energy spread is increased from the nominal value (the maximum bunch length σ_z=6.3cm corresponds to the case where the energy spread is twice larger than the nominal value). Results by the theory and simulations show excellent agreement over the wide range of the bunch length. Both results show an improvement of the threshold current. However, calculation results with the same energy spread, but without higher harmonic cavities, show an even better improvement. For instance, the threshold current with doubled σ_ϵ/E in the single rf system is about 1.4 mA, while the corresponding value in the double rf system is only 0.9 mA.

Table 1: Main LEP parameters used for the calculations.

Beam energy, E_0 (GeV)	20.0
Average machine radius, R (km)	4.2429
Momentum compaction factor, α	0.000387
Harmonic number of the single rf system, h	31320
Rms bunch length in the single rf system, σ_s (cm)	2.5
Rms relative energy spread in the single rf system, σ_ϵ/E_0	0.00137
Radiation damping time, τ_y (sec)	0.405
Synchrotron tune of the single rf system, Q_s	0.09
Ratio of the higher-harmonic frequency to the main frequency, n	3
Peak voltage of the main rf system, V_{1rf} (MV)	84
Peak voltage of the 3rd harmonic rf system, $V_{3rf}(MV)$	27.6
Rms bunch length in the double rf system, $\sigma_\phi R/h$ (cm)	4.44
Beta function at the impedance, β_y (m)	40.7
Resonant frequency of the broadband impedance, f_r (GHz)	2.0
Peak value of the broadband impedance, R_r (MΩ/m)	2.0
Q-factor of the broadband impedance, Q	1.0

4 CONCLUSIONS

The results by the theory and simulations on the TMC instability in the double rf system at LEP show excellent agreement. Both results predict that higher harmonic cavities will not improve the stability limit at injection.

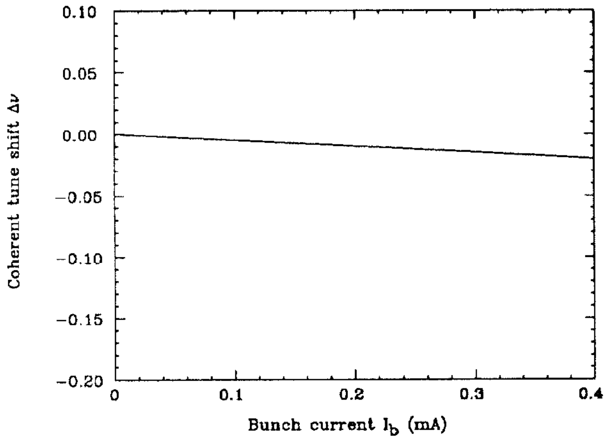

Figure 1: Coherent tune shift as a function of the bunch current.

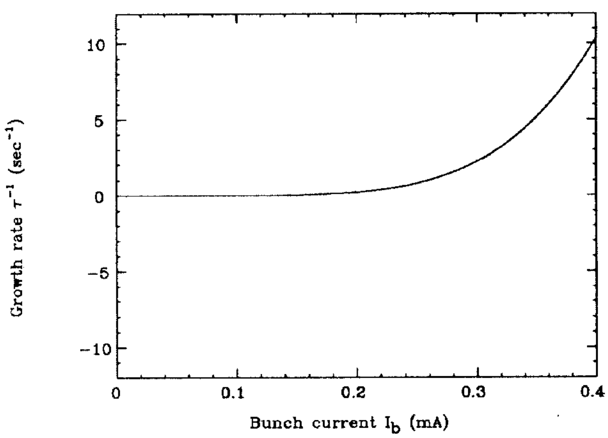

Figure 2: Growth rate as a function of the bunch current.

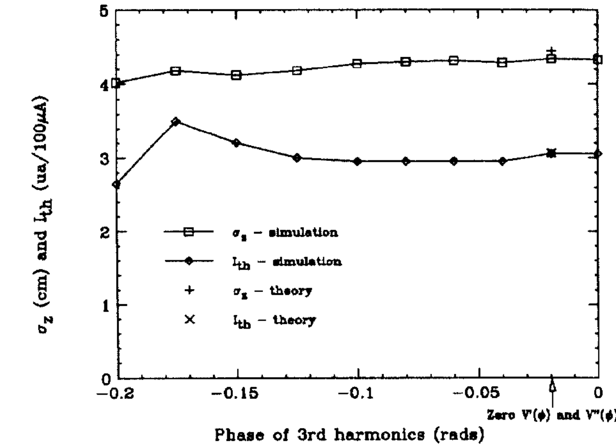

Figure 3: Bunch length and the threshold current versus the rf phase of the 3rd harmonic cavity.

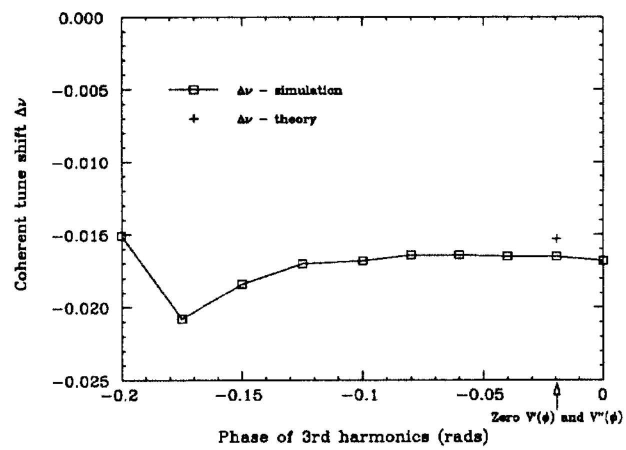

Figure 4: Coherent tune shift versus the rf phase of the 3rd harmonic cavity.

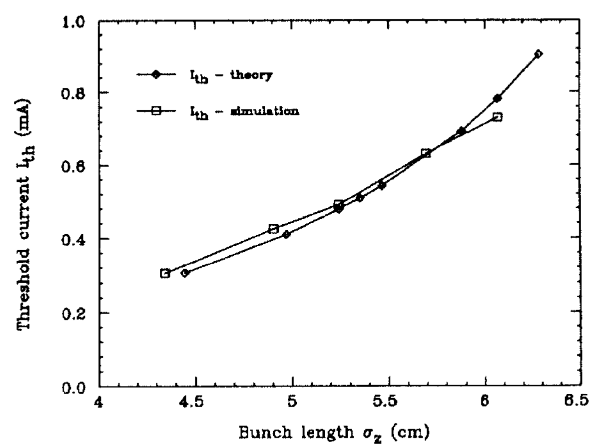

Figure 5: The threshold current as a function of the bunch length.

5 REFERENCES

[1] A. Hofmann, Proc. 3rd Workshop on LEP Performance, Chamonix, 1993, Ed. J Poole, CERN SL/93-19 (DI), p.493 (1993).

[2] Y. H. Chin, "Transverse mode coupling instability in a double rf system", CERN SL/93-03 (AP), 1993, and to be published from Part. Accelerators.

[3] S. Myers, talk at the 3rd Workshop on LEP Performance, Chamonix, 1993.

[4] S. Myers and J. Vancraeynest, CERN-LEP-RF/84-13 (1984).

[5] B. Zotter, Proc. 3rd Workshop on LEP Performance, Chamonix, 1993, Ed. J Poole, CERN SL/93-19 (DI), p.89 (1993).