

# An Impedance Model of the Relativistic Heavy Ion Collider, RHIC \*

S. Peggs, V. Mane, W.W. MacKay, M. Blaskiewicz, R. Connolly,  
D.P. Deng, A. Ratti, J. Rose, T. Shea, J. Wei  
Brookhaven National Laboratory  
Upton, NY 11973

## Abstract

This paper is an abbreviated version of a comprehensive and detailed analysis of RHIC instabilities soon to be published as a RHIC project report[1]. It emphasizes longitudinal impedance modeling and design choices in RHIC, while a companion paper emphasizes instability calculations[2].

## 1 INTRODUCTION

RHIC is designed to collide fully stripped ions from protons to gold, at a maximum energy of 250 GeV (protons) or 100 GeV/amu (gold). The two intersecting superconducting rings, Blue and Yellow, provide the possibility of head-on collisions in up to six interaction regions. The circumference of 3.834 km yields a maximum revolution frequency of 78.2 kHz. There are four phases to the machine cycle: injection, acceleration, rebucketing, and storage. Ions other than protons cross transition during the acceleration part of the cycle, when the Lorentz factor  $\gamma = \gamma_t = 22.89$ . Rebucketing occurs at top energy when beam is transferred from the 26 MHz rf buckets used for injection and acceleration, to the 196 MHz buckets used for storage[3].

Up to twelve bunches of protons are accelerated in one AGS batch, or three bunches per batch for other ion species. Bunches are injected one at a time into RHIC. Successive AGS cycles alternate between filling Blue and Yellow rings, taking about 30 seconds to fill each ring with 57 bunches of protons, and about 110 seconds for other ion species.

Beam acceleration will take about 60 seconds, limited by the rate at which the superconducting magnets can ramp from about 0.4 T to 3.5 T. RHIC will be the first superconducting accelerator to cross transition. It will also employ the first ever *linear bipolar* transition jump[4, 5]. Without a jump, the effective (Jøhnsen) time to cross transition would be about 140 ms, but with a nominal jump of  $\Delta\gamma_t = -0.8$  in 60 ms, the time is only about 17 ms.

Rebucketing from the 26 MHz rf system to the 196 MHz system occurs at top energy. The beam is elongated on an unstable fixed point by shifting the 26 MHz synchronous phase, before a bunch rotation around a stable fixed point. When the bunch is short in time but tall in momentum spread, the 26 MHz system is turned off and the 196 MHz system turned on. The whole process takes about 20 ms.

Storage times will vary from a few hours to about 10 hours. A “low beta squeeze” reduces the collision point beta from about 10 m to as little as 1 m in some experiments soon after storage begins. The dominant multi-particle effect during storage is intrabeam scattering, which is discussed elsewhere[3, 6].

Vulnerability to instabilities is largest during injection and storage, and during transition crossing. For example, emittance blowup at transition is found to be a problem when RHIC is modeled by a constant  $|Z/n|$  value of 1.5  $\Omega$  or more[5]. In reality, of course, the impedance of RHIC will be a much more complicated function of frequency,  $f$ .

## 2 DOMINANT IMPEDANCES

An object resembling a closed cavity can support narrow-band (high  $Q$ ) resonances, and is represented by a set of  $R$  and  $Q$  values for those resonances. Conversely, an open ended device from which energy easily radiates is often represented by a single  $Z/n$  broadband value, implicitly assuming that  $Z/n$  is approximately constant over the frequency range of interest, and that the impedance is inductive. The RHIC goal is to maintain a “reasonably” accurate impedance model up to frequencies of 3 GHz, corresponding to the power spectrum of the shortest bunches expected. This is slightly less than the cutoff frequency of 3.32 GHz corresponding to the standard beampipe diameter of 6.91 cm, and so the broadband assumption is not necessarily valid.

### 2.1 Simple inductive devices

Small vacuum chamber discontinuities in RHIC, such as aperture transitions and expansion bellows, are normally inductive. If the bunch length obeys the relationship  $\sigma \geq d/4$ , where  $d$  is the beampipe diameter, then little of its power spectrum ( $\leq 9\%$ ) is above the cutoff frequency[7]. The beam leaves little energy behind when it passes by, since the beampipe will not support free waves below cutoff. The head of the beam will lose energy to the discontinuity, but the tail will reabsorb most of it. Above transition, the voltage induced when a bunch passes by an inductive object has a slope opposite in sign to the rf system wave slope. It tends to lengthen the bunch.

Simple expressions for the inductive impedance of common elements in RHIC are surprisingly accurate. For example, if the radius of the beampipe increases from  $r_1$  to  $r_2$  by an amount  $\Delta \ll r_1$  in an aperture transition with a

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flare half-angle of  $\theta$ ,

$$\frac{Z}{n} = -iZ_0 \frac{3}{2C} \left( \frac{r_1 \Delta^2}{r_2^2} \right) \left( \frac{2\theta}{\pi} \right)^{1/2} \quad (1)$$

where  $C$  is the circumference and  $Z_0 = 377 \Omega$  is the free space impedance[7]. (Note that this paper adopts the Chao sign and nomenclature conventions[8]). Similarly, a vacuum bellows with a total of  $N$  small rectangular corrugations, each of length  $g$  and depth  $\Delta$ , on a beampipe of nominal radius  $r$ , has an approximate impedance of

$$\frac{Z}{n} = -iZ_0 \frac{N}{C} \left( \frac{g\Delta}{r} \right) \quad (2)$$

A numerical investigation has shown that these expressions are accurate at the 20% level for common RHIC components, up to frequencies of 1.5 GHz[9].

## 2.2 Beampipe aperture transitions

Dipoles and quadrupoles in the arcs have a coil internal diameter (ID) of 8 cm while the Interaction Region quadrupoles have a coil ID of 13 cm. A tapered vacuum chamber segment makes the transition between beampipe IDs of 6.91 cm and 12.28 cm. This segment, often called the “warm to cold transition”, also makes a thermal transition from cryogenic to room temperature. It is a prominent example of the numerous aperture transitions in RHIC, some of which are individually tailored to specialised local needs, and all of which contribute to the total impedance.

Eq. 1 shows that the impedance is significantly reduced if an aperture transition occurs gradually, rather than abruptly. A warm to cold transition with a taper length of 13.4 cm, 5 times as long as the difference in pipe radii, illustrates the “five-to-one” rule that is applied to RHIC aperture transitions, wherever possible. Figure 1 shows predicted contours of constant  $\text{Im } Z/n$  in the space of the beampipe radii, when the five-to-one rule is applied.

Figure 2 shows the imaginary part of the impedance of the warm to cold transition, as calculated by the numerical code MAFIA[10]. The impedance is adequately described by the  $Z/n$  value of  $-0.35 i m\Omega$  predicted by Eq. 1 and

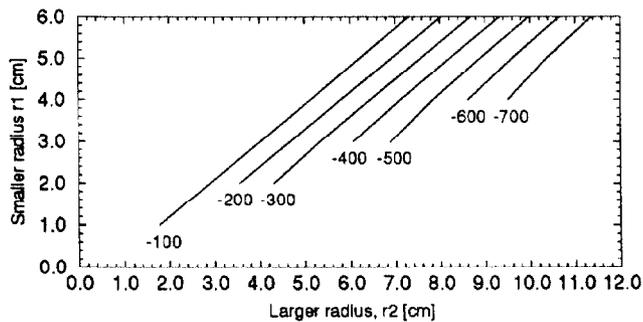


Figure 1: Contours of constant  $Z/n$  [ $i \mu\Omega$ ], for a RHIC aperture transition with a five-to-one flare.

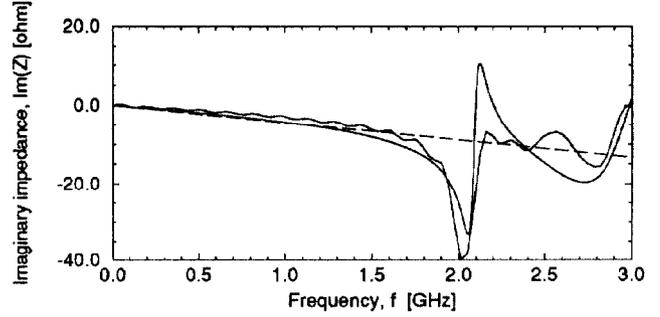


Figure 2: Impedance of a RHIC warm to cold transition. The complex curve is a numerical calculation, the smooth curve is a two resonance fit, and the dashed straight line is  $Z/n = -0.35 i m\Omega$ .

Figure 1. Since there are 16 such transitions in RHIC, they contribute only about  $-0.006 i \Omega$  to the total broadband  $Z/n$ .

## 2.3 Expansion bellows

RHIC has approximately one vacuum bellows per magnet, to allow for thermal expansion and contraction. The most common are the 408 cold bellows with an ID of 6.91 cm, and the 72 warm bellows with an ID of 12.28 cm. There are 30(40) corrugations of depth 0.94(1.27) cm and length 0.25(0.32) cm in the common cold(warm) bellows. These bellows alone would contribute a broadband impedance of  $-1.01 i \Omega$  if left unshielded, an unacceptably large value. Consequently, all of the common bellows (except 10 warm ones) will be shielded, although some of the rarer specialised bellows will be unshielded. The unshielded bellows impedance function measured on the bench using a wire method is in good agreement with calculations[11].

## 2.4 Beampipe chamber wall impedance

The “resistive wall” impedance of the cylindrical vacuum chamber is given for all but the highest frequencies by

$$Z = (1 - i) \sum (L/\pi d \sigma \delta) \quad (3)$$

where  $L$  is the length of a section of beampipe with an inner diameter of  $d$ ,  $\sigma$  is the conductivity, and the skin depth is  $\delta = \sqrt{(1/\mu_0 \sigma \pi f)}$  with  $\mu_0$  the permeability of free space[8]. The resistive wall impedance varies like the square root of the frequency, and so it dominates the low frequency end of the impedance model.

A naive but reasonably accurate representation breaks RHIC into two pipes, one with a length of 2.955 km, an ID of 6.91 cm, and a conductivity of  $2.0 \times 10^6 \Omega m$ , the other with a length of 0.879 km, an ID of 12.28 cm, and a conductivity of  $1.0 \times 10^6 \Omega m$ . This “two pipe model” yields a total resistive wall contribution of

$$Z(f) = (1 - i) f^{1/2} [\text{GHz}^{1/2}] 748 \Omega \quad (4)$$

## 2.5 Beam position monitors

174 of the stripline beam position monitors (BPMs) in RHIC have two strips, while 72 BPMs have four. The impedance of a two strip BPM is directly related to its sensitivity, and has the periodic form

$$Z = 2Z_0 \left(\frac{\phi_0}{2\pi}\right)^2 \left[ \sin^2\left(\frac{\omega l}{c}\right) - i \sin\left(\frac{\omega l}{c}\right) \cos\left(\frac{\omega l}{c}\right) \right] \quad (5)$$

where  $l = 23$  cm is the strip length,  $\phi_0 = 80^\circ$  is the angle the strip subtends, and  $Z_0 = 50 \Omega$  is the characteristic impedance of the BPM. Four strip BPMs have twice this impedance. The impedance amplitude of a two strip BPM is  $4.93 \Omega$ , with a period,  $c/2l$ , of 0.65 GHz. In the low frequency limit, the inductive contribution of all BPMs to the total  $Z/n$  is  $-0.58 i \Omega$ .

## 2.6 Vacuum pump-out ports

About half of the expansion bellows in the arcs have an associated vacuum pump out port, in the form of a cylindrical pipe intersecting the beampipe at right angles to make a "T". In most cases the side pipe ID is 2.2 cm and the beampipe ID is 6.91 cm. Because the side pipe is relatively narrow, very little wake field penetrates into it, and the associated impedance is very small. The net effect of all 224 such ports, left unshielded, is a  $|Z/n|$  value of less than  $0.001 \Omega$ . However, it is necessary to shield the approximately 50 pump-out ports per ring where the side pipe ID is about 12 cm and the beampipe ID is 12.28 cm. A rectangular wire mesh with a coarse spacing of 3 cm placed over the end of the side pipe removes essentially all of the impedance contribution of these "warm pump-out ports", while causing a negligible loss in the pumping conductance[12].

## 2.7 Other devices

Other devices contribute to the total longitudinal impedance, but are not discussed in any detail here.

Vacuum gate valves are numerous, but contribute a negligible impedance because they are all shielded. Radio frequency cavities are the dominant source of narrowband resonances in RHIC[2]. The Y-shaped beam chambers near the interaction region beam splitting dipoles, and in the injection Lambertson septum, both need further study. Beampipe designs for the abort line and for the injection kickers are still being discussed. Effects of the 4 meters of ferrite-dielectric sandwiches in the injection kicker are under study. The finite curvature of the vacuum pipe makes a negligible impedance contribution. Beam collimators are present in small numbers, with a minor contribution to the broadband impedance. They will be shielded to avoid a potentially significant narrowband contribution. The PHO-BOS experiment has a rather unusual vacuum chamber with 6 cavities that is under study.

## 3 IMPEDANCE DATABASE

The aperture and impedance tables in the *beampipe* database contain an inventory of the geometries and impedances of significant RHIC devices, so that an accurate impedance model may be easily maintained. Each object in the machine is described as a *generic slot*, which is further divided into segments. Geometry parameters vary from segment to segment. For example, unshielded bellows are described by the depth of corrugation, length of corrugations, and the pipe aperture. Examples of geometry types are *bellows\_shield*, *bellows\_noshield*, *aperture\_transition*, *pipe*, *rf\_26MHz* and *rf\_196MHz*. The *seginfo* and *impedance\_info* tables give the geometry type and impedance model type for a given segment.

The impedance of a simple object, such as a resistive wall pipe or a BPM, is given by a straightforward analytical expression. The impedance of a complex object, such as a bellows or a pump out port, must be determined numerically, even though approximate expressions may be valid in the low frequency limit. A numerical description is fitted to an appropriate curve, and the fitting parameters stored in one or more database tables. For example, the aperture transition impedance shown in Fig. 2 might be (poorly) represented by the parameters of two resonances in the *resonance* table.

It is practically impossible to trigger a complex external process, such as a MAFIA run with a new input file, when a geometry parameter of a complex object is modified. That is, referential integrity cannot be guaranteed in the simple fashion that comes as an automatic benefit with simpler databases. The capacity to do a post-facto check on referential integrity is kept by maintaining separate aperture geometry and impedance geometry parameters. If the two parameter sets disagree for a complex object, it is time to perform another numerical run.

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