

# Single Bunch Tracking Calculations and Orbit Correction for the SBLC Study

M. Drevlak, R. Wanzenberg  
 DESY  
 Notkestr. 85, 22603 Hamburg, Germany

## Abstract

In linear accelerators, wake field effects and dispersion from orbit distortions can lead to considerable emittance dilution. As the transverse wake fields and the dispersion depend on the orbit in the accelerating structures and quadrupoles constraints are imposed on the resolution of the beam position monitors and on alignment tolerances.

The tracking code L, which is affiliated to the family of MAFIA codes, has been developed to simulate both single- and multi-bunch dynamics in linear accelerators. By use of this code, the impact of misalignments and injection jitter on the dilution of the single bunch emittance is investigated for the S-band accelerator as envisioned in the SBLC study. Further, the performance of several orbit and emittance correction algorithms is examined. From the results, limits for the alignment tolerances are deduced.

## 1 INTRODUCTION

To provide favourable experiment conditions with high luminosity, it is a fundamental requirement for any linear collider to maintain a small emittance of the beam. However, there are several effects which contribute to a strong dilution of this emittance.

A bunch of charged particles traversing an accelerating structure experiences the energy gain

$$\Delta U(s) = \Delta U_0 \cos\left(\varphi_0 + \frac{\omega_0 s}{c_0}\right) + \int_{-\infty}^s \rho(\zeta) W_{\parallel}(s - \zeta) d\zeta \quad (1)$$

If, due to structure misalignment or a non-vanishing trajectory, this structure is traversed off axis, a trailing particle with the charge  $q$  experiences a transverse kick from leading particles

$$p_{\perp}(s) = q \int_{-\infty}^s \rho(\zeta) \Delta r(\zeta) W_{\perp}(s - \zeta) d\zeta \quad , \quad (2)$$

where  $W_{\parallel}$  and  $W_{\perp}$  are the longitudinal and transverse wake functions,  $\rho(s)$  the charge density along the bunch and  $\Delta r(\zeta)$  the displacement of the leading charge with respect to the cavity centerline. Dispersive errors due to the energy spread and transverse kicks, as given by (1) and (2), can lead to a significant increase of the single bunch emittance and establish the necessity of corrective action.

The relevant beam parameters for our case are

- bunch population:  $2.9 \cdot 10^{10}$
- bunch length ( $2\sigma$ ):  $1mm$
- normalized vertical emittance:  $\gamma\epsilon_y = 5 \cdot 10^{-7} rad m$

## 2 LATTICE AND BNS-DAMPING

We have chosen a FODO lattice with  $90^\circ$  phase advance per cell, as such a lattice offers great convenience for the correction methods described below. The  $\beta$ -function

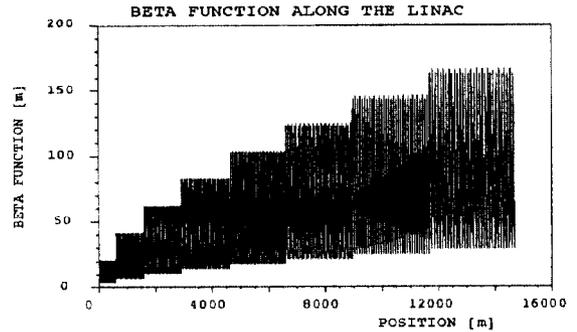


Figure 1:  $\beta$ -function in the main linac

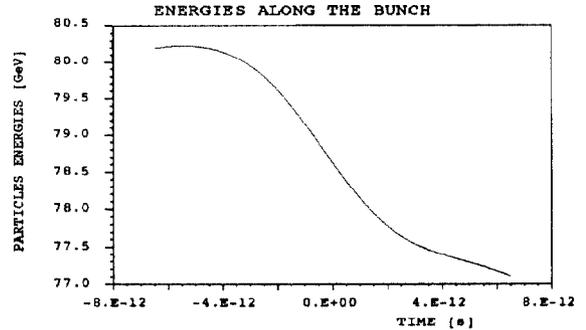


Figure 2: particle energies in the bunch after the first 4 sectors

scales with  $\sqrt{\gamma}$  (see figure 1). It can be seen that the lattice has been divided into 8 sectors of constant focusing strength. In order to minimize the beam blow-up from injection errors and dipole kicks the method of BNS-damping is used ([2]). Here, an energy spread is induced in the bunch by choosing an appropriate RF-phase so that the stronger focusing for the bunch tail compensates with the wake fields caused by non-vanishing offsets in the cavities. The optimum RF-Phase has been found empirically at  $+4^\circ$ , corresponding to an rms energy spread of 0.9%. The first 4 accelerating structures are operated at an rf-phase of  $90^\circ$  to provide an initial energy spread. Figure 2 shows the energy profile in the bunch after the first 4 sectors. To reduce the energy spread to a value

of 0.33% by the end of the linac, the rf-phase in the last 4 sectors of the linac has been set to  $-12^\circ$ . The injection jitter is limited by the bunch height and should not exceed  $\frac{1}{4}\sigma_y$  ( $\approx 6\mu\text{m}$ ).

### 3 BEAM BLOW-UP FROM CAVITY MISALIGNMENT

In this simulation it has been assumed that the cavities are randomly misaligned with  $100\mu\text{m}$  rms. On average the emittance then increases by a factor 2.2. Much of this emittance blow-up can be corrected by trajectory bumps (see [1]). Figure 3 shows a series of 8 non-dispersive bumps (ND-bumps) and then 8 dispersive bumps (D-bumps) at the beginning of a linac. The type of D-bump shown here and to be used later is simply a betatron motion extending over a limited fraction of the linac. Having no dispersive effects, ND-bumps are capable of exactly cancelling the wake field effect from cavity misalignment.

The use of many small bumps is preferable to a single large bump as this reduces possible implications from quadrupole wakes and the effect on the multi-bunch dynamics. Therefore, the bumps to be used in the following will always be distributed over 14 "elementary" bumps.

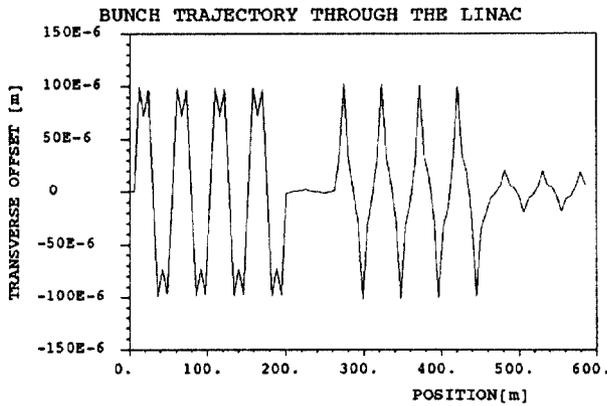


Figure 3: bumps consisting of 8 individual ND-bumps/D-bumps resp.

In the correction procedure the emittance was measured at 4 stations located after the 2nd, 4th, 6th and 8th sector with an accuracy of 3% and the ND-bumps upstream of each station adjusted empirically to optimize the emittance. With this method, the average emittance blow-up was reduced to 16%. The result implies that the cavities must be aligned with an accuracy of  $60\mu\text{m}$ . As an alternative, one could use more ND-bumps and stations for emittance measurement along the linac.

### 4 BEAM BLOW-UP FROM QUADRUPOLE MISALIGNMENT

Quadrupole misalignments lead to considerable trajectory distortions with strong emittance blow-up due to both wake field and dispersive effects. As a cure, various orbit correction techniques are used. In our case the correction

is assumed to be carried out with beam position monitors and correction dipoles located at each quadrupole.

Most current advanced orbit correction schemes attempt to simultaneously minimize the BPM readouts and their variation with changes of selected quadrupole strengths. These algorithms can be expressed as an optimisation problem:

$$\sum_j \left[ \frac{(m_j + \sum_i R_{ji}^{12} \theta_i)^2}{\sigma_m^2 + \sigma_{res}^2} + \sum_{k=1}^K \frac{(\Delta m_j^{(k)} + \sum_i \Delta R_{ji}^{12(k)} \Delta \theta_i^{(k)})^2}{2\sigma_{res}^2} \right] \stackrel{!}{=} \min \quad (3)$$

Here, the  $m_j$  are the orbits as measured by the BPM,  $\theta_i$  the deflection angles from the dipoles,  $R_{ji}^{12}$  the transport coefficients and  $\Delta m_j^{(k)}$ ,  $\Delta \theta_i^{(k)}$ ,  $\Delta R_{ji}^{12(k)}$  the difference values after reducing the strength of a set of quadrupoles and their dipole correctors.  $\sigma_m$  and  $\sigma_{res}$  are the total alignment precision and the resolution of the BPM's.

The algorithm with  $K=0$  is called "one-to-one" and is usually confined to the focusing quadrupoles. The algorithms  $K=1,2$  are called Dispersion-Free and Wake-Free and are due to [1]. In the WF-algorithm, one difference orbit is measured with a reduction of the focusing quadrupoles and another with reduction of the defocusing quadrupoles. The algorithm  $K=3$  uses the difference orbits with reduction of the quadrupoles  $3i+k$ . Its performance turns out to be slightly weaker than the WF correction. The RDF algorithm is a DF algorithm with a strong variation of the focusing strengths: all quadrupole strengths are reversed completely. The performance of this algorithm is virtually equivalent to the WF correction. As an example, figure 4 compares an orbit after one-to-one correction with an orbit after  $K=3$  correction.

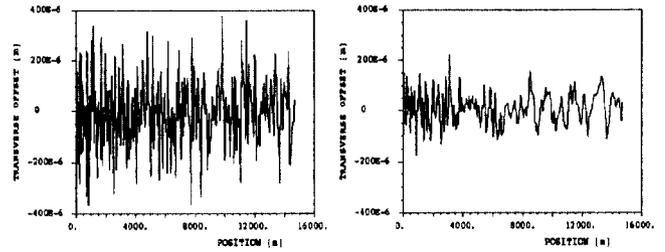


Figure 4: orbit after one-to-one correction and  $K=3$  correction

The table below shows the average emittance blow-ups after application of the various correction techniques. It was assumed that the quadrupoles were aligned with a precision of  $100\mu\text{m}$  rms to an ideal axis and the BPM's were aligned with  $100\mu\text{m}$  rms to the quadrupole centres. The corrections were carried out with  $2 \cdot 10^9$  particles per bunch, the emittances are given for the full bunch charge.

The variation of the quadrupole strengths for DF, WF and K=3 is 10%.

algorithm	mean final emittance after orbit correction + D-bumps	
one-to-one	8.9 $\epsilon_y$	
DF	5.0 $\epsilon_y$	
WF	1.2 $\epsilon_y$	1.03 $\epsilon_y$
RDF	1.2 $\epsilon_y$	1.01 $\epsilon_y$
K=3	1.6 $\epsilon_y$	1.17 $\epsilon_y$

These results suggest a total alignment tolerance for the BPM's of  $\approx 80\mu\text{m}$  or an improved BPM resolution. The emittance blow-up can also be eased by operating a longer fraction of the machine at an rf-phase suitable for BNS-damping.

The emittance blow-up remaining after orbit correction can be improved further by application of trajectory bumps. As the emittance growth from quadrupole misalignment is always some mixture of dispersive and wake effects, we used dispersive bumps to achieve a final emittance minimization. If the machine can rely on this combination of correction techniques, the corresponding alignment tolerances are rather relaxed.

## 5 RESIDUAL EMITTANCE

To demonstrate the effect of single bunch dynamics on the convergence of trajectory correction schemes, a one-to-one correction has been carried out repeatedly on a perfectly aligned machine with  $5\mu\text{m}$  BPM resolution for various bunch charges. Due to the finite BPM resolution, an additional new trajectory error is induced with each correction step. As the bunch population increases over

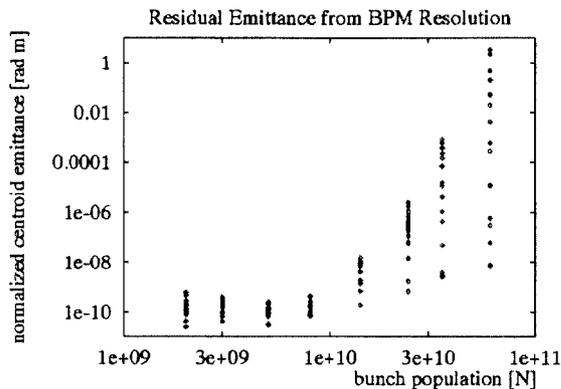


Figure 5:  $\beta$ -function in the main linac

a threshold of  $\approx 10^{10}$ , the new trajectory errors are enhanced by transverse effects and outweigh the corrective effect. This can be seen in figure 5. As a consequence, trajectory corrections must be carried out at reduced beam current. Only the one-to-one algorithm can be carried out at full beam current if only a few dipole correctors are adjusted with each shot. Such a procedure will have to be implemented for maintaining a beam trajectory after successful beam based alignment.

## 6 EMITTANCE LIFETIME

Any linear collider is subject to a slow uncorrelated ground motion that is commonly described by the  $atl$ -law (see [3]). According to this law, the rms relative motion of two points separated by the distance  $l$  over the time interval  $t$  is characterized by its rms

$$\sigma^2 = a \cdot t \cdot l \quad (4)$$

For the coefficient  $a$  we will use the rather pessimistic value  $a = 10^{-16} \frac{\text{m}}{\text{s}}$  throughout this investigation.

This ground motion induces misalignment in a linac, causing continuous deterioration of the emittance and establishing the necessity to reiterate orbit- and emittance corrections after certain time intervals. To investigate the speed of this emittance deterioration, a perfectly aligned linac was subjected to 100s of ground motion and then the centroid emittance for that machine was calculated. Then, the orbit was corrected with a one-to-one algorithm on all quadrupoles and the new emittance calculated. The results suggest the following time scales for repeating the correction measures during linac operation:

correction method	typical time interval
one-to-one	200s
bumps	45min.
full beam based alignment	24h

Here, the one-to-one algorithm does not steer to the zeros of the BPM's but to the BPM readouts as found after successful orbit correction and bump adjustment.

## 7 CONCLUSIONS

Our investigation has shown that the underlying accelerator concept is appropriate for maintaining a small single bunch emittance during acceleration with moderate requirements for the alignment precision of quadrupoles and cavities. For successful operation of the machine, reliable measurement of the transverse bunch positions with a resolution of  $5\mu\text{m}$  and the emittance with a resolution of 3% or better will be necessary. These specifications can be met with present technology. Further it was demonstrated that such a machine can be operated stably and the typical speed of emittance deterioration leaves sufficient time for continuous readjustment of the beam trajectory.

## 8 REFERENCES

- [1] T. O. Raubenheimer "The Generation and Acceleration of Low Emittance Flat Beams for Future Linear Colliders", Ph.D. thesis, Stanford Linear Accelerator Center, Stanford University, Stanford, California 94309, November 1991
- [2] V. Balakin, A. Novokhatsky, V. Smirnov "VLEPP: Transverse Beam Dynamics" Proceedings of the 12th Int. Conf. on High Energy Accelerators, Fermilab, 1983
- [3] B. A. Baklakov et al. "Investigation of Seismic Vibrations and Relative Displacements of Linear Collider VLEPP Elements" Proc. 1991 IEEE Part. Acc. Conf., San Francisco, USA, 3273 (1991)