ISSUES IN MULTI-BUNCH EMITTANCE PRESERVATION IN THE NLC*

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INTRODUCTION

In the linac of the SLAC NLC design 1 nC bunches are accelerated in trains 90 bunches long with an interbunch spacing of 42 cm. In this multi-bunch design one important problem that needs to be controlled is the multi-bunch beam break-up instability. One method of controlling this instability is by detuning the transverse modes of the accelerator cavities. This is accomplished by varying the cell dimensions (specifically the cell and iris radii) as one proceeds down the structure in such a way that the transverse modes are detuned, while the fundamental, accelerating mode is left unchanged. In a properly designed, gaussian detuned structure the transverse wakefield excited by the first bunch in the train can be made to cancel sufficiently at the positions of bunches 2 to 25, after which it begins to grow again. To accommodate the long NLC bunch train it was suggested to build the linac out of 4 types of structures whose modes are interleaved so that the recoherence of the wake does not begin until after the last bunch has passed. More recently, another way of avoiding the recoherence of the wake is being studied: using only one type of structure and introducing weak damping through a manifold coupled by radial slots. In this paper, however, we will consider only the earlier idea. (For a review of earlier work on this subject, see Ref. 1.)

An important consideration in the NLC linac is the tolerance to accelerator structure misalignments. In Ref. 2 numerical tolerance studies were performed for such misalignments. We present here an approximate analytic calculation that works well for the NLC parameters with 4 structure types. Another important question for the detuned structure is the effect of the higher dipole band modes, which have been ignored in most analyses. We find that by slightly varying the thickness of the irises as one proceeds down the structure (in addition to the normal iris radius variation) the effect of these modes can be made acceptably small.

EFFECT OF STRUCTURE MISALIGNMENTS

Analytical Model

Consider a train of bunches passing through an ideal linac in which only the accelerator structures (which for the moment are all identical) are misaligned. The position x_m of the $m^{\rm th}$ bunch at position z in the linac, under the smooth focusing approximation, is given by

$$\frac{1}{E}\frac{d}{dz}\left(E\frac{dx_m}{dz}\right)+\frac{x_m}{\beta^2}=$$
$$\frac{e^2N}{E}\sum_{m'=1}^{m-1}W[(m-m')\Delta s]\left(x_{m'}-\sum_{i=1}^{N_a}x_{ai}L_a\delta[z-z_i]\right) \quad (1)$$

where E is the bunch energy, β the beta function, N the particles per bunch, W the transverse wake function, Δs the bunch spacing, N_a the number of structures, x_{ai} the offset of structure i, and L_a the structure length. We assume that the bunch-to-bunch energy variation is small and can be ignored. Note that x_m, E, and β are, in general, functions of z. Of the driving terms in Eq. (1) the first we will call the betatron term, the second the misalignment term. For the model that we present here we limit ourselves to the case where the betatron term is small compared to the misalignment term, so that it can be dropped. Under this assumption, and now generalizing to the case of discrete focusing, we find that the final position (designated by subscript f) of the $m^{\rm th}$ bunch is given by

$$x_{mf}=-e^2NL_aS_{am}\sqrt{\frac{\beta_f}{E_f}}\sum_{i=1}^{N_a}x_{ai}\sin\mu_{if}\sqrt{\frac{\beta_i}{E_i}} \quad , \quad (2)$$

with μ_{if} the phase advance between positions z_i and z_f, and $S_{am}=\sum_{m'}^{m}W[(m-m')\Delta s]$, a parameter which we will call the sum wake. Note that bunch m's final angle x'_{mf} is given by an expression similar to Eq. (2); in particular, it is also proportional to S_{am}, its only m dependence. Therefore, in phase space all bunches lie on a straight line that goes through the origin. (However, also note that for an accelerator with more than one structure type, as is proposed for the NLC, this will no longer be true.)

The growth in projected emittance, if the fractional growth is small, at a position where $\alpha_f=0$, is

$$(\Delta\epsilon)_f\approx\frac{1}{2\beta_f}\left[\langle x_f^2\rangle-\langle x_f\rangle^2\right]+\frac{\beta_f}{2}\left[\langle x_f'^2\rangle-\langle x_f'\rangle^2\right] \quad , \quad (3)$$

where we let brackets ($\langle\rangle$) represent an averaging over the bunches. Let us consider linear acceleration and an average beta function variation $\bar\beta\sim E^{1/2}$, as in the NLC. Then for an ensemble of machines, each of which has normally distributed, uncorrelated, structure offsets with rms $(x_a)_{rms}$, the final position of the $m^{\rm th}$ bunch will also follow a normal distribution with rms (if $\beta_f\approx\bar\beta_f$, $E_f\gg E_0$)

$$(x_{mf})_{rms}\approx e^2NL_aS_{am}(x_a)_{rms}\sqrt{N_a}\bar\beta_0\left[\frac{1-[E_0/E_f]^{1/2}}{E_0E_f}\right]^{\frac{1}{2}} \quad (4)$$

where subscript 0 designates initial conditions. It follows that the emittance growth will follow a χ^2 distribution of degree 2, *i.e.* an exponential distribution $\exp[-(\Delta\epsilon)_f/\sigma_\epsilon]/\sigma_\epsilon$ with

$$\sigma_\epsilon=e^4N^2\bar\beta_0N_aL_a^2(x_a)_{rms}^2(S_a)_{rms}^2\left[\frac{1-[E_0/E_f]^{1/2}}{E_0^{1/2}E_f^{3/2}}\right] \quad . \quad (5)$$

Here $(S_a)_{rms}=\sqrt{\langle S_a^2\rangle-\langle S_a\rangle^2}$. Assuming we can allow a certain emittance growth $\sigma_{\epsilon t}$ we can obtain a misalignment tolerance $(x_{at})_{rms}=(x_a)_{rms}\sqrt{\sigma_{\epsilon t}/\sigma_\epsilon}$. Fig. 1 displays the sum wake S_{am} for one of the structure types; here $(S_a)_{rms}=7.2$ V/pC/mm/m.

Four Structure Types

In Ref. 2 it was shown that the use of 4 structure types can greatly reduce the emittance growth due to betatron

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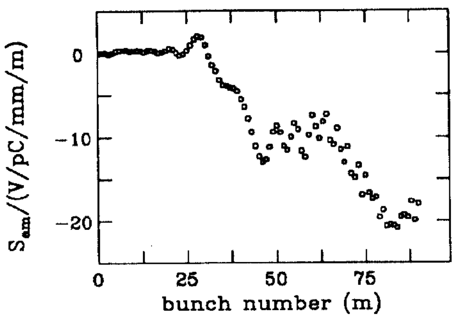

Fig. 1. The sum wake of one structure S_{am} *v.s.* bunch number m with nominal bunch spacing.

oscillations, particularly for the bunches near the end of the train. Therefore the approximation of dropping the betatron term in Eq. (1) will be more valid than before. As for the emittance growth due to the alignment term it can be shown that its expectation value is still given by Eq. (5), provided that now the sum wake is understood to be averaged over the 4 structure types in quadrature; the emittance distribution, however, will no longer follow a simple exponential distribution.

To benefit from the use of 4 structure types in the misalignment term effects (as we did in the betatron term effects) we need to align the structures particularly well within each group of 4. Let us suppose each group is on its individual girder, to facilitate the alignment. To include the effect of girder misalignments on emittance we need to add a second term to Eq. (5), one that differs only in that the combination of parameters $N_a L_a^2 (x_a)_{rms}^2 (S_a)_{rms}^2$ is replaced by the one corresponding to the girder scale. If we take subscript g to represent girder quantities, we have $N_g = N_a/4$, $L_g = 4L_a$, and S_{gm} is found by simply averaging the sum wake over the 4 types. Fig. 2 displays S_{gm} *v.s.* bunch number m; here $(S_g)_{rms} = 0.38$ V/pC/mm/m.

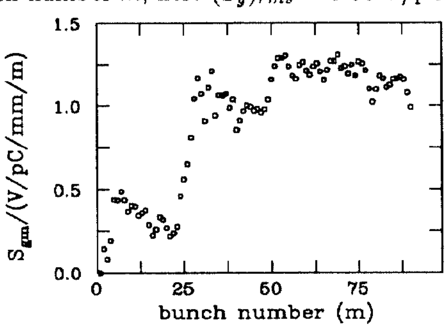

Fig. 2. The sum wake for the combination of the 4 structure types S_{gm} *v.s.* bunch number m.

Sensitivity to Slight Frequency Changes

The distribution of dipole modes is approximately gaussian with a central frequency of 15 GHz, an rms of 2.5%, and a total width of 10%, and the bunch spacing is 42 cm. Therefore a kind of resonance can develop between a mode frequency and the 20$^{\text{th}}$, 21$^{\text{st}}$, or 22$^{\text{nd}}$ harmonic of the bunch frequency. For one structure type the relative mode spacing at the center of the distribution is 4.5×10^{-4} [1]; therefore, a small shift in the mode frequencies relative to the bunch frequency can result in a large change in effect. To show this effect, in Fig. 3 we plot $(S_a)_{rms}$ for the 4 structure types as function of small changes in relative bunch spacing (the dashed curves). We see more than a factor of 5 variation when the relative bunch spacing is changed by only 2.5×10^{-4}. (However, in a real accelerator, where each structure has different, random manufacturing errors, the effect of the fluctuations will be reduced.) The solid line in Fig. 3 gives the sum of the contributions of the 4 structure types added in quadrature. This gives the effect on emittance growth when the errors are on the structure scale and 4 structure types are used. We note that the fluctuations are much smaller. Fig. 4 gives $(S_g)_{rms}$ the sensitivity on the girder scale when 4 structure types are used. We note that the average value of $(S_g)_{rms} \approx 1/20(S_a)_{rms}$, therefore the expected gain in tolerance on the girder scale (remember the factor $\sqrt{N_a/N_g} L_a/L_g$) is about 10.

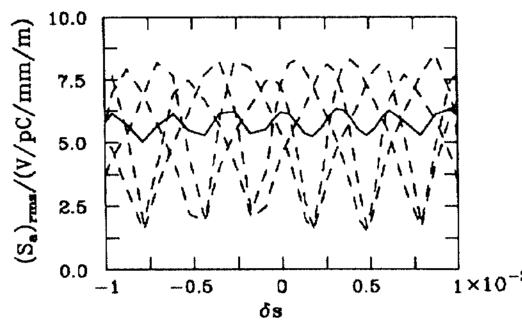

Fig. 3. $(S_a)_{rms}$ for the 4 different structure types, as function of relative change of bunch spacing δs (the dashes). The solid line gives the rms of the four averaged in quadrature.

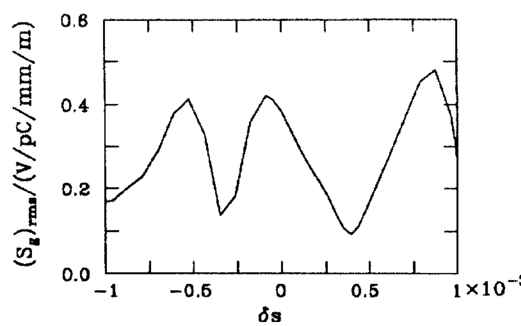

Fig. 4. The sensitivity of $(S_g)_{rms}$ on δs.

Comparison with Numerical Results

For the numerical comparisons we use the NLC parameters: $eN = 1$ nC, $E_0 = 10$ GeV, $E_f = 250$ GeV, $\beta_0 = 8$ m, $L_a = 1.8$ m, $N_a = 3600$, and normalized emittance $\gamma\epsilon = 3 \times 10^{-8}$ rm. The lattice is a piecewise 90 degree-per-cell FODO type; the number of structures between quads is given by the integer part of $\sqrt{4E/E_0}$. Fig. 5 displays the results of numerical tracking when there are 4 independently misaligned structure types, and when $(x_a)_{rms} = 5$ μm. The dashes give an exponential approximation, with σ_ϵ given by Eq. (5). The average growth obtained numerically, 0.091, agrees with that obtained analytically, 0.089, but the distributions differ slightly.

In Table 1 we compare the results of three methods of finding the tolerance for 25% emittance growth: (i) using Eq. (5); (ii) numerically performing the sum Eq. (2)

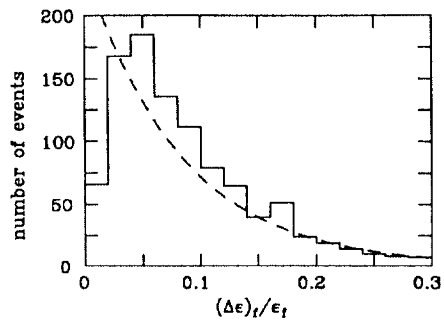

Fig. 5. Tracking results for 4 independently misaligned structure types, with $(x_a)_{rms} = 5\ \mu$m.

(and its counterpart for x'_{mf}), but with x_{ai} replaced by $(x_a)_{rms}$, to find the expected emittance for the real lattice; and (iii) tracking. The error factors give the rms variation due to bunch spacing. We give results for the case of only 1 structure type, and for 4 structure types on both the structure (a) and girder (g) scales. In the first case the analytical tolerances are much larger than the tracking results, indicating that dropping the betatron term of Eq. (1) is not a good approximation. Also, the variation terms are large, showing a great sensitivity to frequency changes. (With random fabrication errors along the linac this sensitivity should decrease.) In case 2 the analytic approximation agrees with the tracking results.* Finally, in case 3, 4 structure types on the girder scale, we note that the accurate inclusion of the amplitude and phase of the betatron sum is important for a good estimate.

Scale	Ntype	Eq. (5)	Num. sum	Tracking
Structure (a)	1	10.9 ± 6.6	11.5 ± 8.4	4.3 ± 3.2
Structure (a)	4	8.3 ± 0.6	8.9 ± 0.6	8.7 ± 0.6
Girder (g)	4	$120. \pm 60.$	32.0 ± 6.4	32.0 ± 4.9

Table 1. Alignment tolerances in microns for 25% emittance growth.

THE HIGHER DIPOLE BANDS

The analysis of the detuned structure has focused almost entirely on the modes of the first dipole band (including the effect of the second band modes on the first) since the kick factors of these modes are at least an order of magnitude larger than those of the other bands. However, when we perform the uncoupled calculation including also the effects of the modes of bands 3-8 we find that the wakefield amplitude is now an unacceptable 10% at $s = 42$ cm, the position of the second bunch, and it decreases only slowly as we move further back in the bunch train.

Fig. 6a illustrates the cause of the problem. In this figure we plot the dispersion curves representing a cell near the beginning, middle, and end of the detuned structure for dipole bands 3 to 8. Note that in the vicinity of the speed of light line the 3rd, 6th, and 7th bands for the three cells are closely spaced, resulting in little detuning. The 3rd band, which contributes about 12% to the total wake near the origin, has only decreased to 8% by $s = 42$ cm; the 6th band wake, from a maximum of 8% of the total has barely changed by this position (The 7th band has a very small kick factor and can be ignored).

* This tolerance is a factor of 2 tighter than that presented in Ref. 2, a discrepancy accounted for by differences in parameters and the tolerance definition used.

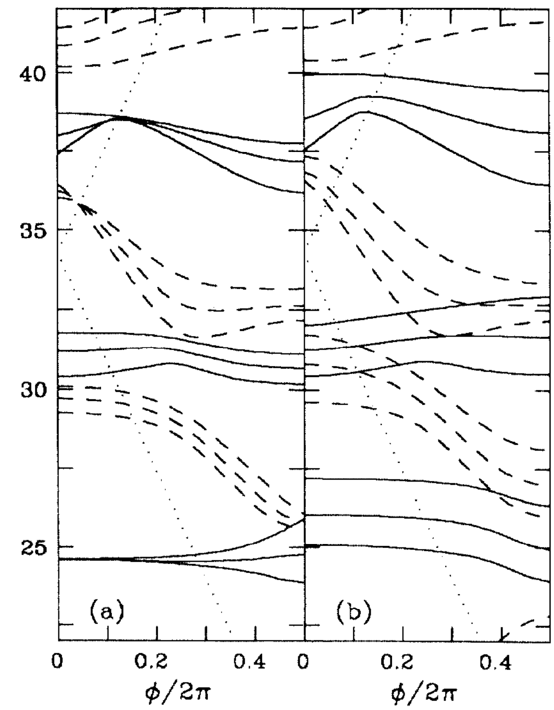

Fig. 6. The dispersion curves (phase advance per cell vs frequency in GHz) representing a cell near the beginning, middle, and end of the detuned structure, for dipole bands 3 to 8, when the iris thickness is kept at 1.46 mm (a), and when it is varied. The bands are alternately given by solid and dashed curves for ease of viewing.

Running URMEL [3] we find that near the light line band 3 is a TM111-like mode, and band 6 a TM121-like mode. This suggests that by varying the iris thicknesses along the structure, with the thinner irises in cells with the larger radii—$i.e.$ near the beginning of the structure—and the thicker ones in cells with smaller radii—$i.e.$ near the end of the structure, we can detune these modes more. Fig. 6b shows the results when the iris thickness is changed to 1.67 mm, 2.06 mm, and 2.45 mm for respectively the representative cell near the beginning, middle, and end of the structure. We see that the 3rd and the 6th band curves have separated near the light line. In the actual structure the average iris thicknesses vary as a gaussian with an average of 1.5 mm, an rms of 0.25 mm, and a total variation of 1 mm. We find the contribution of bands 3–8 to the wake at following bunches has been reduced to 1%.

ACKNOWLEDGEMENTS

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