MULTI MODE APPROACH IN CUMULATIVE BEAM BREAK UP THEORY

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Abstract

Multi mode approach is suggested to evaluate the deflecting gradient of dipole modes. General expression are derived for steady state case followed by numerical calculations. The validity of formulae obtained is discussed.

1. INTRODUCTION

Cumulative beam break up (CBBU) in multi cavity linac manifests itself as the excitation of coherent betatron oscillations of the accelerated particles under the influence of asymmetrical electromagnetic waves, induced by misalined beam in accelerating cavities [1-4]. Unlike a regenerative beam break up (RBBU) there is not the effect of field amplification in the cavities through the feed back mechanism, arising in the system due to cavities coupling. CBBU is the instability of convective type and has not any current threshold. The latter results in important consequence, that the single mode approximation, traditionally used in CBBU theories, cannot be justified in non resonance case of cumulative instability. While in RBBU single mode approximation is quite natural in small interval above the instability threshold, where amplifications conditions are satisfied for one mode only, the choice of any particular mode in non resonance case of CBBU becomes completely uncertain. Many of deflecting modes are equivalent to some extent in this case and multi mode approach suggests itself.

The attempt of generalisation of CBBU theory to multi mode beam cavity interaction is undertaken in present paper. Appropriate expressions, taking into account the infinite set of deflecting dipole modes, are derived. Steady state deflecting gradients, that are of interest for cw as well as for long pulse linac operation are obtained for some particular cases.

2. THE MAIN EQUATIONS

Let us assume, that the linac consists of the cavities electromagnetically decoupled from one another, so that RBBU can not take place. In linear approximation the equation of transverse motion is:

$$\frac{d}{dz}m\gamma\frac{dx}{dz} = -eg_x(z)x+\frac{1}{v}F_e \tag{1}$$

Here, z denotes the particle coordinate along the longitudinal axis; e, m and γ are the particle charge, its mass and the energy in the rest mass units respectively, x is transverse displacement, g_x is the gradient of external focusing system and v is the particle velocity. F_e represents the transverse force, induced in the cavity by accelerated beam. The general approach to the problem used is similar to that of the paper [2].

The objective of this paper is to find the general expression for F_e for steady state solution as well as to apply the results to be obtained to investigate some particular cases.

According to the deflection theorem [5] (MKS units are used throughout this work) the transverse momentum $p_\perp$, imparted to the particle

$$p_\perp = e\int_0^d \nabla_\perp A_z(z,t_k+z/v)dz, \tag{2}$$

where $\vec{A}$ is vector potential and the integration is performed along the cavity of length d. It is assumed that the particle orbit is not substantially affected in its passage through the cavity. Thus, the average transverse force, with which the field inside the cavity acts on a particle, is:

$$\langle f_x\rangle = \frac{ev}{d}\int_0^d \frac{\partial A_z}{\partial x}(z,t_k+\frac{z}{v})dz\ , \tag{3}$$

t_k is the moment at which the k-th particle enters the cavity.

Following [6] let us represent the vector potential in (3) in the form of infinite sum of eigenvectors $\vec{A}_\lambda(\vec{r})$:

$$\vec{A}(\vec{r},t) = \sum_\lambda q_\lambda(t)\vec{A}_\lambda(\vec{r}) \tag{4}$$

with the time dependent amplitudes $q_\lambda(t)$ satisfying the differential equation

$$\ddot{q}_\lambda + \frac{\omega_\lambda}{Q_\lambda}\dot{q}_\lambda + \omega_\lambda^2 q_\lambda = \frac{1}{\varepsilon_0}\frac{\int_V \vec{j}\,\vec{A}_\lambda\,dV}{\int_V \vec{A}_\lambda^2\,dV} \tag{5}$$

and with $\vec{A}_\lambda$ satisfying the condition $div\vec{A}_\lambda = 0$ as well as the Helmholtz equation:

$$\Delta\vec{A}_\lambda + \frac{\omega_\lambda^2}{c^2}\vec{A}_\lambda = 0\,. \tag{6}$$

Here, ω_λ and Q_λ are frequency and quality factor of a mode respectively, ε_0 is electrical permeability of free space, c is the light velocity. We shall assume the beam current to be a sequence of charged bunches of δ-function distribution moving in positive direction of z-axis. Thus, the current density of any bunch is:

$$j_k = qv\delta(x-x_0)\delta(y)\delta[z-v(t-t_k)] \tag{7}$$

The solution of (5)-(7) is

$$q_{\lambda,k}(t)=\frac{x_0 q}{\omega_\lambda}\frac{\exp[\frac{\omega_\lambda}{2Q_\lambda}(t_k-t)]}{\varepsilon_0\int_V \vec{A}_\lambda^2\,dV}\mathrm{Im}(\exp(i\omega_\lambda(t-t_k))\times \qquad (8)$$

$$\int_0^d \frac{d}{dx}A_{\lambda,z}(0,0,z)\exp(-i\omega_\lambda z/v)dz\}$$

In deriving (7) the field change due to damping during cavity transit was neglected.

The final formula for the average transverse force with that the field induced by N-1 particles acts on N-th particle looks like (in linear approximation)

$$\langle f_x\rangle=\frac{eqvx_0}{\varepsilon_0 d}\sum_\lambda\sum_{k=0}^{N-1}\frac{\left|\int_0^d\frac{\partial}{\partial x}A_{\lambda,z}\exp(-i\omega_\lambda z/v)dz\right|^2}{\omega_\lambda\int_V \vec{A}_\lambda^2\,dV}\times \qquad (9)$$

$$\exp\frac{\omega_\lambda}{2Q_\lambda}(t_k-t)\sin\omega_\lambda(t-t_k).$$

For steady state solution $(N\to\infty)$ it can be transformed to

$$\langle f_x\rangle=\frac{1}{2}\frac{eqvx_0}{\varepsilon_0 d}\sum_\lambda\frac{\left|\int_0^d\frac{\partial}{\partial x}A_{\lambda,z}\exp(-i\omega_\lambda z/v)dz\right|^2}{\omega_\lambda\int_V \vec{A}_\lambda^2\,dV}\times \qquad (10)$$

$$\frac{\sin(\omega_\lambda\tau)}{\cosh\frac{\omega_\lambda\tau}{2Q_\lambda}-\cos\omega_\lambda\tau}.$$

One can obtain the solution for direct current beam putting in (10) $\tau\to 0$, $q/\tau=I$, I being the beam current value. For large Q $(Q>>1)$:

$$\langle f_x\rangle_{dc}=\frac{eIvx_0}{\varepsilon_0 d}\sum_\lambda\frac{1}{\omega_\lambda^2}\frac{\left|\int_0^d\frac{\partial}{\partial x}A_{\lambda,z}\exp(-i\omega_\lambda z/v)dz\right|^2}{\int_V \vec{A}_\lambda^2\,dV} \qquad (11)$$

Comparison of the above formulae with those obtained in the assumption of the single mode approximation [2,4] shows that it is sufficient to replace the expression for the mode transverse impedance with the sum of the transverse impedance's of the individual modes with appropriate coefficients, depending on modes characteristics as well as on temporary structure of beam current. It is worth to notice that the exponent in the denominators of the series members in the case of δ-bunch beam provides the series converging. It is not the case for the expression for direct current beam, the conclusion of converging or diverging of the series may be done for concrete case only. At the same time, as it follows from (11), multi mode approach gives rise to the higher deflecting gradient in the case of direct current beam, since (11) represents the sum of positive members.

3. RF GRADIENTS FOR CYLINDRICAL CAVITIES

Concrete expressions for eigenvectors $\vec{A}_\lambda$ are necessary to understand the quantitative difference between two approaches in cumulative beam break up theory (traditional single mode and multi mode suggested). We shall consider the simplest accelerator structure, namely, the series of single sell cylindrical resonators. For such a resonator the components of eigenvectors are

$$\begin{aligned}A_r&=-\frac{k_z}{k_c}J_n'(rk_c)\cos n\varphi\sin k_z z,\\A_\varphi&=\frac{k_z n}{k_c^2}\frac{J_n(rk_c)}{r}\sin n\varphi\sin k_z z,\\A_z&=J_n(rk_c)\cos n\varphi\cos k_z z,\end{aligned} \qquad (12)$$

and the formulae for deflecting gradients $\langle g_x\rangle_{rf}=\langle f_x\rangle/evx_0$ are turned to ($v=c$ is assumed)

$$\langle g_x\rangle_{rf}=\frac{1}{4}\frac{I}{\varepsilon_0 c^2 d\Lambda}G_x, \qquad (13)$$

where

$$G_x=\sum_{p,m}\frac{k_{m,p}[1-(-1)^p\cos\delta k_{m,p}]\sin k_{m,p}}{J_{m,p}\left[\cosh\frac{k_{m,p}}{2Q_{m,p}}-\cos k_{m,p}\right]} \qquad (14)$$

for δ-bunches beam and

$$G_x^{dc}=2\sum_{m,p}\frac{1-(-1)^p\cos\delta k_{m,p}}{J_{m,p}} \qquad (15)$$

for direct current beam. Here,

$$J_{m,p}=\frac{\pi^3\rho^2}{2\delta}\frac{p^2}{v_m^2}(J_{1m}+J_{2m})+\pi\delta\times\begin{Bmatrix}1, p=0\\ \frac{1}{2}, p\neq 0\end{Bmatrix}\times J_{3m},$$

$$J_{1m}=\int_0^{v_m}J_1'^2(x)x\,dx,\ J_{2m}=\int_0^{v_m}\frac{J_1^2}{x}dx,\ J_{3m}=\int_0^{v_m}J_1^2(x)x\,dx, \qquad (16)$$

$$k_{m,p}=\sqrt{\frac{v_m^2}{\rho^2}+\frac{\pi^2 p^2}{\delta^2}},$$

and ρ and δ are normalised cavity radius and its length respectively, $R=\rho\Lambda$, $d=\delta\Lambda$, Λ is the wavelength of the accelerating mode, v_m is the m-th null of Bessel function of the first order $J_1(x)$, $k_z=\pi p/d$, $k_c=v_m/R$, $p=0,1,\ldots,$

m=1,... . Since linear approximation is used, only dipole modes (n=1) have been confined in the sums (14) and (15). The series (15) is diverging relative m like the sum of $(1 \pm \cos m)/m$. The reason of this is the approximation used rather than physical essence of the phenomenon under consideration. The relations $A_{\lambda,z}(x) \approx x\partial A_{\lambda,z}/\partial x(0)$ as well as $\partial A_{\lambda,z}/\partial x(x) \approx A_{\lambda z}/\partial x(0)$ are valid for x (or for radial modes $J_1(k_c r)$) satisfying the condition $x << R/v_m$. One should use the exact expression

$$\int_0^d A_{\lambda,z}(x,z)\exp(-\frac{i\omega_\lambda z}{v})dz \int_0^d \frac{\partial}{\partial x}A_{\lambda,z}(x,z)\exp(\frac{i\omega_\lambda z}{v})dz \quad (17)$$

instead of its approximate value in order to use the infinite set of radial modes in the expression for deflecting force. It follows from this remark that multi mode approach leads to non linear character of beam cavity interaction.

4. NUMERICAL RESULTS

Fig.1 illustrates the dependence of normalised gradient for δ-bunch beam on the number of modes included in the sum (14), n being equal to the upper limit in this sum; the plot was calculated for the case $n = m_{\max} = p_{\max} + 1$. Fig.2 is the plot of the dependence of deflecting gradient on cavity length, calculated for the case of "mode saturation", that is for the case, when adding any number of new higher modes does not change the deflecting gradient. Such behaviour, seen on fig.1, follows from the decay of modes with large wave number $k_{m,p} >> Q$ between the successive bunch passes through the cavity, quality factor being assumed to be the same for all modes. Comparison with single mode approximation show clearly the significance of the approach suggested.

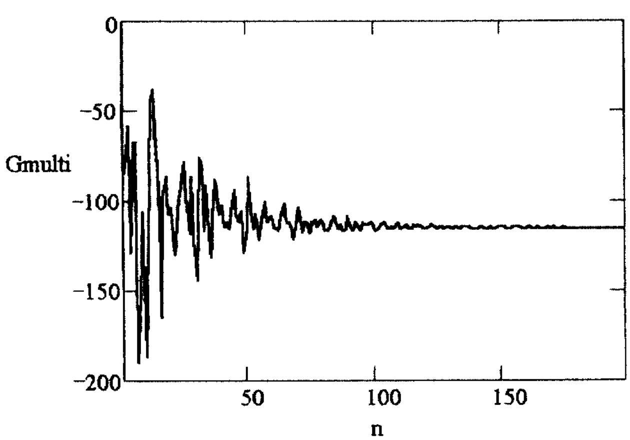

Fig.1. The dependence of deflecting gradient on the number of dipole modes. Q=100, Gsingle= -1.15, δ = 0.5, ρ = 0.383.

5. CONCLUSION

Multi mode approach is the most logical in cumulative beam break up theory. The results obtained for some particular steady state cases show its significance for beam cavity interaction. Moreover, such approach results in non linear character of instability, that is especially true for direct current beam due to weak dependence of mode contribution on its quality factor. It is worth to emphasise that in practical use of formulae obtained one should take into account a non zero beam size as well as mode dependence of Q-value.

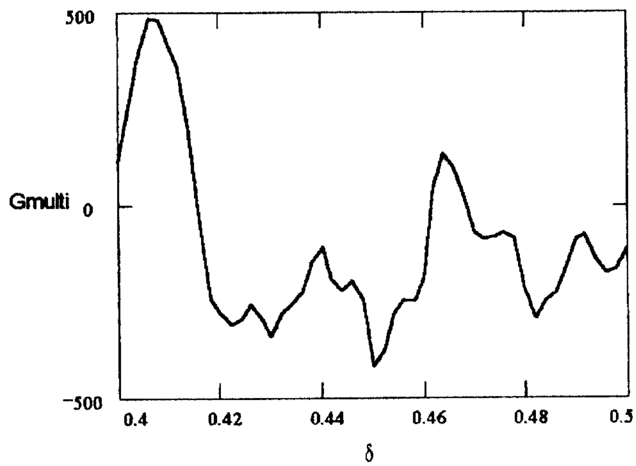

Fig.2. The dependence of deflecting gradient on cavity length. Q = 100 for all modes.

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