Head-Tail Instabilities Enhanced by the Beam-Beam Interaction.

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Abstract

The bunch intensity limit for two counter-rotating beams in LEP is smaller than for one beam. Experiments showed that the intensity limit for two beams is not just due to the tune spread induced by the residual beam-beam interaction in the eight crossing points where the beams are vertically separated. It was observed that the intensity limit is caused by horizontal head-tail instabilities induced by the presence of a second beam. A simple calculation, based on a two particle representation for the head-tail motion of each of the two beams with a linear beam-beam coupling between them, is presented. It shows the influence of the beam-beam interaction on the growth rate of m=0 and m=1 head-tail modes. The results are compared with experimental data.

Introduction

LEP can be operated with four against four or eight against eight bunches of e^+ and e^-. In the first case the counter rotating beams are separated during injection in the eight crossing points by local vertical electrostatic bumps. In the case of eight against eight, the bunches are separated in the additional crossings by means of a horizontal electrostatic pretzel. In both cases the bunch intensity limit with two counter rotating beams is smaller than that with one single beam. This intensity reduction was first thought to be due to tune spread induced by the residual beam-beam effect in the crossing points (1). Further investigation, however, showed the presence of second beam enhanced coherent head-tail instabilities. This observation was important. The single beam current is also limited by coherent head-tail instabilities (2), thus, if a cure can be found in order the raise the single beam current (bunch lengthening, increasing Qs,...) the current limit for two beams might increase as well. In the case where the two beams are limited by the beam-beam tune spread, the intensity limit would be absolute and independent of the single beam current limit.

Experiments were carried out in LEP in order to study the interplay between head-tail modes and beam-beam interaction. A simple model was developed in order to understand this interplay.

Observations

When LEP is filled with a single beam the bunch current is limited by the vertical transverse mode coupling instability. The vertical m=0 and m=-1 modes merge together and the bunch becomes unstable. For a Qs=.08 and a bunch length of 18mm this happens at a bunch current of 0.640 mA(2). The impedance in the horizontal plane is smaller so the transverse-mode-coupling instability in the horizontal plane occurs at higher currents. However, with vertical and horizontal chromaticities equal to +1, the m=-1 modes are slightly excited in both planes, from 0.400mA per bunch. The oscillation stabilises at a limited amplitude and no intensity is lost (3). Only if the chromaticity is raised to +3 do the m=-1 modes grow in amplitude and beam losses occur. Therefore both chromaticities are carefully kept at +1 during filling. When a second beam is injected, the m=-1 mode in the horizontal plane becomes unstable at a lower current. In fig. 1 the oscillation amplitude of the horizontal m=-1 mode is shown as function of intensity. The lowest curve is for single beam, the second curve is for two beam filling. At the intensity indicated the horizontal chromaticity was lowered by 1 unit in order to stabilise the m=-1 mode a little more. The amplitude was measured with a pickup that can only detect the dipole component of the motion. For the m=-1 mode this component is small compared to the full motion.

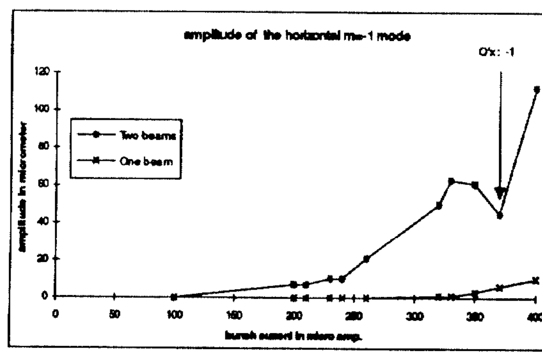

fig. 1 Amplitude of the horizontal m=-1 mode as a function of bunch intensity. The lower curve is with one beam, the higher curve is with two beams.

Model

In order to understand the effect of the beam-beam interaction on the head-tail motion, a simple model was developed. When the bunch is oscillating with the m=+-1 mode the head and tail of the bunch pass at a different distances to a counter rotating bunch. This results in a different beam-beam kick. The effect of this can be described in the following 2-particle head-tail model (4). Since only the m=0 and m=1 modes are considered, a two particle model should be sufficient. A bunch of positrons is then described in the following way :

$$\frac{d^2x_1}{dt^2} + \omega_\beta x_1 = W(\tau)x_2$$
$$\frac{d^2x_2}{dt^2} + \omega_\beta x_2 = W(-\tau)x_1$$

Where x_1 and x_2 are the horizontal coordinates of the head and tail particle , ω_β is the betatron frequency which can be modulated by the chromaticity:

$$\omega_\beta = \omega o_\beta - C \cdot \frac{\Delta P}{P} \sin(2\pi Q_s)$$

Qs is the synchrotron tune and $\Delta P/P$ is the relative momentum spread. τ is the difference in longitudinal coordinates of the head and tail, being approximated by :

$$\tau = \sigma_s \cos(2\pi \cdot Q_s)$$

σ_s is the bunch length in seconds. $W(\tau)$ is the wake function depending on the distance between head and tail. The wake is approximated by a broad band resonator (2). These equations represent two coupled oscillators in which the coupling term is modulated with Qs. The solution is shown in fig 2.

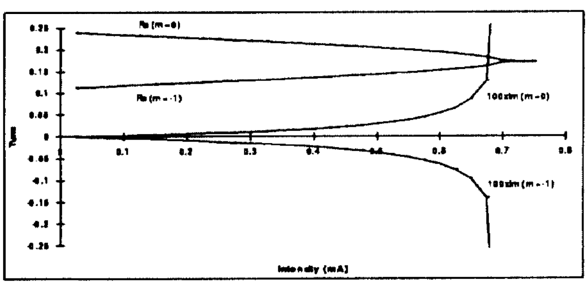

Fig. 2 : Real and imaginary frequencies of the m=0 and m=-1 calculated by a two particle head-tail model.

The real parts are the frequencies of the m=0 and m=-1 mode. The imaginary parts show whether the modes are damped (positive) or anti-damped (negative). For a positive chromaticity (LEP is normally running with a chromaticity of +1) the m=0 mode is damped and the m=-1 mode is anti-damped.

The beam-beam interaction is represented by a linear coupling between two such head-tail oscillators (x_1,x_2), (u_1,u_2) :

$$\frac{d^2x_1}{dt^2} + \omega_\beta^2 x_1 = W(\tau)x_2 - k(2x_1 - u_1 - u_2)$$
$$\frac{d^2x_2}{dt^2} + \omega_\beta^2 x_2 = W(-\tau)x_1 - k(2x_2 - u_1 - u_2)$$
$$\frac{d^2u_1}{dt^2} + \omega_\beta^2 u_1 = W(\tau)u_2 - k(2u_1 - x_1 - x_2)$$
$$\frac{d^2u_2}{dt^2} + \omega_\beta^2 u_2 = W(-\tau)u_1 - k(2u_2 - x_1 - x_2)$$

By putting $S_i = x_i + u_i$ and $D_i = x_i - u_i$ we can rewrite the above equation :

$$\frac{d^2S_1}{dt^2} + (\omega_\beta^2 - k)\cdot S_1 = (W(\tau) - k)\cdot S_2$$
$$\frac{d^2S_2}{dt^2} + (\omega_\beta^2 - k)\cdot S_2 = (W(-\tau) - k)\cdot S_1$$
$$\frac{d^2D_1}{dt^2} + (\omega_\beta^2 - 3k)\cdot D_1 = (W(\tau) + k)\cdot D_2$$
$$\frac{d^2D_2}{dt^2} + (\omega_\beta^2 - 3k)\cdot D_2 = (W(-\tau) + k)\cdot D_1$$

This change in coordinates results in two independent sets of coupled differential equations (one in S and one in D) which have essentially the same form as the equations for the simple two-particle head-tail motion but with a different betatron tune and a different impedance. These equations result in four modes : One mode in S and D which represent the classical m=0 (or σ-mode) and the classical π-mode. The two other solutions in S and D are m=-1 modes. The m=-1 is in fact shifted up by half the distance between σ and π-mode. For a positive chromaticity and a focusing beam-beam force the m=-1 mode becomes unstable at a lower current (compare Fig 3. and Fig 2.).

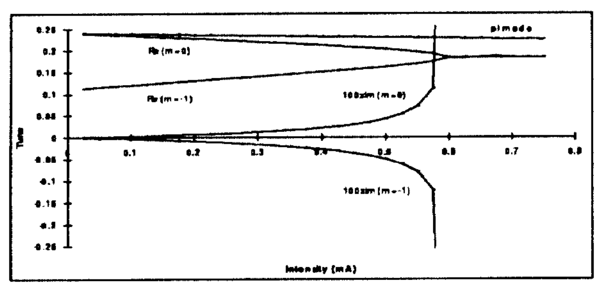

Fig. 3 : Head-tail modes taking the beam-beam coupling into account. The m=-1 mode is shifted up and is becoming unstable for a lower current.

Experimental results

In order to check in more detail the validity of this model a few simple experiments were conducted. In order to be able to recognise the modes the experiments were performed with only one bunch against one bunch. In a first experiment both bunches were filled with a fixed current of 0.4 mA . The vertical separation was reduced in the two crossing points in steps so as to increase the residual horizontal beam-beam tune shift. In Fig 4. one can see how the m=-1 mode varies by about half the tune shift. of the π-mode.

In Fig. 5 and Fig. 6 one can compare how these modes behave when filling one beam (Fig 5.) and when filling two beams (Fig 6.). Note that at 0.420 mA the m=-1 and m=0 modes are closer together in the two beam case. Again these experiments were performed with one bunch against one bunch, keeping the intensity of the two bunches equal.

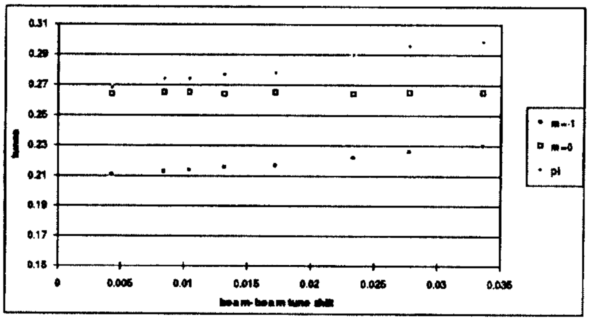

Fig. 4: Tunes of the m=0, m=-1 and p-modes as function of residual beam-beam tune shift.

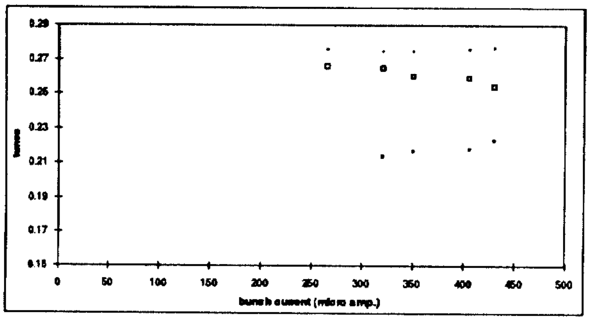

Fig. 5 : m=-1, m=0 and p-mode as function of current with two beams.

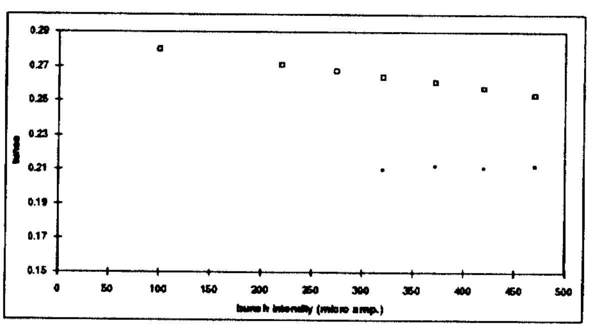

Fig. 6 *m=0 and m=-1 modes as function of current for a single beam.*

Experiments were also performed with four on four bunches, but here the tune spectra reveal many modes. A simulation program, using localised impedances and localised beam-beam kicks between different number of bunches is being developed. The aim is to understand how the effect of different crossing points add up. The results are still being analysed.

Conclusions

It was observed in LEP that the horizontal m=-1 mode is enhanced by the presence of a counter rotating beam. A simple model was developed in order to explain the influence of the residual beam-beam interaction on the head-tail modes. The model was successfully used in explaining the results of some beam-beam experiments we performed. The main result is the fact that according to this model the present maximum current for two beams in LEP is not an absolute limit. The model predicts that the two beam limit is a certain fraction of the single beam current limit. Fig. 7 gives an example how the two beam current increases together with the single beam current limit as function of Qs. The "x's" and the "+'s" are bunch currents that could be obtained during different experiments.

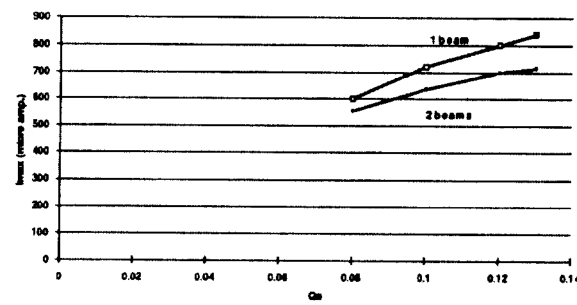

Fig. 7 : *Intensity limits for 1 beam and 2 beams as function of the synchronous frequency.*

References

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