Chaotic Behaviour Induced by Space Charge

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Abstract

In numerous non-linear dynamical systems studied in various disciplines (fluid dynamics, celestial mechanics, chemistry, biology, economy, ecology ...), chaotic motions are generated by the dynamics itself whereas no random force is present. This phenomenon, already studied in the particle accelerator field to understand the beam-beam effect, is also observed in numerical experiments on space-charge dominated beams. Stochasticity threshold and halo formation are discussed for a continuous focusing channel (1D beam) and for a FODO channel (2D beam) with the possibility to take into account the defocusing effects of RF gaps localized between the quadrupoles.

1. INTRODUCTION

High-current linear accelerators are needed as drivers for tritium production, spallation neutron sources, material testing facilities, new concepts of energy production and nuclear waste transmutation. At least for the two last applications which concern crutial points for the evolution of human life on earth, accelerator physicists focus their attention on such machines.

For this new generation of high-power linear accelerators, the most important aim is to reduce beam losses to an extremely low fraction of the total beam ($< 10^{-7}$/m) in order to limit the radioactivity in the machine area. This can be reached if the basic phenomena which lead to emittance growth and halo formation are understood. The objective of this paper is to demonstrate that the chaotic behaviour of the particles induced by space charge is one of them, probably the most important. Section 2 summarizes briefly the first observations and analyses of this behaviour [1][2]. In the following sections, a mechanism leading to particle diffusion is studied (Sec.3) and a way to reduce its effects is proposed (Sec.4).

2. FROM CELESTIAL MECHANICS TO BEAM DYNAMICS

The bases of the chaotic dynamics go back to the end of the last century when Poincaré and others studied the planetary motion. Nevertheless, important progress have been done only since the 1960s thanks to theoretical developments associated with numerical experiments obtained using computers (Kolmogorov, Arnol'd, Moser, Chirikov, Feigenbaum, Lorenz, Hénon, Smale, Mandelbrot, Ruelle, Takens , Prigogine...). The classical dynamics validity domain, already limited to nonrelativistic and nonquantal systems, has been still reduced. Chaos theory has demonstrated that deterministic systems can induce unpredictable motions. This science is now used to analyse the behaviour of complex systems in all the disciplines. Percolation theory, spin glass theory, self-organization (synergetics) ... are studied in many laboratories. It will be The Science of the next century.

A.N. Kolmogorov said : "It is not so much important to be rigorous as to be right". In most of the numerical experiments, the mathematical model used to study chaos is simplified as far as the basic characteristics of the physical system are preserved. The analyses presented here follow this principle in using the particle-core model first introduced in ref.[3]. This method can be briefly summarized by :

- at first, computation of the evolution of the *beam core* envelope in the focusing channel to be studied,
- then, analysis of the behaviour of *test particles* injected into or around the beam core.

The first numerical experiments done using this method [3][1] concern the evolution of a zero-emittance beam in a continuous focusing channel. When the beam core is mismatched, its envelope oscillates and the space charge induces a periodic nonlinear *perturbing force* which is added to the continuous focusing force. This system is similar to the restricted problem of three-bodies of the celestial mechanics as pointed out in ref. [1]. In this paper, it is demonstrated that it is the *resonance overlap mechanism* [4][5] which leads to the formation of a halo area where the particle trajectories are stochastic. This chaotic behaviour is clearly observed when the Poincaré surface of section technique is used. Intermittencies and sensitive dependance on initial conditions which characterize chaotic systems are also shown. In addition, the different sorts of trajectories are described and the leading role played by the $\upsilon = \omega_{particle}/\omega_{core} = 1/2$ resonance is pointed out. When the mismatching factor (a free parameter) is increased, the perturbing force becomes stronger and this main resonance can overlap with low order resonances also present at the core vicinity. The *stochastic thresholds* are given in ref.[1].

The particle-core model has also been used to study the behaviour of a matched beam evolving in a FODO channel [2]. In this paper, the beam core dynamics is computed using the 2D KV envelope equations but the test particle trajectories are studied in only one phase plane in order to make easy the use of the Poincaré surface of section technique. Like for 1D study, the beam envelope oscillates periodically, it is therefore not surprizing to observe similar phenomena. Nevertheless, for a FODO channel, the perturbing force is no longer a free parameter, both number and order of the resonances which are present around the beam core are determined by the choice of the phase advances with (σ_t) and without (σ_{0t}) space charge. Actually, test particles which travel near the core have tunes close to $\upsilon = \sigma_t/2\pi$ and tending towards $\sigma_{0t}/2\pi$ when the transverse energy is increased because the effect of space charge becomes more and more negligeable. The test particle phase advances (σ) are then such that $\sigma_t \leq \sigma \leq \sigma_{0t}$.

Figure 1 shows a typical phase portrait for the overlapping of the $\upsilon = 1/4$ ($\sigma = 90^o$) and $\upsilon = 1/5$ ($\sigma = 72^o$) resonances when $\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$. The stochastic areas surrounded by KAM curves can be clearly observed. This example is chosen to display phenomena prominently and the reader must keep in mind that for lower σ_{0t} values, the stochastic areas are always present. This is shown in ref. [2] where the stochastic thresholds are also given.

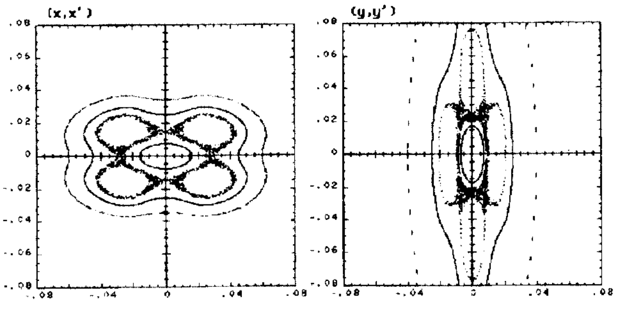


Fig.1: Poincaré surface of section for $\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$ (test particles without (x,x')-(y,y') coupling)

The influence of the RF gap defocusing effects which induce a strong coupling between transverse and longitudinal planes is also studied in ref.[2]. Even for a choice of phase advances estimated to be "safe" for a FODO channel ($\sigma_{0t} = 60^o$ and $\sigma_t = 24^o$) it is shown that the synchrobetatron coupling can lead to the formation of a large stochastic area. To add the "synchrotron oscillator" to the system deeply modifies its dynamics. When new resonances and a new source of excitation are added, "tremendous" effects can be observed. Misalignment and mismatching which add other "oscillators" (also coupled by space charge) will probably induce similar phenomena. Such studies are in progress.

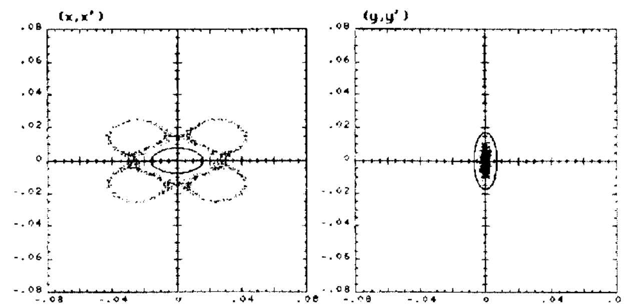
3. ARNOL'D DIFFUSION

The true logic of this world is in the calculus of probabilities
James Clerk Maxwell

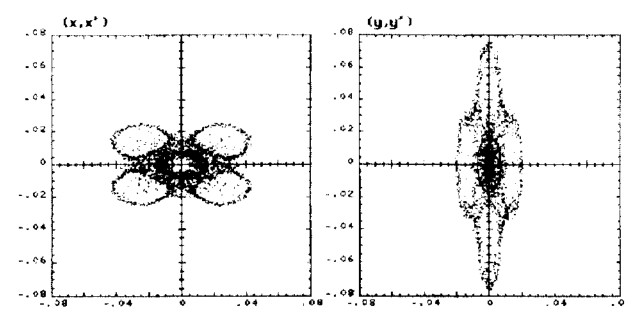
The two previous studies concern systems with 1.5 degrees of freedom. It is shown that they lead to the formation of stochastic areas (stochastic layers) isolated by KAM surfaces. For more than two degrees of freedom, these layers, no longer isolated by KAM surfaces, intersect. They form a dense "Arnol'd web" into which the stochastic motion can drive the particules [4][5]. This universal mechanism of diffusion in phase space is called Arnol'd diffusion.

For a FODO channel, the coupling force induced by space charge leads to the analysis of a nonautonomous system with 2.5 degrees of freedom : (x,x')+(y,y')+z . The successive stages of the Arnol'd diffusion are shown in figure 2 which must be interpreted taking into account the two following remarks : -i- The test particle initial condition has been chosen inside the stochastic area of the (x,x') phase plane but very close to y=y'=0 in order to obtain a slow diffusion more easily observable than a fast one. -ii- This figure is drown with $\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$ but it must be noted that numerical experiments done with other choices of phase advances (with $\sigma_{0t} = 60^o$ for example) lead to similar results.

Looking at this figure, it is obvious that Arnol'd diffusion induces a strong mixing between the two transverse phase planes and we can postulate that <u>this is a mechanism which leads to equipartitioning</u>. Diffusion along the web drives the particle up to the main (y,y') stochastic area and spreads its position in (x,x'). Motions appear as random, we are then pushed towards a calculus of probabilities. We must "go from the dynamical description in terms of trajectories to a description in terms of processes", "to achieve a picture that unifies dynamics and thermodynamics" [6]. Beam equipartitioning and thermodynamics approach, first introduced at the 1968 LINAC conference by P. Lapostolle, seem therefore to be connected with Arnol'd diffusion.



-(a)-



-(b)-

Fig. 2 : Arnol'd diffusion for $\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$
One particle injected at : x = .028, x' = .0, y = .001, y' = .0
is followed during 1000 (a) and 2000 (b) periods.

Arnol'd diffusion is an universal mechanism but the diffusion rate is very weak for systems with nonoverlapping resonances. In high-current linear accelerators, the space charge and synchrobetatron coupling forces are high, it will be difficult to avoid the formation of stochastic areas which increase the diffusion rate. The random walk of particles along the web can drive them far from the beam core but the width of this web decreases exponentially as they move away from its center. The probability of diffusion towards the accelerator aperture is then lower and lower when the amplitude increases.

To be able to produce a "safe" design, it is important to find a method to compute the quantitative values of the diffusion rates. This work is in progress.

Figure 3 shows the evolution of the main characteristics of a cloud of 2000 test particles uniformly distributed inside an hyper ellipsoid which emittance is three time larger than the beam core one. This figure displays only the parameters related to the (x,x') phase plane, similar results are observed in (y,y').

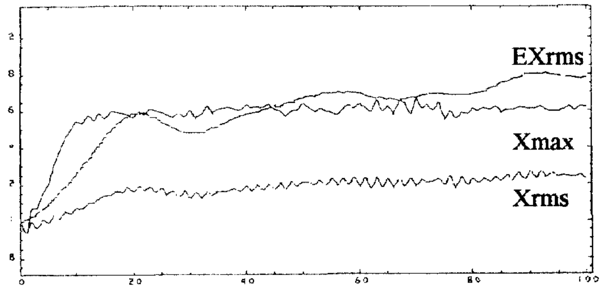


Fig. 3 : emittance (EXrms), rms dimension (Xrms) and maximum position (Xmax) as a function of the cell number for a cloud of 2000 particles ($\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$).

4. THE USE OF OCTUPOLES

In high-energy circular accelerators, the space charge is negligible. Octupoles are used to give amplitude dependant tune spread in order to damp the transverse collective instabilities (Landau damping). For a space-charge dominated focusing channel, they can be used to induce opposed effect i.e. to reduce the tune spread in the beam core vicinity in order to avoid overlap of resonances. Figure 4 shows how the tune spread evolves when the strength of octupolar correctors located in the quadrupoles is increased.

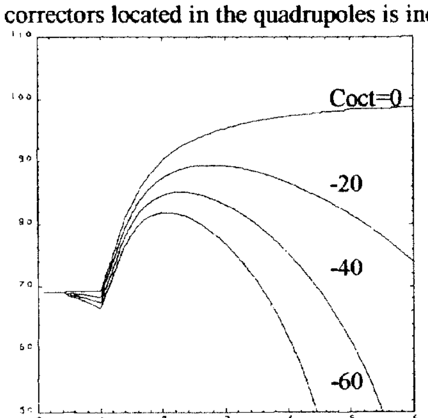


Fig. 4 : Phase advances (°) versus amplitudes
$\sigma_{0t} = 100^o$ and $\sigma_t = 70^o$
Coct = octupole strength / quadrupole strength

For Coct < -40, the two main resonances $\upsilon = 1/4$ and $\upsilon = 1/5$ no longer overlap. Simulations done in the same conditions than the ones used to draw figure 3 with Coct = -50 have demonstrated that the use of octupoles enables to cancel the emittance growths.

Nevertheless, figure 4 shows that an octupolar field is not well adapted to the space-charge field. For large amplitudes, low order resonances overlap and an instable area is formed when $\sigma < 0$.This problem can be avoided using the modified octupoles described in ref.[7] .

5. CONCLUSION

The 1D study has been very useful to discover the basic phenomena which can lead to halo formation from space-charge dominated beams. The important role played by the resonance overlap mechanism is one of the main results obtained with this study. For a matched beam evolving in a FODO channel, the behaviour of the particles is more complex. The main results of the first studies presented here are the followings :

-a- $\sigma_{0t} < 72^o$ and $\sigma_t/\sigma_{0t} > 0.4$ are constraints well known for several years to avoid large emittance growth. Nevertheless, $\upsilon = 1/6$ ($\sigma = 60^o$) and low order resonances can be present. Larger is the tune spread $\sigma_{0t} - \sigma_t$, more numerous are the resonances which can overlap.

-b- When the synchrobetatron coupling is taken into account, the dynamics is much more perturbed. The addition of both new resonances and a new source of excitations induce the formation of large stochastic areas. This is more worrying because synchrobetatron coupling, mismatching and misalignment will be present in a real linac.

-c- Arnol'd diffusion is a mechanism which leads to equipartitioning and, theoretically, to unbounded motions. Particules can diffuse far from the beam core through the Arnol'd web. Effective diffusion rates remain to be computed.

-d- It is possible to minimize the number of resonances which can overlap using modified octupoles.

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