6D-High Current Beam Matching at RFQ Entrance

A.I. Balabin and G.N. Kropachev
ITEP, 25, Bolshaja Cherjomushkinskaja, Moscow 117259, Russia

Abstract

Envelope equations for the bunched beam with electrostatic interaction of bunches represented as uniformly charged ellipsoids are derived. On the base of the equations solution injection conditions for a matched beam are formulated at the RFQ entrance which fulfilment necessitates beam modulation in the longitudinal velocities. Introduction of the initial sinusoidal velocity modulation at the linac operating frequency is shown by the macroparticle method to permit considerable reduction of high current beam emittance growth in RFQ.

1 INTRODUCTION

The 6D-beam matching in RFQ is realized in the initial matching section. The transverse matching is achieved by monotonic increasing of the focusing strength from zero to the specified value. The longitudinal matching is provided by changing of the longitudinal tune and synchronous phase.

Earlier injection conditions for a matched beam have been determined by a solution of the Kapchinskij–Vladimirskij (K-V) equations along the matching section neglecting particle phase oscillations [1].

Below a set of envelope equations for a bunched beam with electrostatic interaction of bunches represented as uniformly charged ellipsoids is derived. On the base of the equations solution injection conditions for a matched beam are formulated at the RFQ entrance which fulfilment necessitates beam modulation in the longitudinal velocities. Introduction of the initial sinusoidal velocity modulation at the linac operating frequency is shown by the macroparticle method to permit considerable reduction of high current beam emittance growth in RFQ.

2 ENVELOPE EQUATIONS FOR A BUNCHED BEAM WITH ELECTROSTATIC INTERACTION OF BUNCHES

We derive the envelope equations for a bunched beam approximating it by a sequence of uniformly charged ellipsoids following each other at distance L. The beam self-field potential is determined by the field superposition of all ellipsoidal charges. If we neglect the metal boundary influence the Coulomb potential inside the considered bunch in linear approximation to space-charge forces is described by the quadratic form

$$U(x,y,\zeta) = -\frac{\rho}{2\varepsilon_0}[M_x^* x^2 + M_y^* y^2 + M_z^* \zeta^2]. \qquad (1)$$

Here ρ is the space-charge density; ε_0 is the electric constant; x, y, ζ are the coordinates originated from the given bunch centre; $M^*_{x,y,z}$ are the ellipsoid form factors with mutual influence of the bunches.

Factors $M^*_{x,y,z}$ may be represented in the form

$$M^*_{x,y,z} = M_{x,y,z} + \Delta M_{x,y,z} \qquad (2)$$

where $M_{x,y,z}$ are the single ellipsoid form factors [2, 3]; $\Delta M_{x,y,z}$ are the corrections to the form factors caused by electrostatic bunch interaction.

Further the considered bunch is assumed to be an ellipsoid with semiaxes r_x,r_y,r_z and all the rest of bunches are spheroids with semiaxes $r_t = \sqrt{r_x r_y}$, r_z.

The effects of electrostatic bunch interaction have been studied in book [2] through approximation of the beam by a sequence of uniformly charged spheroids. Using the results of this work the following expressions for form factor corrections may be easily obtained

$$\Delta M_{x,y} = -\frac{1}{2}\Delta M_z;$$
$$\Delta M_z = -2\frac{(1-\varepsilon^2)}{\varepsilon^3}\sum_{k=1}^{\infty}\frac{2k}{2k+1}\zeta(2k+1)\left(\frac{f}{L}\right)^{2k+1} \qquad (3)$$

where $\varepsilon = f/r_z$ and $f = \sqrt{r_z^2 - r_t^2}$ are the eccentricity and the focal length of the spheroid; $\zeta(s)$ is the ζ-Rieman function.

Within the framework of the accepted beam model relations (3) are accurate for axisymmetric focusing. In a quadrupole focusing linac these formulas take into account average influence of the bunch train.

The linear equations of particle motion with the beam self-field potential (1) have the form

$$\frac{d^2x}{d\tau^2} + Q_x(\tau)x - \frac{\alpha M_x^*}{r_x r_y r_z}x = 0,$$
$$\frac{d^2y}{d\tau^2} + Q_y(\tau)y - \frac{\alpha M_y^*}{r_x r_y r_z}y = 0, \qquad (4)$$
$$\frac{d^2\zeta}{d\tau^2} + Q_z(\tau)\zeta - \frac{\alpha M_z^*}{r_x r_y r_z}\zeta = 0.$$

Functions $Q_{x,y,z}(\tau)$ are proportional to the external force gradients with an opposite sign; $\tau = t/T_F$; T_F is the transition time of the focusing period; $\alpha = \frac{3\lambda^3}{\gamma^3}\frac{I}{I_0}k_F^2$; I is the beam pulsed current; λ is the wavelength of the accelerating field; γ is the Lorentz factor; I_0 is the characteristic current [2]; $k_F = S_F/\beta\lambda$; S_F is the length of the focusing period.

From motion equations (4) we receive the following set of envelope equations for a bunched beam ($r_z < L/2$)

$$\begin{aligned} \frac{d^2 r_x}{d\tau^2} + Q_x(\tau) r_x - \frac{F_t^2}{r_x^3} - \frac{\alpha M_x^*}{r_y r_z} &= 0, \\ \frac{d^2 r_y}{d\tau^2} + Q_y(\tau) r_y - \frac{F_t^2}{r_y^3} - \frac{\alpha M_y^*}{r_x r_z} &= 0, \\ \frac{d^2 r_z}{d\tau^2} + Q_z(\tau) r_z - \frac{F_\ell^2}{r_z^3} - \frac{\alpha M_z^*}{r_x r_y} &= 0 \end{aligned} \qquad (5)$$

where F_t and F_ℓ are the transverse and the longitudinal beam emittances on the phase planes $(x, dx/d\tau)$ and $(\zeta, d\zeta/d\tau)$ respectively.

Equations (5) together with relations (2),(3) generalize the results of paper [3] to case when mutual influence of the bunches is not ignored.

3 NUMERICAL STUDY OF 6D-BEAM MATCHING

We find conditions of 6D-beam matching at RFQ entrance by the envelope method based on the solution of equations (5). Let $K^2(\tau)$, $\gamma_0(\tau)$ and $\varphi_s(\tau)$ be laws of the focusing strength, defocusing factor and synchronous phase along the matching section respectively. Then functions $Q_{x,y,z}(\tau)$ in the matching section containing τ_0 focusing periods are defined by expressions

$$\begin{aligned} Q_{x,y}(\tau) &= \pm 4K^2(\tau)\cos[2\pi(\tau - \tau_0) + \varphi_s(\tau)] - \frac{1}{2}Q_z(\tau), \\ Q_z(\tau) &= -4\gamma_0(\tau)\sin\varphi_s(\tau). \end{aligned}$$

The set of equations (5) was solved in the segment from the gentle buncher input to the matcher input, i.e. from $\tau = \tau_0$ to $\tau = 0$. The initial conditions for system (5) at $\tau = \tau_0$ are the values of envelopes and their derivatives corresponding to the matched beam at the gentle buncher input. Numerical solution of system (5) was conducted for the proton beam with pulsed current 200 mA and transverse normalized emittance 0.2π cm·mrad. The longitudinal emittance is determined by the ratio of bunch length to linearized zero current separatrix length at the gentle buncher input. This ratio is chosen to be equal to 0.7 with a space charge bucket width depression. The matching section parameters are taken to be the same as for 3 MeV proton RFQ [4].

Normalized beam envelopes $\rho_{x,y} = r_{x,y}/\sqrt{F_t}$ and the relative bunch length $2r_z/\beta\lambda$ as functions of the dimensionless longitudinal coordinate τ inside the matching section are shown in figs. 1,2. When $\tau \approx 0.9$ bunches approach closely each other ($2r_z = L = \beta\lambda$) and the bunched beam becomes dc one (fig.2). Therefore in the following at $\tau < 0.9$ the length of each bunch is assumed to be constant and equal to $\beta\lambda$ (fig.2).

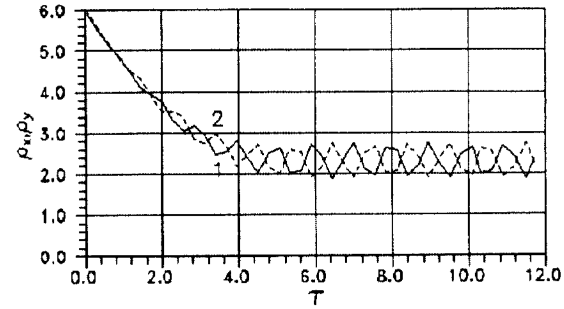

Figure 1: The normalized beam envelopes ρ_x (1), ρ_y (2) vs the dimensionless longitudinal coordinate τ.

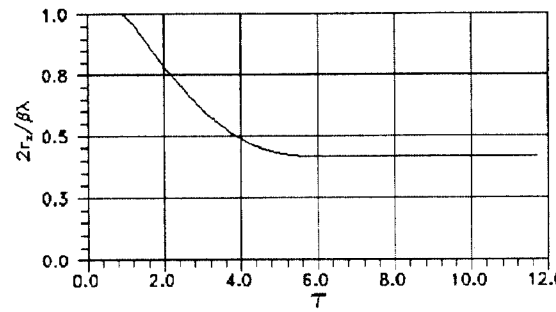

Figure 2: The relative bunch length $2r_z/\beta\lambda$ vs the dimensionless longitudinal coordinate τ.

As seen in fig.1, the matching conditions of transverse emittance with the channel acceptance are consistent with the convergent axisymmetrical beam at the linac entrance.

The representative ellipse of the matched beam in the longitudinal phase coordinates $(\zeta, d\zeta/d\tau)$ is plotted in fig. 3 at the touch position of bunches. To obtain the longitudinal phase portrait of the matched beam at the linac input it is necessary to take into account the following. Firstly, the used linear model adequately describes small longitudinal oscillations therefore at $|\zeta| \ll 1$ the phase volume boundary of the matched beam must coincide with the representative ellipse. Secondly, the requirement of the bunch length constancy at $\tau < 0.9$ means that particles located on the beam envelopes ($\zeta = \pm 0.5\beta\lambda$) must have zero slope of their trajectories ($d\zeta/d\tau = 0$). Taking these circumstances into consideration we come to the longitudinal phase portrait of the matched beam at the linac input shown in fig. 3 where the emittance is set equal to zero for drawing simplicity.

Thus to achieve a matching of 6D-phase volumes at the RFQ entrance it is necessary to inject in the linac a dc axisymmetrical convergent beam modulated in the longitudinal velocities.

From the practical point of view it is most simple to realize sinusoidal modulation in the longitudinal velocities at the linac input. In this case the sinusoid amplitude is determined by a unique manner by representative ellipse coefficients.

By the macroparticle method we consider the matching of 6D-phase volumes in RFQ at injection of the beam

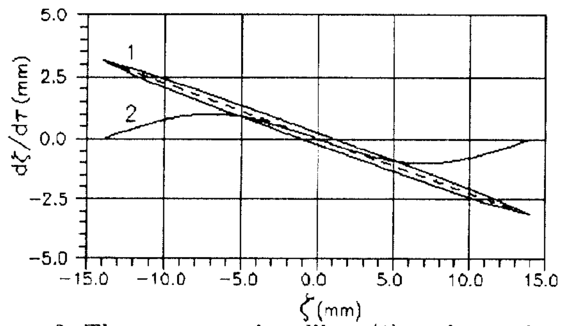

Figure 3: The representative ellipse (1) at the touch position of bunches and the phase portrait of matched beam (2) at the linac entrance for zero longitudinal emittance.

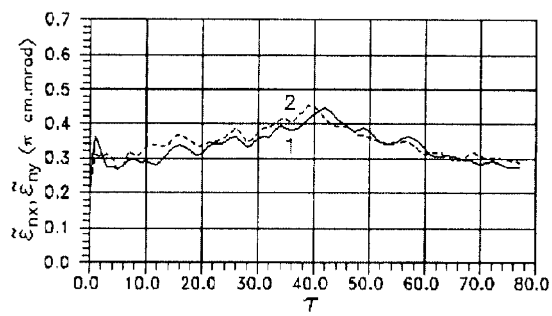

Figure 4: The normalized rms emittances $\tilde{\varepsilon}_{nx}$ (1), $\tilde{\varepsilon}_{ny}$ (2) vs the dimensionless longitudinal coordinate τ at the beam injection with sinusoidal modulation in longitudinal velocities.

with initial conditions obtained from the solution of the envelope equations (5).

We use the K–V macroparticle distribution in the input transverse phase space to reduce nonlinear space-charge effects on the calculated beam parameters. On the input longitudinal phase plane macroparticles coordinates were uniformly distributed in segment $\beta\lambda$ and their velocities were modulated in the sinusoidal law. The input beam energy spread with respect to the sinusoid was $\pm 1.35\%$. The energy spread value was calculated from equality condition of the longitudinal phase volume of the injected beam to the representative ellipse area.

The 6D-beam matching study was conducted in the 3 MeV proton RFQ by the code package [5]. The space-charge field calculations were carried out on the mesh $16 \times 16 \times 32$ in cylindrical coordinate system with 3600 macroparticles in the whole beam. The time integration step of motion equations was $\omega\Delta t = 2\pi/20$ where ω is the cyclic frequency of the rf field.

The normalized transverse rms emittances $\tilde{\varepsilon}_{nx,y}$ as functions of the dimensionless coordinate τ along the linac are shown in figs. 4,5 at the beam injection with sinusoidal modulation (fig. 4) and without modulation (fig. 5) in the longitudinal velocities.

Comparison of the curves in figs. 4,5 shows that the introduction of the initial sinusoidal modulation in the longitudinal velocities at the linac operating frequency allows to

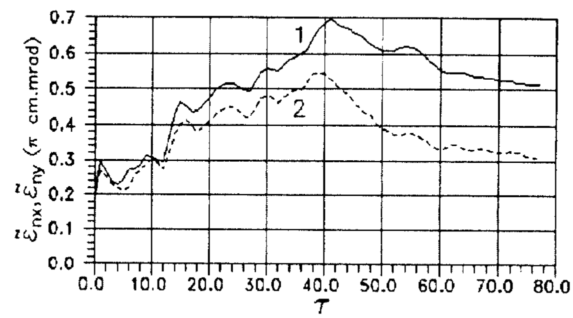

Figure 5: The normalized rms emittances $\tilde{\varepsilon}_{nx}$ (1), $\tilde{\varepsilon}_{ny}$ (2) vs the dimensionless longitudinal coordinate τ at the beam injection without modulation in longitudinal velocities.

reduce the beam transverse emittance growth in the RFQ approximately 2 times. Moreover the linac transmission efficiency is increased from 85.6 to 90%.

4 CONCLUSION

A set of envelope equations for a bunched beam with electrostatic interaction of bunches represented as uniformly charged ellipsoids is derived. On the base of the equations solution the behaviour of envelopes in the matching section of the 3 MeV proton RFQ is studied. The necessity for the dc beam modulation in the longitudinal velocities at the linac entrance is substantiated. During the beam injection with phase density 1 A/cm·mrad introduction of the initial sinusoidal velocity modulation at the linac operating frequency is shown by the macroparticle method to permit approximately 2 times reduction of the emittance growth in RFQ. Moreover the linac transmission efficiency is increased from 85.6 to 90%.

5 REFERENCES

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