Simulation of the Touschek Effect for BESSY II – A Monte Carlo Approach

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Abstract

A detailed Monte Carlo simulation of the Touschek effect is presented and the validity of simplifications usually made in analytical calculations as in the code ZAP (e.g. the non-relativistic approximation) is discussed.

For the BESSY II storage ring, the Touschek lifetime is calculated as a function of beam energy, momentum acceptance, horizontal aperture and beam polarization.

1 INTRODUCTION

For electron storage rings with very dense bunches (low-emittance light sources, Φ-factories), the scattering between electrons within a bunch (Touschek effect [1]) may significantly reduce the beam lifetime. Viewed in the bunch rest frame, this scattering process transfers transverse momentum into the longitudinal direction, which leads to a considerable momentum deviation in the lab system. Electrons are lost if the momentum deviation exceeds the momentum acceptance of the machine or if their displaced orbit – due to dispersion – exceeds the aperture.

Two effects contribute to a horizontal displacement:
1) Each off-momentum particle is displaced by dispersion.
2) If Touschek scattering occurs in a dispersive region, the betatron amplitude is increased.

The Touschek effect has been studied for the BESSY II storage ring currently under construction [2].

2 THEORETICAL DESCRIPTION

2.1 The Touschek Beam Lifetime

Since two electrons are involved, the Touschek loss rate $\dot{N}$ depends on the square of the number of electrons per bunch N:

$$\dot{N} = -aN^2 \rightsquigarrow N(t) = N(0)/(1 + N(0)\,a\,t). \qquad (1)$$

The half-lifetime $\tau_{1/2} = -N(0)/\dot{N}(0)$ depends on time and is further modified by current-dependent changes of the bunch size (multiple intrabeam scattering, bunch lengthening). All results below are given for a current of 100 mA in 320 bunches at a revolution frequency of 1.25 MHz, i.e. $N = 1.6 \cdot 10^9$.

2.2 Semi-Analytical Approach

In the non-relativistic approximation and with restriction to the horizontal momentum component, the Touschek loss rate is given by [3]

$$\dot{N} = \frac{N^2\, r_e^2\, c \cdot D(\xi)}{8\pi\gamma^2\, \sigma_x\sigma_y\sigma_z\, (\Delta p/p)^3} \quad \text{with} \quad \xi = \left(\frac{\Delta p/p}{\gamma\sigma_{x'}}\right)^2, \qquad (2)$$

where r_e is the classical electron radius, c is the velocity of light, $\Delta p/p$ is the momentum acceptance and γ is the Lorentz factor. The function D (≈ 0.3, typically) is evaluated by numerical integration. The symbol σ denotes one standard deviation of the coordinate in its subscript. The code ZAP [4] can be used to compute the Touschek lifetime by evaluating equation 2 along the circumference of a machine.

2.3 Monte Carlo (MC) Approach

In order to test the validity of the non-relativistic approximation and to include other features (non-linear effects, beam polarization), the scattering process is simulated event by event in the following way (for details see [5]):

1) The position along the circumference, 3 spatial coordinates (x, y, z), 6 momentum coordinates $(x'_{1,2}, y'_{1,2}, z'_{1,2})$ and 2 scattering angles (Θ, Φ) are chosen randomly.

2) The resulting momentum deviations $\delta p/p$ and the corresponding horizontal displacements δx are calculated.

3) For a given momentum acceptance $\Delta p/p$ and a horizontal aperture Δx, the summation over events with $\delta p/p > \Delta p/p$ or with $\delta x > \Delta x$ yields the Touschek loss rate

$$\dot{N} \approx \frac{\Delta V}{n}\frac{N^2}{\gamma^2 C}\sum_k 2v_k\sigma_k \sin\Theta_k\,(\rho_1\ \rho_2)_k \qquad (3)$$

where ΔV is the volume in the multi-dimensional space within which the random events are selected, n is the number of events, C is the circumference, v is the electron velocity in the center-of-momentum frame, ρ_i is the electron density and σ is the differential Møller cross section.

The mathematical simplicity of the MC approach allows to use relativistic kinematics and the relativistic differential Møller cross section (see [6] and references therein)

$$\sigma = \frac{r_e^2}{4}\left(1 - X^{-2}\right)\left[(X+1)^2\left(\frac{4}{\sin^4\Theta} - \frac{3}{\sin^2\Theta}\right) + 1 + \frac{4}{\sin^2\Theta} - P^2\left\{\frac{X^2-1}{\sin^2\Theta} + 2(X-1)\left(\frac{\sin^2\Phi}{\sin^2\Theta} + \cos^2\Phi\right) + \cos(2\Phi)\right\}\right] \qquad (4)$$

with $X \equiv (c/v)^2$. The second term depends on the beam polarization P. For large X, equation 4 reduces to the non-relativistic expression used e.g. in [3].

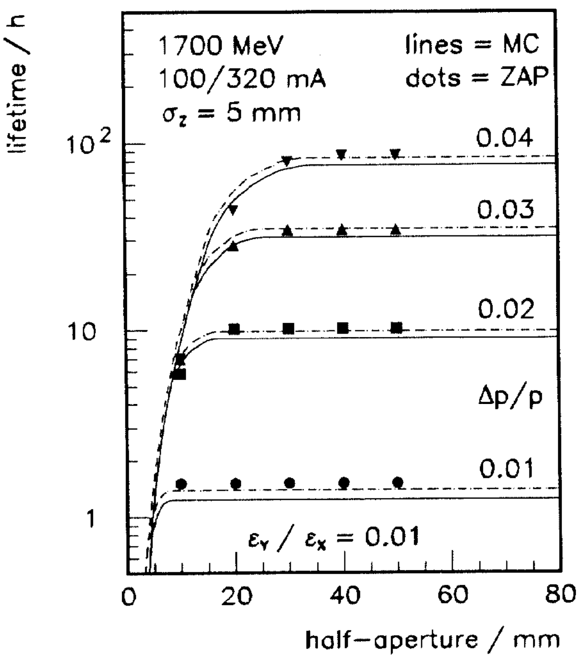

Figure 1: Touschek lifetime as a function of aperture and momentum acceptance $\Delta p/p$. The dots are lifetimes obtained using ZAP, the solid (dashed) lines are relativistic (non-relativistic) MC results.

3 APPLICATION TO BESSY II

All results shown below are obtained with 10^7 MC events. The statistical accuracy can be estimated by performing the same calculation with different random numbers. The resulting variation of the lifetimes presented here is $\approx 2\%$.

3.1 *Multi-bunch Operation at 1700 MeV*

At a current of $I = 100$ mA in $n_b = 320$ bunches, estimates of turbulent bunch lengthening (using ZAP) suggest a bunch length of $\sigma_z = 5$ mm with only little dependence on the beam energy. A coupling of $\varepsilon_y/\varepsilon_x = 0.01$ is assumed. The dependence of the lifetime τ on these parameters is

$$\tau \sim \sqrt{\varepsilon_y/\varepsilon_x}\ \ \sigma_z \cdot n_b \cdot I^{-1}. \tag{5}$$

Figure 1 shows the Touschek lifetime as a function of the horizontal half-aperture for BESSY II operating at 1700 MeV. The curves correspond to different values of momentum acceptance $\Delta p/p$. The lifetimes obtained from the relativistic MC simulation (solid lines) are about 10% shorter than the ZAP results (dots) and the lifetimes obtained from a non-relativistic MC calculation (dashed lines).

For a given momentum acceptance, there is a minimum aperture which does not influence the lifetime. According to figure 1, $\Delta p/p = 3\%$ yields a satisfactory Touschek lifetime of 31 hours, if the half-aperture is 25 mm or larger.

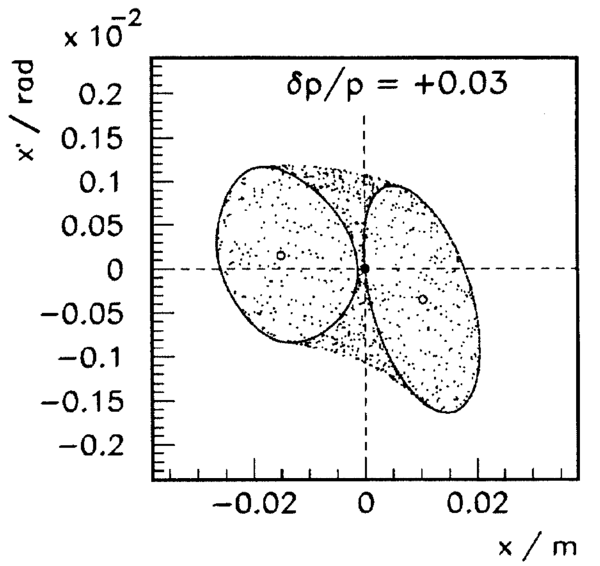

Figure 2: Motion of an off-momentum electron started with $\delta p/p = 0.03$ in a dispersive region and performing synchroton oscillations. The lack of symmetry between the dispersive orbits (small circles) and between the enhanced ellipses corresponding to $\delta p/p = \pm 0.03$ exhibits higher-order effects i.e. deviations from equation 6.

In this calculation, the maximum horizontal displacement δx of an off-momentum electron was treated in linear approximation:

$$\delta x = \sqrt{\varepsilon_x \cdot \beta_x} + |D_x \cdot \delta p/p|, \tag{6}$$

where β_x, D_x is the worst-case combination of beta function and dispersion. The motion of an electron after Touschek scattering in a dispersive region was simulated using the tracking code TRACY [7]. The simulation shows that higher-order chromatic effects (see figure 2) lead to $\approx 10\%$ larger displacements and the required half-aperture for $\Delta p/p = 3\%$, for example, is 29 mm instead of 25 mm.

3.2 *Single-bunch Operation at 1700 MeV*

For single-bunch operation at a current of 10 mA, ZAP estimates suggest a bunch length of 15 mm. According to equation 5, this leads to Touschek lifetimes which are 11 times shorter compared to the multi-bunch case with 100 mA, i.e. three hours for $\Delta p/p = 3\%$.

3.3 *Touschek Lifetime versus Beam Energy*

The BESSY II storage ring is optimized for a beam energy of 1700 MeV. However, operation at energies ranging from 900 to 1900 MeV is envisaged.

The dependence of the Touschek lifetime on the beam energy E is governed by two effects:

1) The factor γ^2 in equations 2 and 3, which can be interpreted as Lorentz-contraction of the bunch length and of the longitudinal projection of the Møller cross section.

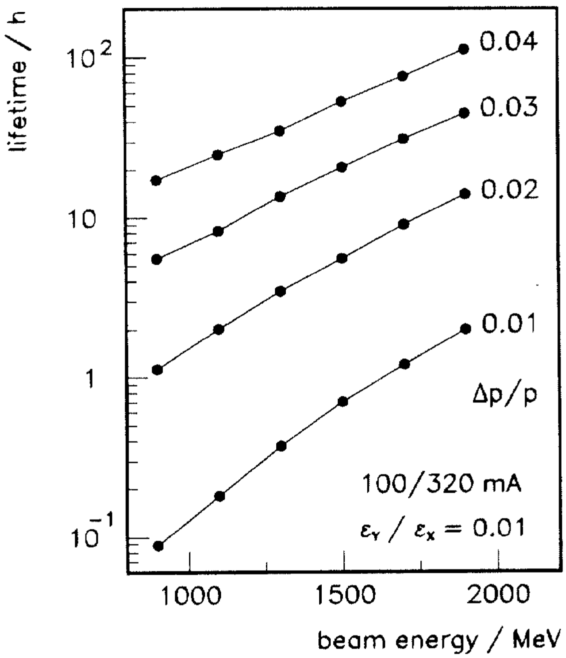

Figure 3: Touschek lifetime as a function of beam energy E and momentum acceptance $\Delta p/p$.

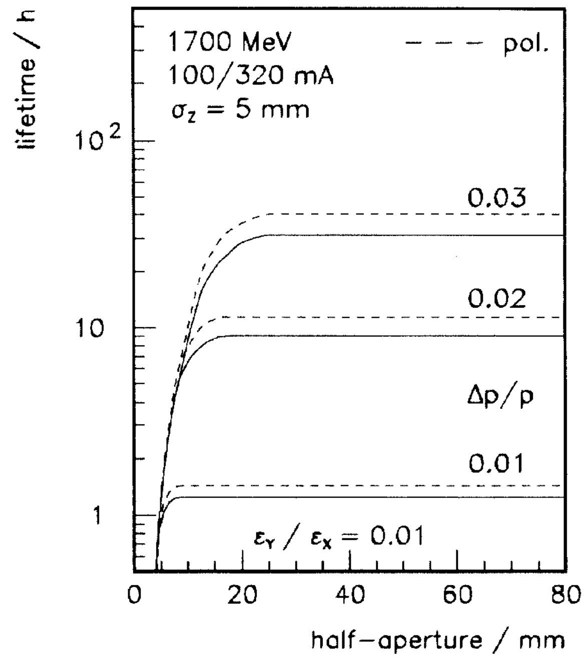

Figure 4: Touschek lifetime as a function of aperture and momentum acceptance $\Delta p/p$. The solid (dashed) lines represent the lifetimes of an unpolarized (fully polarized) beam.

2) The quadratic energy-dependence of the emittance, which determines the beam size and the transverse momentum spread.

Furthermore, the emittance blow-up due to multiple intrabeam scattering is significant at low beam energies. As a result, the energy-dependence of the Touschek lifetime is between $\tau \sim E^3$ and $\tau \sim E^4$, where the exponent decreases with increasing momentum acceptance as demonstrated in figure 3 for the case of multi-bunch operation.

3.4 *Beam Polarization*

At BESSY I, the beam energy is routinely determined by depolarizing the beam while monitoring the Touschek loss rate with scintillation counters in coincidence [8]. The signature of depolarization is an increase of the Touschek rate due to the spin-dependence of the Møller cross section (equation 4). According to the MC results shown in figure 4, the Touschek lifetime of a fully polarized beam at BESSY II is $\approx 20\%$ larger than the lifetime of an unpolarized beam. Lifetime measurements with a recently installed current monitor at BESSY I, where the effect is $\approx 10\%$, are in good agreement with the MC prediction [9].

4 SUMMARY

The Tousckek lifetime of the BESSY II storage ring was estimated using analytical and MC methods. The MC technique allows to simulate the scattering process in any detail. For most purposes, the non-relativistic approximation turns out to be sufficiently accurate ($\approx 10\%$ for BESSY II), but the possibilities to study the process in detail and to include e.g. beam polarization and higher-order effects make the MC approach worthwhile.

For a satisfactory operation of BESSY II, the beam lifetime should be of the order of 1 day. To achieve this, the Touschek effect requires a momentum acceptance of 3%, which also helps to reduce losses due to Bremsstrahlung. In this case, ± 3 cm is the minimum horizontal aperture which does not influence the lifetime.

5 REFERENCES

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