Numerical and Analytical Computations of Undulator/Wiggler Radiation

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Abstract

We present analytical and numerical methods of computation of the undulator/wiggler radiation. the techniques we discuss include the effects of the e-beam energy spread, emittances and matching conditions. We also analyze the possibility of exploiting the undulator/wiggler radiation as a diagnostic tool.

1. INTRODUCTION

Radiation emitted in magnetic undulators (U) or wiggler (W) is a versatile source and can be exploited in many different ways. Whatever application one has in mind, it is always necessary to know in a detailed way the characteristics of the U/W spectrum. Deviations from the ideal shape may be due to a number of reasons. Several computational schemes have been proposed [1,2]. The use of generalized Bessel functions, of two or more variables, allows the analytical computation of the brightness of *exhotic* forms of undulators [3]. Even though the brightness expansion in terms of generalized Bessel functions yields results hardly achievable using classical means, the inclusion of all the contributions providing the W/U spectrum modifications cannot be easily accomplished within an analytical framework.

In this contribution we compare analytical and numerical results. The analysis we develop refers to the brightness of linearly polarized U/W and includes: the dependence of the magnetic field on the transverse coordinates, the electron betatron motion, the e-beam emittances and the energy spread.

2. ANALYTICAL TREATMENT

It is rather difficult to perform an accurate analytical computation accounting for all the brightness distortion effects. A reasonably accurate computation should include: a) the magnetic field dependence on the transverse coordinates, b) the transverse e-beam phase-space distribution and the relevant optical functions, c) the e-beam relative energy distribution.

In Refs 2 an expression providing the U/W brightness including a-c) effects, has been obtained. We do not quote the explicit results, but comment on the general consequences of the analysis.

The on axis spectrum profile for the ideal case is

$$F(\nu_n) = \left(\frac{\sin \nu_n/2}{\nu_n/2}\right)^2 \qquad \nu_n = 2\pi N\,(n - \omega/\omega_1)\,; \qquad \omega_1 = 2\pi c/\lambda_1 \;,\quad \lambda_1 = \lambda_u/2\gamma^2(1+k^2/2) \tag{1}$$

where n is the order of the harmonic, N, k and λ_u the number of undulator periods, the parameter strength and period length respectively, finally γ denotes the electron relativistic factor. When the non ideal features are included and the e-beam is assumed to have the following phase-space and relative energy distribution

$$f(x,x';y,y';\varepsilon) = f(x,x')\cdot f(y,y')\cdot \frac{1}{\sqrt{2\pi}\,\sigma_\varepsilon}\exp\left[-\frac{\varepsilon^2}{2\sigma_\varepsilon^2}\right]$$

$$f(\eta,\eta') = 1/2\pi\varepsilon_\eta \exp\left[-\frac{\gamma_\eta \eta^2 + 2\alpha_\eta\,\eta\,\eta' + \beta_\eta \eta'^2}{2\varepsilon_\eta}\right] \tag{2}$$

with $(\beta_\eta, \alpha_\eta, \gamma_\eta)$ being the Twiss parameters and $\varepsilon_\eta \sigma_\varepsilon$ the e-beam emittance and energy spread, $F(\nu_n)$ modifies as follows

$$F(\nu_n) = 2\,\mathrm{Re}\int_0^1 \frac{dt\,(1-t)\,e^{i\nu_n t}}{\sqrt{R_x^{(n)}(t)\,R_y^{(n)}(t)}}\, e^{-1/2\,(\pi n \mu_\varepsilon)^2 t^2} \tag{3}$$

where

$$R_\eta^{(n)} = (1+\alpha_\eta^2)\,(1+i\pi n\mu_\eta t)\,(1+i\pi n\mu_{\eta'} t) - \alpha_\eta^2$$

$$\mu_\eta = \frac{2N\,\gamma^2\varepsilon_\eta}{(1+k^2/2)\beta_\eta}\;,\; \mu_{\eta'} = \frac{2N\,\gamma^2}{(1+k^2/2)}\,\Omega_\eta^2\,\frac{\varepsilon_\eta}{\gamma_\eta} \tag{4}$$

$$\Omega_\eta = \pi\lambda_u/k\gamma \;,\quad \mu_\varepsilon = 4N\,\sigma_\varepsilon$$

The above expression yields an idea of the U/W brightness spectrum distorsion, due to the μ-parameters.

Equation (3) is rather accurate for the first harmonics. The accuracy decreases with increasing n and when the matching conditions deviate from perfect matching ($\alpha_\eta = 0, \beta^*_\eta = 1/\gamma_\eta = 1/\Omega_\eta$).

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An e-beam with emittance radiates even on axis harmonics, the relevant analytical treatment is affected by rather crude approximations a numerical procedure seems therefore more appropriate.

3. NUMERICAL ANALYSIS OF THE LINEARLY POLARIZED U/W

The numerical calculation is performed by means of a Monte Carlo sampling of the mean electron radiation in function of the frequency. The initial energy of the electrons is obtained from the normal distribution having $\sigma_\varepsilon = \Delta\gamma/\gamma \cdot \gamma_0$, with γ_0 being the mean energy. The initial values of the position and velocity (x_0, x'_0, y_0, y'_0) of each electron history are obtained from the density distribution $f(\eta,\eta')$ (see Eq. (2)), using a multi-dimensional rejection procedure. The solid angle integration is sampled uniformly over a sphere surface. For each electron we solve numerically the system of ordinary differential equations

$$\frac{d f^{(r)}}{ds} = \left[\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})\right]_\eta \cos\left[\frac{\omega}{c}(s-\mathbf{n}\cdot\mathbf{r})\right]$$

$$\frac{d f^{(i)}}{ds} = \left[\mathbf{n}\times(\mathbf{n}\times\boldsymbol{\beta})\right]_\eta \sin\left[\frac{\omega}{c}(s-\mathbf{n}\cdot\mathbf{r})\right] \tag{5}$$

$$\frac{d^2 x}{ds^2} = k_u \frac{k}{\gamma}\left[A(x,y)\frac{dz}{ds}\sin(k_u z) - k_u y(s)\frac{dy}{ds}\cos(k_u z)\right]$$

$$\frac{d^2 y}{ds^2} = k_u^2 \frac{k}{\gamma}\left[\frac{dx}{ds}\cos(k_u z) - k_u/2\,\delta x(s)\frac{dz}{ds}\sin(k_u z)\right] y(s)$$

$$\frac{d^2 z}{ds^2} = k_u \frac{k}{\gamma}\left[\frac{k_u^2}{2}\delta\, x(s)\, y(s)\frac{dy}{ds} - A(x,y)\frac{dx}{ds}\right]\sin(k_u z)$$

where $\eta = x,y,z$, $s = ct$, $k_u = 2\pi/\lambda_u$ an $\mathbf{n}$ is the direction of observation. The quantity $A(x,y)$ accounts for the transverse coordinate dependence of U/W field and reads

$$A(x,y) = 1 + k_u^2/4\left[\delta x^2(s) + (2-\delta)\, y^2(s)\right] \tag{6}$$

and for the equal focussing case $\delta = 1$. The initial values of (5) are

$$f_\eta^{(r)}(0) = 0,\quad x(0) = x_0,\quad \left.\frac{dx}{ds}\right|_{s=0} = x'_0 - \frac{k}{\gamma}$$

$$f_\eta^{(i)}(0) = 0,\quad y(0) = x_0,\quad \left.\frac{dy}{ds}\right|_{s=0} = y'_0 \tag{7}$$

$$z(0) = z_0,\quad \left.\frac{dz}{ds}\right|_{s=0} = (\beta^2 - x_0'^2 - y_0'^2)^{1/2}$$

and the solution is found in the interval $[0, N\lambda_u]$ using the subroutine RKF45 [4]. The brightness derived from the electron history is calculated using the Lienard-Wiechert formula [5]

$$\frac{\partial^2 I}{\partial\omega\partial\Omega} = \frac{e^2}{4\pi^2 c}\left(\frac{\omega}{c}\right)^2 \sum_\eta \left\{\left[f_\eta^{(r)}(0) - f_\eta^{(r)}(L_u)\right]^2 + \left[f_\eta^{(i)}(0) - f_\eta^{(i)}(L_u)\right]^2\right\} \tag{8}$$

$$\eta = x, y, z$$

The brightness of the first and third on axis harmonic, calculated with the above quoted procedure, is shown in Figs 1,2 containing also a comparison with the analytical approximations.

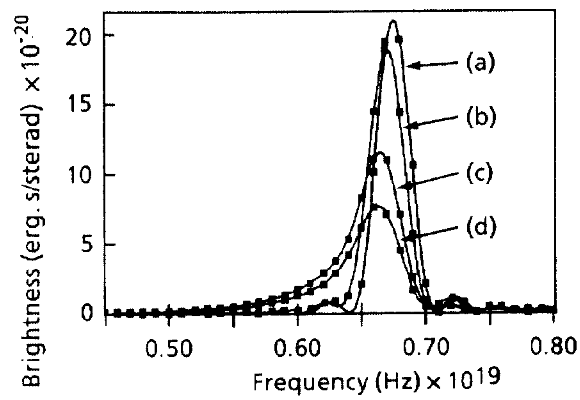

Figure 1. First harmonic brightness vs frequency parameters. a) analytical approximation, b) Numerical analysis, $\alpha_x = \alpha_y = 0$, $\beta_x = \beta_y = \beta^*$, c) analytical approximation, d) Numerical analysis $\alpha_x = \alpha_y = 1$, $\beta_x = \beta_y = \beta^*/10$. $E = 7$GeV, $\lambda_u = 5$cm, $K = 1.48$, $\varepsilon_x = 7\cdot 10^{-7}$ cm·rad, $\varepsilon_y = 8\cdot 10^{-8}$ cm·rad,

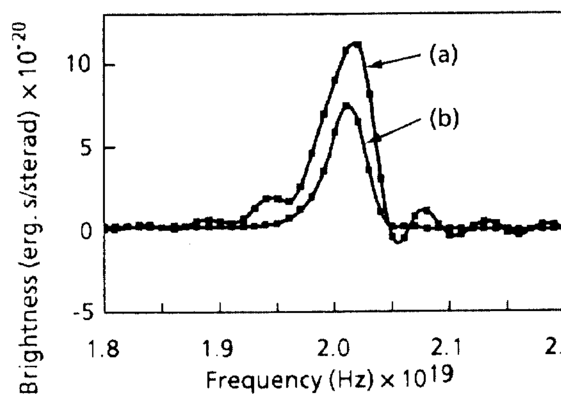

Figure 2. Third harmonic brightness vs frequency parameters of Fig. 1. a) analytical approximation, b) numerical analysis, $\alpha_x = \alpha_y = 0.5$, $\beta_x = \beta_y = \beta^*/5$,

In Fig. 3 we show the on axis second harmonic brightness for a matched and non matched case.

Regarding the first two figures it is evident that the reliability of the analytical approximation decreases with increasing order of the harmonics and when the matching conditions deviate from the perfect matching.

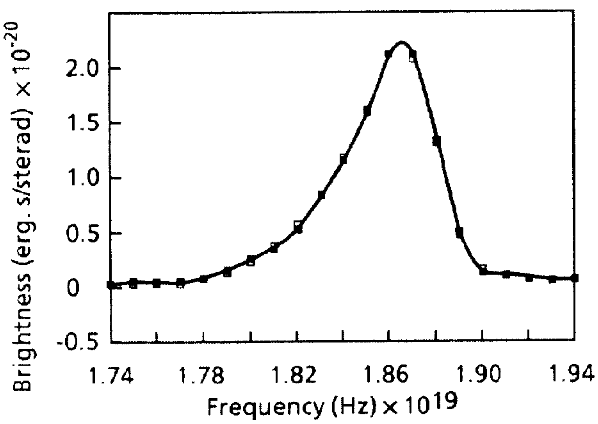

Figure 3. Second harmonic on axis brightness k=1, E=7 GeV, N=34, λ_u=5 cm, $\varepsilon_x=7\cdot10^{-7}$ cm·rad, $\varepsilon_y=8\cdot10^{-8}$ cm·rad.

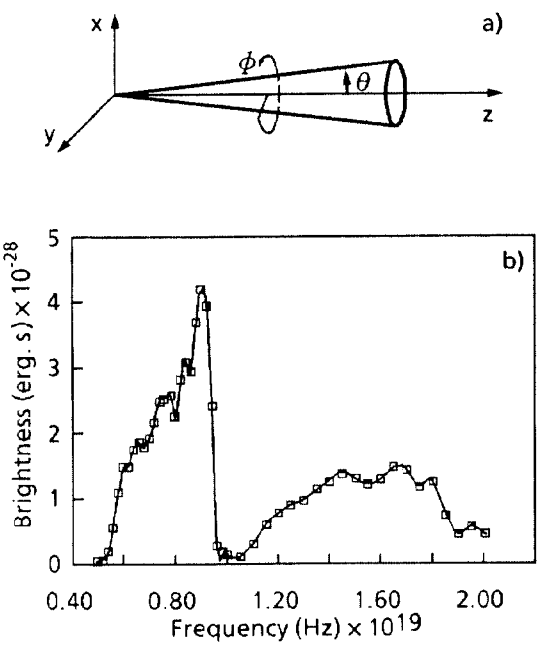

Figure 4. a) Solid angle integration cone. b) Brightness integrated over the solid angle vs frequency optimum matching conditions. k=1, E=7 GeV, N=34, λ_u=5cm, $\varepsilon_x=7\cdot10^{-7}$ cmrad, $\varepsilon_y=8\cdot10^{-8}$ cmrad, $\sigma_\varepsilon=10^{-3}$ integration intervals $\phi\in(0,2\pi)$, $\theta\in(0,k/\gamma)$.

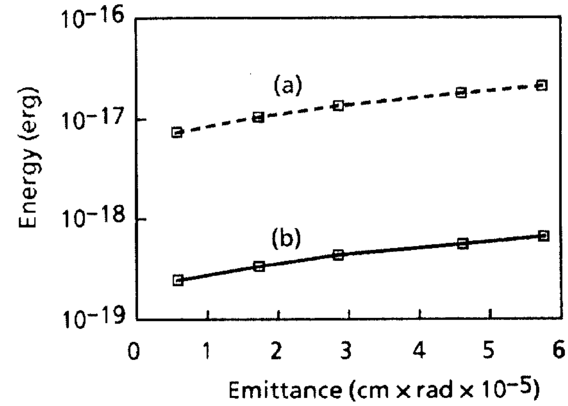

Figure 5. Energy vs emittance for a single macroelectron with distribution (2) vertical and radial emittances have been assumed to be identical. Optimum matching conditions k=1,N=20, λ_u=5cm, $\sigma_\varepsilon=10^{-3}$. a) E=56 MeV, integration interval $\Delta\omega\in$ (1.12,1.29) $\cdot10^{15}$ Hz, $\phi\in(0, 2\pi)$, $\theta\in(0,5\cdot10^{-4})$; b) E=28 MeV, integration interval, $\Delta\omega\in$ (2.78,3.24) $\cdot10^{14}$ Hz, $\phi\in(0,2\pi)$, $\theta\in(0,5\cdot10^{-4})$

In Fig. 4 we show the solid angle integration and it is evident that in this case the spectrum does not contain any information on the e-beam distribution. In Fig. 5 we show the energy vs emittance radiated by a low energy beam in a frequency interval around the second on axis harmonic and over a solid angle having ϕ and θ ranging from $[0, 2\pi]$ and $[0,5\times10^{-4}<k/\gamma]$ respectively. The radiated energy increases with emittance almost linearly and this fact is a precise indication that the undulator radiation can be quantitatively exploited for diagnostic tools.

4. REFERENCES

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