Computation of Eddy Currents in a Vacuum Chamber inside a Kicker Magnet.

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Abstract

The paper describes an algorithm for computation of eddy currents an accelerator vacuum chamber during a fast magnetic field variation cycle inside a kicker magnet. The approach is based on boundary integral equations method. Computer code BKICK is presented in which this algorithm is implemented. The program can calculate eddy-current contribution, screening of the external magnetic field, and heat rate inside the chamber walls. It is possible to define an arbitrary shape of the chamber and a complicated time dependence of the external magnetic field via the input data file. Efficiency of the computational algorithm all vs one to perform calculations on a small personal computer like IBM PC/AT. As an example of practical application of the program, the paper contains computational results for kicker magnet of the SSC Medium Energy Booster beam abort system.

1 INTRODUCTION

Eddy currents in vacuum chamber walls can produce noticeable screening of magnetic field and result in heating of a chamber. This effects are most valuable inside kicker magnets here abrupt jumps of magnetic fields occur. Experimental investigations of eddy currents are costly and time consuming. Therefore, numerical simulations are useful for that purposes.

In this paper boundary integral equation method is used to solve transient eddy currents problem. The approach involves discretization of the chamber walls only. This provides a good computational effectiveness and simplicity of usage. The paper describes the algorithm formulation, its program implementation and an example of application.

2 PROBLEM FORMULATION AND SOLUTION ALGORITHM

Consider piece of vacuum chamber inside a magnet. Let us suppose that it is a thin metallic cylindrical tube parallel to z direction. The magnet imposes time-dependent field $\mathbf{B}_0(t)$. This alternate field drives eddy currents in the chamber walls. The typical structure of such currents is shown in Fig. 1.

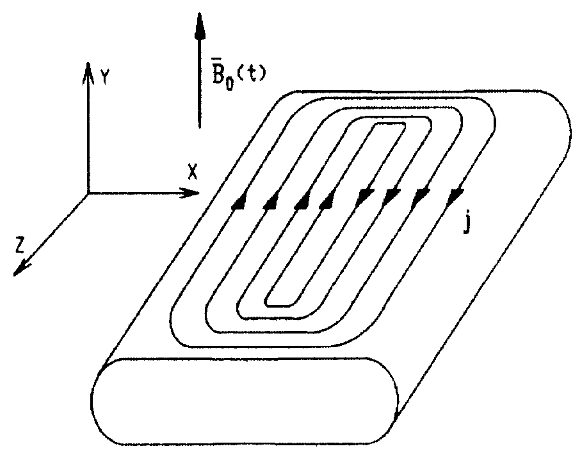


Figure 1: Structure of eddy currents in chamber walls.

These eddy currents disturb field inside the magnet by a value $\delta\mathbf{B}$. In the most part of the chamber the currents are parallel to z-direction, and resulting magnetic field lies entirely in the XY plane. This allows us to consider the problem as homogeneous one in z-direction by disregarding relatively small regions near ends of the magnet. Thus, the problem can be solved in an arbitrary cross-section at $z = $const.

Let us divide boundary of the chamber with N small segments like the ones shown in Fig. 2.

Equation of electromagnetic induction for contour 1-1'-2'-2 reads:

$$\oint E ds = -\frac{1}{c}\frac{d\Phi}{dt}. \qquad (1)$$

On using relation $j = \sigma E$ inside walls with conductivity σ, l.h.s. of this equation can be rewritten as

$$\oint E ds = \oint \frac{j}{\sigma} ds \approx \frac{L}{\sigma}(j_2 - j_1). \qquad (2)$$

Magnetic flux through the contour is

$$\Phi = l \cdot L \cdot \mathbf{n} \cdot (\mathbf{B}_0 + \delta\mathbf{B}) \qquad (3)$$

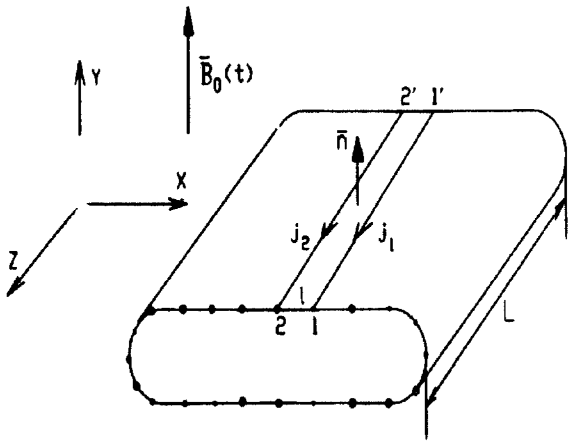
where L is length of the contour, l is its width and **n** is vector of unit normal. Here we use an approximate assumption that magnetic field is constant within a segment.

Additional field $\delta\mathbf{B}$ produced by eddy currents can be expressed as $\delta\mathbf{B} = curl\mathbf{A}$ where vector potential has only z-component $\mathbf{A} = (0, 0, A)$, and satisfies the following equation

$$-\Delta A = \frac{4\pi}{c} j.$$

For a central point of the segment value of A can be found by Green's formula

$$A(\mathbf{r}_0) = -\frac{4\pi d}{c}\int_S G(\mathbf{r}_0, \mathbf{r}) j(\mathbf{r}) d\mathbf{r} \qquad (4)$$



gure 2: Discretization of chamber walls.

where d is the wall's thickness and the integral is computed over chamber boundary cross-section. $G(\mathbf{r}_0, \mathbf{r})$ is Green's function which for 2D Laplace operator is

$$G(\mathbf{r}_0, \mathbf{r}) = \frac{1}{2\pi} ln|\mathbf{r}_0 - \mathbf{r}|.$$

On applying *curl* operator to (4) one gets

$$\mathbf{n} \cdot \delta\mathbf{B}(\mathbf{r}_0) = -\frac{2d}{c} \int_S F(\mathbf{r}_0, \mathbf{r}) j(\mathbf{r}) d\mathbf{r} \qquad (5)$$

where

$$F(\mathbf{r}_0, \mathbf{r}) = n_x \frac{y - y_0}{|\mathbf{r}_0 - \mathbf{r}|^2} - n_y \frac{x - x_0}{|\mathbf{r}_0 - \mathbf{r}|^2}.$$

On substituting (5) into (3) and (2),(3) into (1), one obtains

$$j_2 - j_1 = -\frac{2\sigma l d}{c^2} \int_S F(\mathbf{r}_0, \mathbf{r}) \frac{dj(\mathbf{r})}{dt} d\mathbf{r} + \frac{\sigma l}{c} \big(\mathbf{n} \cdot \frac{d\mathbf{B}_0}{dt}\big). \quad (6)$$

Let us associate current density value $j_k(t)$ with each k-th node of subdivision, and take a linear approximation to current variation inside each segment. Then the integral in r.h.s. of equation (6) can be approximated as a linear combination of nodal values $\dot{j}_k(t)$. Formulation of such relations for all segments of the chamber boundary yields a linear algebraic system of equations

$$j_{i+1} - j_i = \sum_k H_{ik} \dot{j}_k(t) + R_i, \quad i = 1, ..., N, \qquad (7)$$

with the matrix H being

$$H_{ik} = -\frac{2\sigma d l_i}{c^2} \left(\int_{S_k} F(\mathbf{r}_0, \mathbf{r}) \frac{|\mathbf{r} - \mathbf{r}_{k+1}|}{|\mathbf{r}_k - \mathbf{r}_{k+1}|} d\mathbf{r} + \right.$$
$$\left. + \int_{S_{k-1}} F(\mathbf{r}_0, \mathbf{r}) \frac{|\mathbf{r} - \mathbf{r}_{k-1}|}{|\mathbf{r}_k - \mathbf{r}_{k-1}|} d\mathbf{r} \right), \qquad (8)$$

and the load vector being

$$R_i = \frac{\sigma l_i}{c} \big(\mathbf{n} \cdot \frac{d\mathbf{B}_0(\mathbf{r}_0)}{dt}\big).$$

Here $\mathbf{r}_0 = (\mathbf{r}_i + \mathbf{r}_{i+1})/2$ is a central point of the i-th boundary segment; S_k, S_{k+1} are segments adjacent to node k; l_i is a length of the i-th segment.

Integrals in (8) contain weak singularity at $k = i, i-1$. These singularities can be integrated by technique described in [1].

System (7) rewritten in a matrix form reads

$$\mathbf{KJ} + \mathbf{H\dot{J}} + \mathbf{R} = 0. \qquad (9)$$

To solve it, we discretize the time interval under study with small enough step Δt, and apply Crank-Nicholson scheme (refer to [2]):

$$\left[\frac{1}{2}\mathbf{K} + \frac{\mathbf{H}}{\Delta t}\right]\mathbf{J}_{n+1} + \left[\frac{1}{2}\mathbf{K} - \frac{\mathbf{H}}{\Delta t}\right]\mathbf{J}_n + \frac{1}{2}\mathbf{R}_{n+1} + \frac{1}{2}\mathbf{R}_n = 0. \qquad (10)$$

This iterative algorithm yields vector $\mathbf{J}$ at moment t_{n+1} on each step by using the values at previous instant t_n. The initial vector is $\mathbf{J}_0 = 0$.

As a result, we have an approximation to the current density distribution in the whole time interval. This approximation can be used to obtain magnetic field induced by eddy current at any point and at any time moment with formula (5). We can also compute energy delivered by the eddy current as

$$U = \frac{Ld}{\sigma} \int \int j^2(\mathbf{r}, t) dS\, dt,$$

and estimate average temperature rise in the walls

$$\Delta T = \frac{U}{L d L_b c_w \rho_w}$$

where L_b is the boundary perimeter, c_w is specific heat and ρ_w is density of the wall material.

3 ALGORITHM IMPLEMENTATION AND APPLICATION EXAMPLE

Algorithm described above is implemented in the FORTRAN code BKICK. The program calculates eddy-current distribution, magnetic field and heat rate in a chamber wall. It allows to define an arbitrary shape of a chamber cross-section and a complicated time dependence of the external magnetic field via the input data file.

As an example of the practical application of the program, computations are performed for a model of the abort magnet of Medium Energy Booster at Superconducting Super Collider which employs the ceramic chamber with titanium coating. It has an aperture of 97.5mm×46.7mm. Thickness of the coating is about 1μm. Magnetic field rises up to the value of about 0.065T during 1μs.

The chamber wall is discretized with 30 segments. Whole time 2μs is subdivided into 60 steps. Computations

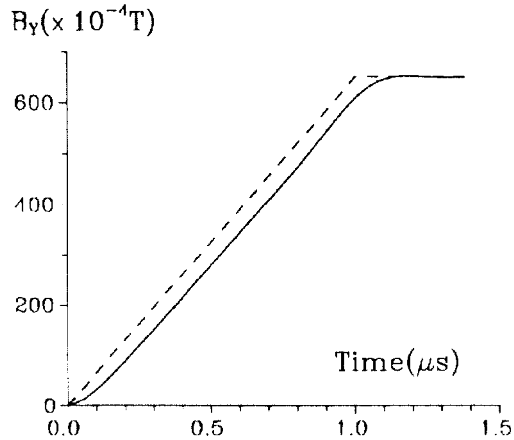


Figure 3 Magnetic field variation inside kicker magnet.

consume 30 s on an IBM PC/AT computer. Maximum of the current density is $0.8\times10^6 A/cm^2$. Resulting magnetic field in the center of the chamber is presented in Fig. 3. Dashed line corresponds to the field without eddy currents while the solid one — to the computed field in the presence of eddy currents inside the chamber walls. Energy delivery is about 2.6 J per meter of the chamber length. Average temperature rise is $\Delta T\approx 6K$.

4 REFERENCES

[1] Banerjee P.K., *Butterfield R. Boundary Element Methods in Engineering Science*. McGraw-Hill, 1981.

[2] Zienkiewicz O.C. *Finite Element Method*. McGraw-Hill, 1977