Some Aspects of TSD and TRL Calibration for a network-analyzer

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Abstract

For the TSD-/TRL-calibration of a network analyzer standard methods like *Through-Reflect-Line* and *Through-Short-Delay* are regarded with respect to the problems that arise when applying them. Methods to deal with these problems are presented and evaluated according to their applicability.

1 INTRODUCTION

For the removal of systematical errors in the measurement on high frequency devices numerous methods have been presented in the last years. Since normally measurements on such devices are carried out with an automatic NWA (Net Work Analyzer) different theoretical approaches were made especially for such measurements ([2], [3], [4]). In general a simple error model is regarded as shown in Figure 1, where the systematical errors are represented by two erroneous two-ports (error-two-ports) without caring about any crosstalk. To disregard crosstalk is justified, because it can be removed later on with a simple subtraction.

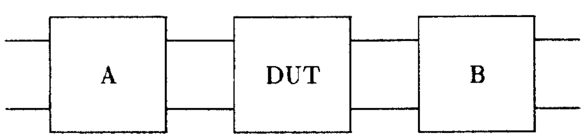

Figure 1. Error model

Since the behaviour of the twoports is uniquely determined by either their S-parameters (scattering parameters) or their T-parameters (wave transmission parameters, in this case non-vanishing transmission has to be assumed) the parameters of the DUT (Device Under Test) can be easily derived by the following matrix equations:

$$T_{DUT,M} = T_A\, T_{DUT}\, T_B \;\Leftrightarrow\; T_{DUT} = T_A^{-1}\, T_{DUT,M}\, T_B^{-1} \qquad (1)$$

Since measurements are carried out for a (countable) set of frequencies, these equations have to be read for one single, fixed frequency. The same applies for all of the following considerations.

2 CALIBRATION TECHNIQUES

In equation (1) the parameters of the error twoports are the unknowns of a set of linear equations. To obtain information about the other parameters in these equations is per se not possible since every measurement will be affected by the above mentioned errors. Therefore standards have to be introduced with known parameters. It is done on the base of their physical composition. With sufficient measurements on these standards enough data should be obtained to solve the linear equations evolving from (1). In praxi only a limited number of standards can be used with a sufficient accuracy of the postulated parameters, these include the ideal *Through* (direct connection), *Delay* or *Line* (line connection), *Short* or *Reflect* (a load with postulated value). The lack of a broader variety is not that uncomfortable since the minimization of the number of standards used is a goal demanded by statistical errors and inaccuracies emerging for example from the assembling and disassembling of the measurement configuration. Two particular sets of standards or calibration techniques are regarded here, TSD (*Through-Short-Delay*) and TRL (*Through-Reflect Line*). The corresponding calculation methods of the error twoports have already been regarded ([2], [5]), yet the particular problems arising when applying them is rarely payed attention to. To unroll these problems let us first discuss the different methods.

3 CALCULATION METHODS

Since T- and S-parameters are uniquely related (save for the above mentioned restrictions), we will mix from now on the two terms as is convenient. The corresponding matrices will be denoted by the appropriate capital letter, of the two different lines by the indices $D1$ and $D2$ (a *Through* is really a line with length zero) and of the load by the index S. Furthermore the adherent measured matrices will be indicated by the additional index M. Note that the length of the lines should be smaller than $\lambda/4$ at this frequency for a unique relation between phase shifts and lengths. The postulated T-matrices resp. S-matrices can be written as follows:

$$T_{D1} = \begin{pmatrix} e^{-\delta_1} & 0 \\ 0 & e^{\delta_1} \end{pmatrix} \quad , \quad T_{D2} = \begin{pmatrix} e^{-\delta_2} & 0 \\ 0 & e^{\delta_2} \end{pmatrix}$$

$$S_S = \begin{pmatrix} r & 0 \\ 0 & r \end{pmatrix} \; .$$

We get with equation (1)

$$\begin{aligned} T_A T_{D1} T_B = T_{MD1} &\;\Rightarrow\; T_{MD1}^{-1} T_A T_{D1} T_B = E \\ T_A T_{D2} T_B = T_{MD2} &\;\Rightarrow\; T_{MD2}^{-1} T_A T_{D2} T_B = E \end{aligned}$$

$$\begin{aligned} \Rightarrow\; T_{MD1}^{-1} T_A T_{D1} &= T_{MD2}^{-1} T_A T_{D2} \\ \Rightarrow\; T_A (T_{D1} T_{D2}^{-1}) &= (T_{MD1} T_{MD2}^{-1}) T_A \; . \qquad (2) \end{aligned}$$

Analogously we find

$$(T_{D2}^{-1}T_{D1})T_B = T_B(T_{MD2}^{-1}T_{MD1}) \quad . \tag{3}$$

We define $H := (T_{MD1}T_{MD2}^{-1})$, $K := (T_{MD2}^{-1}T_{MD1})$. Then we write $(H)_{ik}$ and $(K)_{ik}$ as H_{ik} and K_{ik} and $(T_A)_{ik}$ and $(T_B)_{ik}$ as a_{ik} and b_{ik}, Additionaly we abbreviate $\vartheta := \delta_2 - \delta_1$. The elements of the matrices H and K can be calculated from measured data and are therefore known (and constant) parameters from now on, the lengths of the lines are not necessarily known but constant numbers as well. So we finally obtain:

$$\begin{pmatrix} a_{11}e^{\vartheta} & a_{12}e^{-\vartheta} \\ a_{21}e^{\vartheta} & a_{22}e^{-\vartheta} \end{pmatrix} = \\ = \begin{pmatrix} a_{11}H_{11} + a_{21}H_{12} & a_{12}H_{11} + a_{22}H_{12} \\ a_{11}H_{21} + a_{21}H_{22} & a_{12}H_{21} + a_{22}H_{22} \end{pmatrix} \tag{4}$$

The equation for the b_{ik} is similar and is not explicitly written. Since the T parameters of either error twoport are nonzero, the linear equations for the coefficients a_{ik} and b_{ik} are linear dependent and one has to find other means to determine the coefficients. Let us first regard error twoport A. By dividing the equations in the first and the second column of (4) we obtain

$$\left(\frac{a_{11}}{a_{21}}\right)^2 H_{21} + (H_{22} - H_{11})\left(\frac{a_{11}}{a_{21}}\right) - H_{12} = 0 \tag{5}$$

$$\left(\frac{a_{12}}{a_{22}}\right)^2 H_{21} + (H_{22} - H_{11})\left(\frac{a_{12}}{a_{22}}\right) - H_{12} = 0 \; . \tag{6}$$

Note that both of these quadratic equations have the same coefficients. Because they are known from the measurements we find solutions for the values a_{11}/a_{21} and a_{12}/a_{22} which are characteristiques of error twoport A. Now the problem arises which of the two possible solutions to assign to a_{11}/a_{21} and which to a_{12}/a_{22}. Since (once again) the T-matrix of A has to be nonsingular they have to be different, but there are no means to determine the right root choice solely by the physical properties of A itself. Three methods to do this shall now be presented. After applying a proper root criterion, all the S parameters of the error twoports can be easily calculated with the additional information of the measurement values of the load (and, in the case of the TRL-method the information about the the lengths of the lines). Finally, when trying to deimbed the measured device with equation (1) there occurs the problem of phase stability. We will refer to this in the last section.

4 ROOT CHOICE

We shall now introduce three methods to choose the root correctly and evaluate and compare them afterwards:

1. One possibility is to consider the incident waves on the errorports to be roughly proportional to the emerging ones. This allows an assessment of the two solutions by their magnitude.

$$b_1 = a_{12}\left(\frac{a_{11}}{a_{12}}a_2 + b_2\right) \quad , \quad b_1 \overset{!}{\sim} a_2 \quad \Rightarrow \quad \left|\frac{a_{11}}{a_{12}}\right| \gg 1$$

$$a_1 = a_{22}\left(\frac{a_{21}}{a_{22}}a_2 + b_2\right) \quad , \quad a_1 \overset{!}{\sim} b_2 \quad \Rightarrow \quad \left|\frac{a_{21}}{a_{22}}\right| \ll 1$$

therefore

$$\left|\frac{a_{11}}{a_{21}}\right| \overset{!}{>} \left|\frac{a_{12}}{a_{22}}\right| \quad . \tag{7}$$

2. If the roots have been assigned correctly (i.e. physicaly true) the calibration of the used standards should give some additional information about the validity of the used calibration procedure. So for instance $(S_{D1})_{12}$ from the *Through* measurement should yield a number with positive real part (remember $\lambda < 1/4$). Therefore one arbitrarily assignes the roots of the quadratic equations (5) and (6) to a_{11}/a_{21} resp. a_{12}/a_{22}, continues the algorithm with these values and checks whether the just mentioned physical condition is fulfilled. If not, the root choice must have been wrong and by repeating the whole procedure with swapped values of a_{11}/a_{21} and a_{12}/a_{22} the correct values should be obtained.

3. By dividing the lower left equation by the upper left equation in (4) it can be found after minor remodeling and inserting the solution for the quadratic equation (5) one finds

$$e^{2\vartheta} = \frac{H_{11} + H_{22} \pm \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}}{H_{11} + H_{22} \mp \sqrt{(H_{11} - H_{22})^2 + 4H_{12}H_{21}}} \tag{8}$$

Examining the real and imaginary part of ϑ yields $|e^{2\vartheta}| < 1$ since the absolute value of $e^{\,\mathrm{i\,Im}\,\vartheta}$ is always one and the real part describes dampening because of losses in the lines. Therefore one can test the validity of the assigned root choices by inserting them into (8).

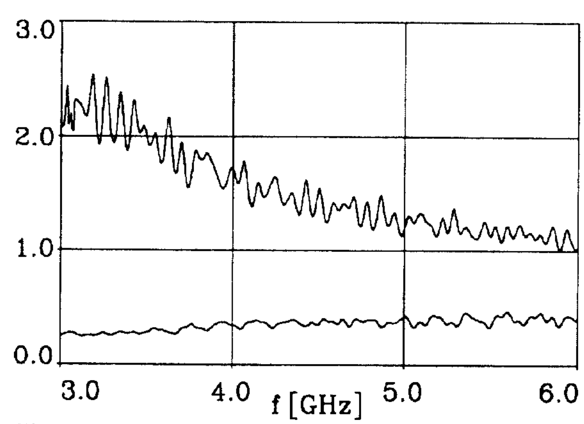

Figure 2: Minimum and maximum value of the solutions of (7) in 4.1)

For a wide scope of measurements we used high quality standards and had the opportunity to test the different criteria. For one exemplary calibration procedure figures

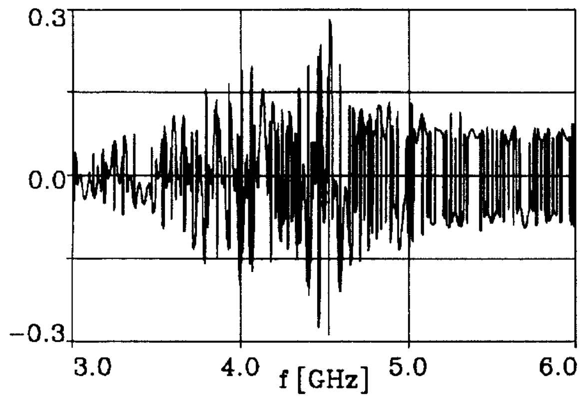

Figure 3: Imaginary part of $(S_{D1})_{21}$, deimbedded, in 4.2)

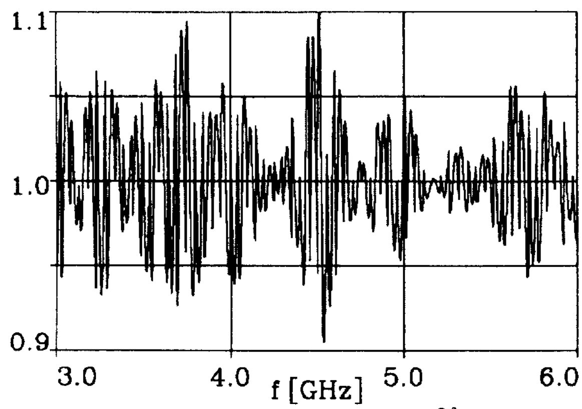

Figure 4: Absolute value of $e^{2\vartheta}$ in 4.3)

2, 3 and 4 allow to compare them. The third criterion is surely the most definitive when regarding the mathematical argumentation, no additional assumptions have to be made. We did in fact use it successfully. Yet the differences from one are at times quite small and noise or other errors could mask them. Therefore we combined it with the first criterion which, with reasonable additional assumptions, gives clearer indication. These two criteria have the advantage of giving a unique statement about the correct root choice.

The second criterion now is one of the most frequently used, with nowadays equipment it is no problem to redo the whole calculation even in the case of many frequency points. Yet it is only a mandatory and not a sufficient criterion and should therefore be regarded with suspicion. Hence it is suggested to mainly use it in combination with one of the other criteria.

5 PHASE STABILITY

The deimbedding of a device should be no problem when applying equation (1) with the obtained values for the error twoports A and B. Yet an unnerving phase instability is observed when applying the transformed equation, as can be seen in figure 5. This goes for the TSD calibration procedure ([2]) as well as for the TRL method ([5]). One remedy that shows good results is to exert the continuity of the phase, the data from figure 5 corrected with

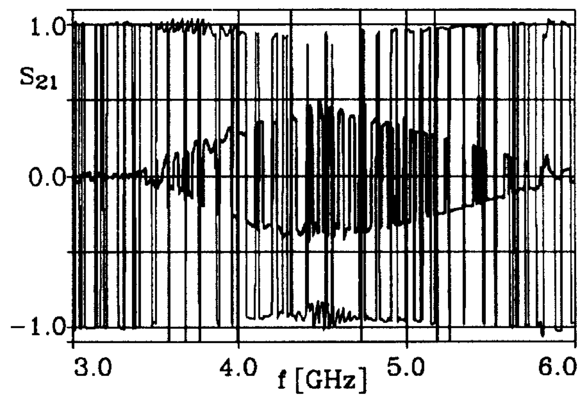

Figure 5: Imaginary and real part of S_{21} of a capacity loaded *Through*, deimbedded, without phase correction

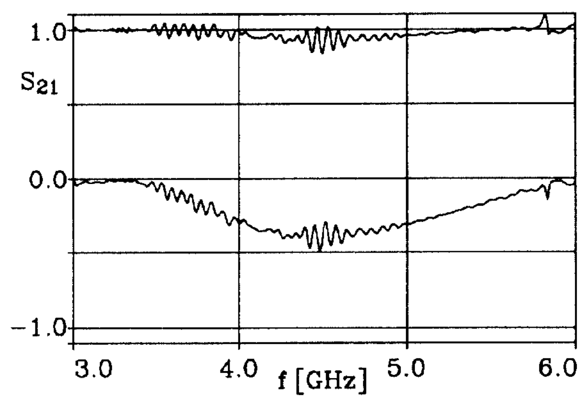

Figure 6: Imaginary and real part of S_{21} of a capacity loaded *Through*, deimbedded, with phase correction

this method is shown in figure 6. Anyhow a more definite treatment would be pleasant since this method very much depends on the initial values and is not very reliable for small values.

6 REFERENCES

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