Comparison between Impedance Measurements and Analytical Calculations

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Abstract

For the calibration of experimental datas using TRL and TSD calibration there are different algorithms. The shunt impedance derived from the measurements with these algorithms has to be verified by theoretical results. For this purpose a cylindrical pill-box with and without a coaxial wire is evaluated using the mode-matching technique. The shunt impedance is evaluated with the power loss method. Furthermore the analytical results are compared with URMEL simulations.

1 INTRODUCTION

To estimate the impedance of a microwave structure the so called coaxial wire method commonly is used. The electromagnetic field of the real structure is disturbed by the wire simulating the beam. To get an idea of the deviation of the impedance for the coaxial structure in contrast to the original one a simple cylindrical cavity is treated. An analytic calculation using the mode matching method is done. It is compared with some datas from the numerical code URMEL. Then some results from measurements are presented.

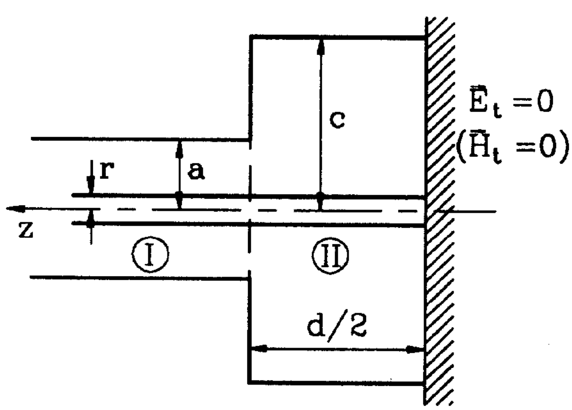

Figure 1: half part of a coaxial pill box

2 ANALYTICAL CALCULATION OF ELECTROMAGNETIC FIELDS IN A PILL-BOX

In the two spaces the following Ansatz is made:

- space(I) $\vec{E}_{t,1}(\varrho,z) = \vec{\mathbf{W}}_1^T(\varrho)e^{-\mathrm{j}\mathbf{B}_1 z}\mathbf{C}_1$, (1)

$$\vec{e}_z \times \vec{H}_{t,1}(\varrho,z) = -\vec{\mathbf{W}}_1^T(\varrho)\mathbf{Y}_1 e^{-\mathrm{j}\mathbf{B}_1 z}\mathbf{C}_1 \qquad (2)$$

- space(II) $\vec{E}_{t,2}(\varrho,z) = \vec{\mathbf{W}}_2^T(\varrho)\sin(\mathbf{B}_2 z)\mathbf{C}_2$, (3)

$$\vec{e}_z \times \vec{H}_{t,2}(\varrho,z) = -\vec{\mathbf{W}}_2^T(\varrho)\mathbf{Y}_2\cos(\mathbf{B}_2 z)\mathbf{C}_2 \qquad (4)$$

with

$$\vec{W}_{1;2n}(\varrho) = \beta_{1;2n,\varrho}\{N_0(\beta_{1;2n,\varrho}r)J_0'(\beta_{1;2n,\varrho}\varrho) - J_0(\beta_{1;2n,\varrho}r)N_0'(\beta_{1;2n,\varrho}\varrho)\}\vec{e}_\varrho ,$$

$$Y_{1;2n} = \frac{\omega\varepsilon}{\beta_{1;2n,z}} , \; B_{1;2n} = \beta_{1;2n,z} , \; \beta_0^2 = \beta_{1;2n,\varrho}^2 + \beta_{1;2n,z}^2 .$$

Because of the coaxial wire we additionally have to consider the TEM-wave:

- $$\vec{E}_{t,1;TEM}(\varrho,z) = Z_0 C_{10}\frac{a}{\varrho}e^{-\mathrm{j}\beta_0 z}\vec{e}_\varrho ,$$

$$\vec{e}_z \times \vec{H}_{t,1;TEM}(\varrho,z) = -C_{10}\frac{a}{\varrho}e^{-\mathrm{j}\beta_0 z}\vec{e}_\varrho ,$$

- $$\vec{E}_{t,2;TEM}(\varrho,z) = Z_0 C_{20}\frac{c}{\varrho}\sin(\beta_0 z)\vec{e}_\varrho ,$$

$$\vec{e}_z \times \vec{H}_{t,2;TEM}(\varrho,z) = -C_{20}\frac{c}{\varrho}\cos(\beta_0 z)\vec{e}_\varrho .$$

The following boundary conditions have to be fulfilled:

$$1)\; E_{\varrho,1}(\varrho, z=\frac{d}{2}) = \begin{cases} E_{\varrho,2}(\varrho, z=\frac{d}{2}) & \text{für } r \le \varrho \le a \\ 0 & \text{für } a < \varrho \le c \end{cases} \qquad (5)$$

$$2)\; \vec{e}_z \times \vec{e}_\varphi H_{\varphi,1}(\varrho, z=\frac{d}{2}) =$$

$$= \vec{e}_z \times \vec{e}_\varphi H_{\varphi,2}(\varrho, z=\frac{d}{2}) \quad \text{für} \quad r \le \varrho < a . \qquad (6)$$

Using the functions (1) to (4), multiplying (5) with $\vec{\mathbf{W}}_2$ and (6) with $\vec{\mathbf{W}}_1$ and integrating over the orthogonal interval yields the following equations:

$$\mathbf{E}\sin(\mathbf{B}_2\frac{d}{2})\mathbf{C}_2 = \mathbf{M}e^{-\mathrm{j}\mathbf{B}_1\frac{d}{2}}\mathbf{C}_1 + \mathbf{S}e^{-\mathrm{j}\beta_0\frac{d}{2}}C_{10} ,$$

$$\mathbf{M}^T\cos(\mathbf{B}_2\frac{d}{2})\mathbf{C}_2 = \mathbf{H}^T e^{-\mathrm{j}\mathbf{B}_1\frac{d}{2}}\mathbf{C}_1 .$$

with

$$\mathbf{M} = \int_r^c \vec{\mathbf{W}}_2\vec{\mathbf{W}}_1^T\varrho\,\mathrm{d}\varrho , \quad \begin{matrix}\mathbf{E} \\ \mathbf{H}\end{matrix} = \int_r^c \begin{matrix}\vec{\mathbf{W}}_2\vec{\mathbf{W}}_2^T \\ \vec{\mathbf{W}}_1\vec{\mathbf{W}}_1^T\end{matrix}\,\varrho\,\mathrm{d}\varrho ,$$

$$\mathbf{S} = \int_r^a \vec{\mathbf{W}}_2 Z_0\,\mathrm{d}\varrho .$$

For determining the TEM-constants the two boundary conditions must be integrated directly to adjust the mean of the function at the boundary of both spaces:

$$T_1 \sin(\beta_0 \frac{d}{2}) C_{20} = T_2 e^{-j\beta_0 \frac{d}{2}} C_{10} \, ,$$

$$\mathbf{T}^T \cos(\mathbf{B}_2 \frac{d}{2}) \mathbf{C}_2 + T_3 \cos(\beta_0 \frac{d}{2}) C_{20} = T_4 e^{-j\beta_0 \frac{d}{2}} C_{10} \, .$$

with

$$T_{1,2} = \int_r^{c,a} Z_0 \frac{c, a}{\varrho} \, d\varrho, \; \mathbf{T} = \int_r^a \vec{\mathbf{W}}_2 \, d\varrho, \; T_{3,4} = \int_r^a \frac{c, a}{\varrho} \, d\varrho.$$

From this you get the defining equation:

$$\Big\{ \mathbf{E} \sin(\mathbf{B}_2 \frac{d}{2}) - \frac{1}{\xi} \mathbf{S} e^{-j\mathbf{B}_2 0.5 d} \mathbf{T}^T \cos(\mathbf{B}_2 \frac{d}{2})$$

$$\mathbf{M} \mathbf{H} \mathbf{Y}_1 \mathbf{M}^T \mathbf{Y}_2 \cos(\mathbf{b}_2 \frac{d}{2}) \Big\} = 0 \qquad (7)$$

with

$$\xi = e^{-j\beta_0 \frac{d}{2}} \left[T_4 - T_2 \frac{T_3 \cos(\beta_0 \frac{d}{2})}{T_1 \sin(\beta_0 \frac{d}{2})} \right] \, .$$

To get solutions for this linear system the determinant of the coefficient matrix has to be zero. Scanning over the frequency range yields the Eigenvalues. For each one can estimate the field in the cavity.

Changing the sine into cosine and vice versa in (3) and (4) gives the cavity modes with odd symmetry. In terms of figure 1 one has to change the electric conducting wall at $z = 0$ into a magnetic one.

Removing the TEM-terms and taking the simple Bessel-function J_0 instead of the combination of J_0 and N_0 this calculation gives the fields in a simple tube loaded pill box. Figure 2 shows the lines of forces of the fundamental even mode in the simple, figure 3 in a coaxial cavity.

For the coaxial cavity one has loss of energy due to the TEM wave, so the fields in the cavity are evanescent with time.

The shunt impedance is defined as:

$$R_{s,lg} = \frac{[U_{lg}(\varrho_0)]^2}{P_V} = \frac{\left[\int_{-\infty}^{\infty} E_z(\varrho_0) e^{j\beta_0 z} \, dz \right]^2}{\frac{1}{2} \frac{1}{\kappa \delta} \int_{\text{surface}} \vec{H}_t \cdot \vec{H}_t^* \, dF}$$

where ϱ_0 is the radius at which the integral is performed.

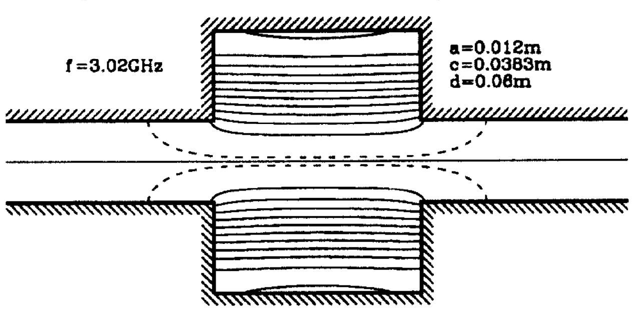

Figure 2: fundamental mode in a simple pill box

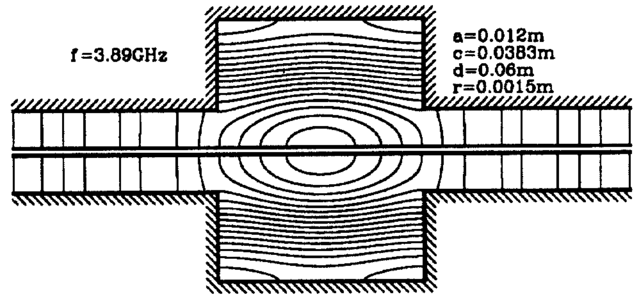

Figure 3: fundamental mode in a coaxial pill box

The values for the simple and coaxial pill-box are given in table 1. Here the voltage is calculated at $\varrho = a$.

simple cavity		
mode	frequency [GHz]	R_s [kΩ]
TM010	3.023	146.8
TM011	3.953	891.5
TM012	5.855	322.7
coaxial cavity		
mode	frequency [GHz]	R_s [kΩ]
TM010	3.893	98.63
TM011	4.371	571.54
TM012	5.976	150.10

Table 1: analytical results

3 NUMERICAL ESTIMATION OF SHUNT IMPEDANCE

With the numerical code URMEL using finite differences the resonant modes of closed structures can be determined. But because of the TEM-wave the wall closing the tubes will affect the field in the cavity forcing the tangential electric field to zero. To avoid this error one has to move the wall to a position of a zero position for the TEM-wave. Then one gets the shunt impedance of the tube loaded coaxial cavity.

For the simple cavity this problem doesn't exist because the field in the tubes is damped. Results are given in table 2.

simple cavity				
mode	frequency [GHz]	R_s [kΩ]	Q	R_s/Q Ω
TM010	3.021	147.5	10488	14.06
TM011	3.941	900.8	8672	103.9
TM012	5.838	320.8	10513	30.51
coaxial cavity				
mode	frequency [GHz]	R_s [kΩ]	Q	R_s/Q Ω
TM010	3.801	100.79	8200	12.29
TM011	4.370	242.01	7688	31.48
TM012	5.802	122.63	9600	12.77

Table 2: numerical URMEL results

4 MEASUREMENT OF SHUNT IMPEDANCE

In figure 4 the measurement setup is shown. For TRL-calibration three standards are needed, one short and two lines with different length. Once the arbitrary two-port is calibrated, the scattering parameters of the DUT can be measured. The measurement of the devices and the calibration is done with a code in the language C [2] controlling the network analyzer.

Figure 5 shows the S_{12}-parameter for the first three resonances in a pill-box like the one shown in Figure 3, figure 6 the impedance for the fundamental mode. Parameters for the first three modes are shown in table 3. The Q-values are loaded ones.

coaxial cavity				
mode	frequency [GHz]	R_s [kΩ]	Q	R_s/Q Ω
TM010	3.841	88.11	2564	34.36
TM011	4.214	543.7	14046	38.71
TM012	5.958	129.3	4186	30.89

Table 3: measurements results

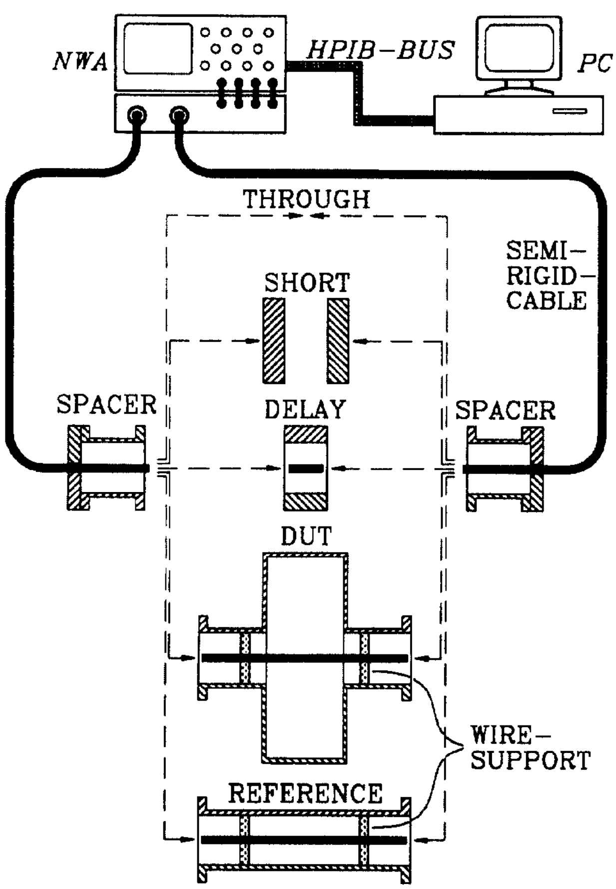

Figure 4:setup for coaxial wire measurement

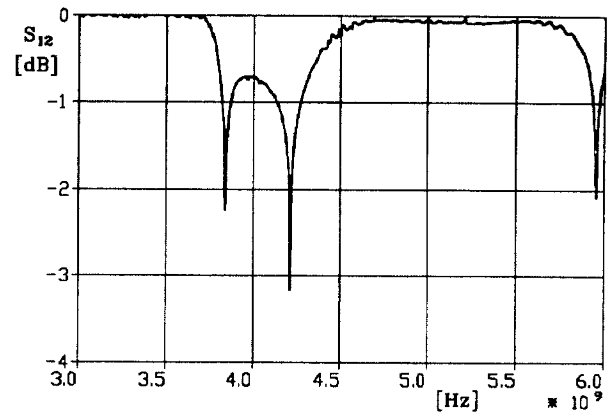

Figure 5: S_{12} parameter for the coaxial pill box

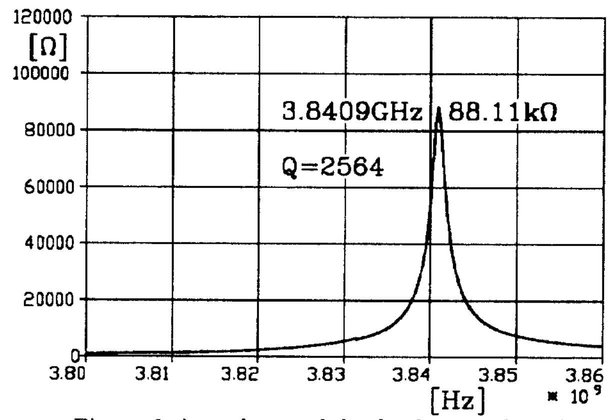

Figure 6: impedance of the fundamental mode

5 CONCLUSION

Comparison between calculations and measurements correspond quite good. But the difference of the values between the simple and the coaxial pillbox indicates the problem using the coaxial wire method. The measurement has to be corrected to higher values.

6 ACKNOLEDGEMENT

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7 REFERENCES

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