Impedance Calculations for a Coaxial Liner

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Abstract

Cold beam pipes as foreseen in superconducting hadron machines require shielding tubes with pumping holes called liner. As an extension of an existing rotational symmetric calculation now a more realistic liner structure with rectangular holes is treated. The electromagnetic field of one rectangular hole with arbitrary dimensions is determined by means of the residuum calculus in combination with orthogonal expansions. Numerical results are presented for the longitudinal coupling impedance and compared to results obtained with Bethe's small hole approximation.

1 INTRODUCTION

In [1] as a preliminary consideration we have treated a coaxial waveguide with rotational symmetric interruptions in the inner conductor.

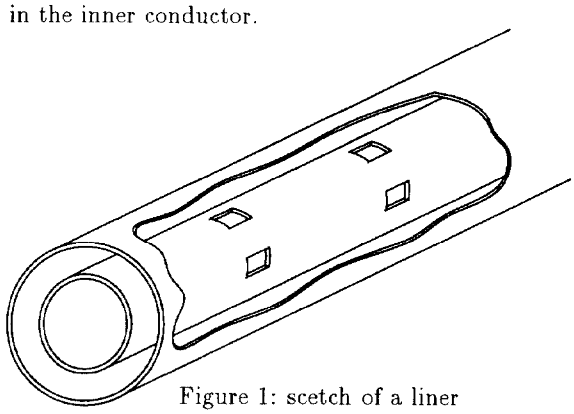

Figure 1: scetch of a liner

The solution was based on mode matching in transverse planes at the discontinuities. It is also possible to match the electromagnetic field on a surface with constant radius. But due to lack of boundary conditions in the longitudinal direction the fields are now represented by a continuous spectrum of waveguide modes. Especially in the case of an inner conductor with vanishing thickness this method is more convenient. So we will use it here for the non rotational symmetric problem of one rectangular hole in the inner conductor. Although we have restricted ourselves to one single hole the present analysis can be extended for any number of holes without fundamental problems. Adding holes equidistantly arranged and with the same longitudinal position the numerical evaluation will become more convenient because of additional symmetries. Holes with different longitudinal positions will merely cause more CPU time.

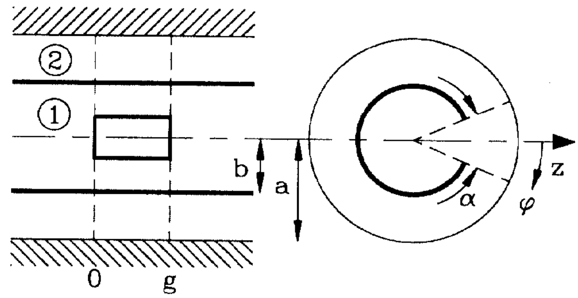

Figure 2: coaxial waveguide with a rectangular hole in the inner conductor

2 FIELDS IN THE DIFFERENT REGIONS

Figure 2 shows a longitudinal and transverse cut of the structure under consideration including all dimensions. On the axis we assume a point charge Q travelling with velocity of light which produces an exciting TEM-field. This reads in frequency domain

$$H_\varphi^{(e)} = \frac{E_\varrho^{(e)}}{Z_0} = \frac{Q}{2\pi\varrho}\,\mathrm{e}^{-\mathrm{j}k_0 z}\,,\; k_0 = \frac{\omega}{c_0}\,. \tag{1}$$

Because of the inhomogeneous boundary the scattered fields have to be a superposition of TE- and TM-waves which can be calculated from scalar functions V and W:

$$\mathbf{E} = \mathrm{rot}(\mathbf{e}_z V) - \mathrm{j}\frac{Z_0}{k_0}\,\mathrm{rot}\,\mathrm{rot}(\mathbf{e}_z W)\,,$$

$$\mathbf{H} = \mathrm{rot}(\mathbf{e}_z W) + \mathrm{j}\frac{1}{Z_0 k_0}\,\mathrm{rot}\,\mathrm{rot}(\mathbf{e}_z V) \tag{2}$$

with $\Delta(V, W) + k_0^2(V, W) = 0$. Taking into account the symmetry and boundary condition at the outer surface $\varrho = a$ we find the following solutions of the wave equation in region 1 and 2 respectively

$$V^{(1)} = Z_0 Q \int_{-\infty}^{\infty} \sum_m \sin(m\varphi) F_m^{(1)}(k_z) J_m(\lambda\varrho)\,\mathrm{e}^{-\mathrm{j}k_z z}\,\mathrm{d}k_z$$

$$W^{(1)} = Q \int_{-\infty}^{\infty} \sum_m \cos(m\varphi) G_m^{(1)}(k_z) J_m(\lambda\varrho)\,\mathrm{e}^{-\mathrm{j}k_z z}\,\mathrm{d}k_z$$

$$V^{(2)} = Z_0 Q \int_{-\infty}^{\infty} \sum_m \sin(m\varphi) F_m^{(2)}(k_z) R_m(\lambda\varrho)\,\mathrm{e}^{-\mathrm{j}k_z z}\,\mathrm{d}k_z$$

$$W^{(2)} = Q \int_{-\infty}^{\infty} \sum_m \cos(m\varphi) G_m^{(2)}(k_z) S_m(\lambda\varrho)\, \mathrm{e}^{-\mathrm{j}k_z z}\, \mathrm{d}k_z \tag{3}$$

with

$$\lambda^2 = k_0^2 - k_z^2$$

$$R_m(\lambda\varrho) = J_m(\lambda\varrho) N_m'(\lambda a) - N_m(\lambda\varrho) J_m'(\lambda a)$$

$$S_m(\lambda\varrho) = J_m(\lambda\varrho) N_m(\lambda a) - N_m(\lambda\varrho) J_m(\lambda a) ,$$

where J_m, N_m, J_m', N_m' are the Bessel- and Neumann-functions and their derivations and $F_m^{(1),(2)}(k_z)$, $G_m^{(1),(2)}(k_z)$ the searched for spatial spectrum.

In the aperture there are various possibilities to describe the tangential electric field. One of them is to split the aperture in $M \times N$ elements and to approximate the fields in each element using simple, e.g. pulse- or triangle-shaped functions $P(\zeta, \eta)$

$$E_z^{(b)} = Z_0 Q \sum_{\mu,\nu}^{M,N} A_{\mu\nu} P(z - z_\nu, \varphi - \varphi_\mu)$$

$$E_\varphi^{(b)} = Z_0 Q \sum_{\mu,\nu}^{M,N} B_{\mu\nu} P(z - z_\nu, \varphi - \varphi_\mu) . \tag{4}$$

Especially for pulse-shaped functions the necessary integration in the complex plane described below becomes more convenient. For $\alpha = 2\pi$, i.e. $E_\varphi^{(b)} = 0$, this choice turns out to be very efficient. In this case one already gets for $M = N = 1$ results comparable with [1].

Another way is to use trigonometric functions in the aperture:

$$W^{(b)}(z, \varphi) = Q \sum_{\mu,\nu} A_{\mu\nu} \cos\frac{(2\mu - 1)\pi\varphi}{\alpha} \cos\frac{\nu\pi z}{g}$$

$$\frac{\partial V^{(b)}}{\partial \varrho} = Z_0 Q \sum_{\mu,\nu} B_{\mu\nu} \sin\frac{(2\mu - 1)\pi\varphi}{\alpha} \sin\frac{\nu\pi z}{g} . \tag{5}$$

This expansion is more suited for the 3 dimensional case $\alpha \neq 2\pi$. Vanishing of E_z at the edges $|\varphi| = \alpha/2$ and of E_φ at the edges $z = 0, g$ is included. If we exchange the eigenvalues $2\mu - 1$ with 2μ we get also a complete field expansion. This choice is more suitable for the limiting case $\alpha \longrightarrow 2\pi$.

3 FIELD MATCHING

Performing the inverse Fourier transformation the boundary condition of the tangential electric field at $\varrho = b$ yields

$$F_m^{(1)} = \frac{1}{\pi^2 \lambda J_m'(\lambda b)} \left\{ \frac{\mathrm{j} m k_z}{\lambda^2 b (1 + \delta_0^m)} \sum_{\mu,\nu} \tilde{A}_{\mu\nu} Ce_{\mu\nu}(m, k_z) + \right.$$

$$\left. + \sum_{\mu,\nu} \tilde{B}_{\mu\nu} Se_{\mu\nu}(m, k_z) \right\}$$

$$G_m^{(1)} = \frac{\mathrm{j} k_0}{\pi^2 \lambda^2 (1 + \delta_0^m)} \frac{1}{J_m(\lambda b)} \sum_{\mu,\nu} \tilde{A}_{\mu\nu} Ce_{\mu\nu}(m, k_z)$$

where we have introduced the following abbreviations:

$$\tilde{A}_{\mu\nu} = \frac{\mathrm{j}}{k_0} A_{\mu\nu} \left\{ k_0^2 - \left(\frac{\nu\pi}{g} \right)^2 \right\} ,$$

$$\tilde{B}_{\mu\nu} = \left\{ B_{\mu\nu} + \frac{\mathrm{j}}{k_0 b} \frac{\pi}{\alpha} (2\mu - 1) \frac{\nu\pi}{g} A_{\mu\nu} \right\} ,$$

$$Ce_{\mu\nu}(m, k_z) = \int_0^g \int_0^{\frac{\alpha}{2}} \cos\frac{(2\mu - 1)\pi\varphi}{\alpha} \times$$

$$\times \cos\frac{\nu\pi z}{g} \cos m\varphi\, \mathrm{e}^{\mathrm{j}k_z z}\, \mathrm{d}\varphi\, \mathrm{d}z ,$$

$$Se_{\mu\nu}(m, k_z) = \int_0^g \int_0^{\frac{\alpha}{2}} \sin\frac{(2\mu - 1)\pi\varphi}{\alpha} \times$$

$$\times \sin\frac{\nu\pi z}{g} \sin m\varphi\, \mathrm{e}^{\mathrm{j}k_z z}\, \mathrm{d}\varphi\, \mathrm{d}z .$$

Similiar formulas are valid for $F_m^{(2)}$ and $G_m^{(2)}$. Matching the magnetic field results in the following system of linear equations:

$$\sum_{\mu,\nu} \left\{ \tilde{A}_{\mu\nu} M_{\mu\nu,\mu'\nu'}^{11} + \tilde{B}_{\mu\nu} M_{\mu\nu,\mu'\nu'}^{12} \right\} = \Gamma_{\mu'\nu'}$$

$$\sum_{\mu,\nu} \left\{ \tilde{A}_{\mu\nu} M_{\mu\nu,\mu'\nu'}^{21} + \tilde{B}_{\mu\nu} M_{\mu\nu,\mu'\nu'}^{22} \right\} = 0 . \tag{6}$$

$\Gamma_{\mu'\nu'}$ is calculated from the exciting magnetic field. The matrix elements $M_{\mu\nu,\mu'\nu'}^{ik}$ can in principle all be expressed in terms of the following integrals:

$$I_\nu = I_\nu^I - (-)^\nu I_\nu^{II} =$$

$$= \int_{-\infty}^{\infty} \left(f_\nu(k_z)\, \mathrm{e}^{-\mathrm{j}k_z z} - (-)^\nu f_\nu(k_z)\, \mathrm{e}^{\mathrm{j}k_z (g - z)} \right) \mathrm{d}k_z . \tag{7}$$

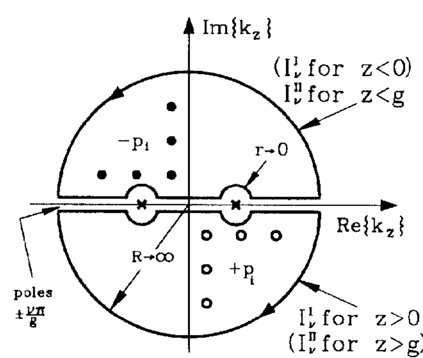

Figure 3: integration in the complex plain

If we close the integration path as shown in Figure 3, these integrals can be solved using the residuum calculus. The functions $f_\nu(m, k_z)$ essentially contain ratios of linear combinations of Bessel functions and due to lack of place cannot be discussed in detail here. The location of their poles is shown in Figure 3 assuming small dielectric losses. Furthermore we have to take into account the poles $\pm\nu\pi/g$

located on the real axis. So an arbitrary solution of (7) can be written as:

$$-\frac{I_\nu(m,z)}{2\pi \mathrm{j}} = \frac{1}{1+\delta_0^\nu} \times$$

$$\times \cos\frac{\nu\pi z}{g}\left\{\Psi_\nu\left(m,\frac{\nu\pi}{g}\right) + \Psi_\nu\left(m,-\frac{\nu\pi}{g}\right)\right\} -$$

$$-\mathrm{j}\sin\frac{\nu\pi z}{g}\left\{\Psi_\nu\left(m,\frac{\nu\pi}{g}\right) - \Psi_\nu\left(m,-\frac{\nu\pi}{g}\right)\right\} +$$

$$+\sum_i\left\{\Psi_\nu(m,p_i)\,\mathrm{e}^{-\mathrm{j}p_i z} + (-)^\nu\Psi_\nu(m,-p_i)\,\mathrm{e}^{-\mathrm{j}p_i(g-z)}\right\} \tag{8}$$

with

$$\Psi(m,\eta) = \lim_{k_z\to\eta}(k_z-\eta)f_\nu(m,k_z)\ .$$

4 THE LONGITUDINAL COUPLING IMPEDANCE

After truncating the infinite system (6) we get the Fourier coefficients for the tangential electric aperture field. It can easily be shewn that the longitudinal coupling impedance is given by

$$\frac{Z(\omega)}{Z_0} = +\frac{\mathrm{j}k_0}{\pi^2}\alpha\sum_{\mu\nu}\tilde{A}_{\mu\nu}\frac{(-)^\mu}{2\mu-1}\frac{(-)^\nu\,\mathrm{e}^{\mathrm{j}kg}-1}{\left(\frac{\nu\pi}{g}\right)^2-k_0^2} \tag{9}$$

with $Z_0 = \sqrt{\mu_0/\varepsilon_0}$. As expected for a stable numeric solution the ratio of μ and m of the azimuthal field dependence has to correspond to the ratio $\alpha/2\pi$.

Figures 4-7 show for different angles α the real and imaginary part of the impedance. As expected for small holes at lower frequencies the imaginary part is much greater then the real one and goes linear with frequency. Assuming a circular hole with about the same cross section Bethes theory of diffraction by small holes gives the impedance [2]:

$$Z \approx \mathrm{j}\,6.37\frac{r^3}{b^2}\ , \quad r:\ \text{radius of the hole}$$

For $\alpha = 10^o$ we choose $r = g/\sqrt{\pi}$ and obtain

$$Z(k_0 = 0.2\mathrm{mm}^{-1}) = \mathrm{j}0.024\ .$$

This value agrees very good with Figure 4. Increasing the angle α (Figure 5-7) the behaviour of the impedance more and more looks like in the limit $\alpha = 2\pi$ in [1] except for low frequencies. For $\alpha = 350^o$ we have an inductive low frequency behaviour which differs from the capacitive impedance for $\alpha = 360^o$, where the inner conductor is completely divided.

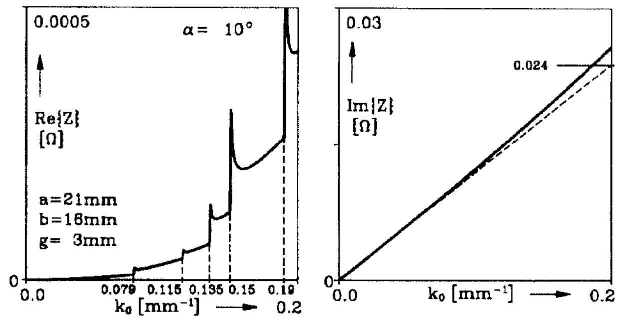

Figure 4: impedance for $\alpha = 10^o$

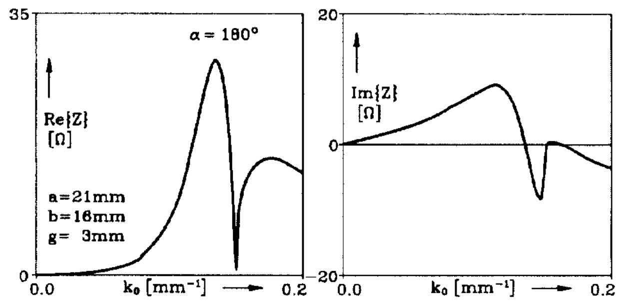

Figure 5: impedance for $\alpha = 180^o$

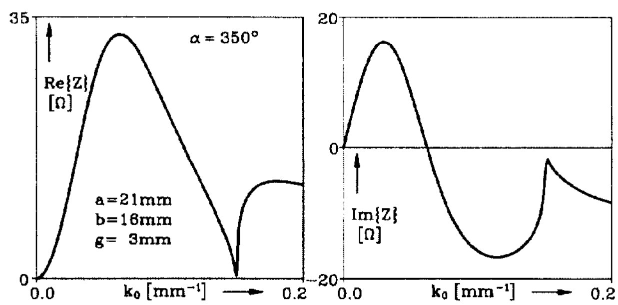

Figure 6: impedance for $\alpha = 350^o$

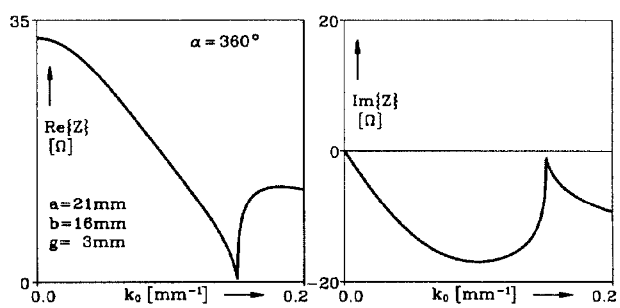

Figure 7: impedance for $\alpha = 360^o$

5 REFERENCES

[1] M. Filtz, T. Scholz, Impedance Calculations for a Coaxial Liner, Proceedings of the PAC 93, pp. 1036-1038

[2] S. S. Kurennoy, Beam Coupling of Holes in Vacuum-Chamber Walls, IHEP Preprint 92-84