Simulation of Beam-Beam Lifetime for LEP*

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ABSTRACT

A program that uses a new technique to simulate the beam halo in circular e^+e^- colliders is applied to the LEP. This technique makes it possible to investigate tail particle behavior in reasonable CPU times. The program now includes four interaction points and errors at or between the IP's. Analytical analysis shows that the errors break the symmetry and introduce extra low-order resonances that are forbidden by symmetry. Simulation results show the expected phenomenon. Another effect, the quadrupole mode coherent motion of the strong beam, is also studied.

I. INTRODUCTION

A simulation method[1] was proposed to look into rare particles in the beam tail (halo) while saving a factor of hundreds, or even thousands, on CPU time as compared to brute-force tracking. A program based on this method was written, tested and applied to PEP-II to understand the halo from the beam-beam interaction[2]. The study concluded that the resonance streaming dominates the beam-beam lifetime.

This technique was also applied to LEP. The results showed that the resonance streaming by the beam-beam kick is relatively weak in the LEP, but the lattice nonlinearities, in combination with the beam-beam interaction, can dramatically change lifetimes in the LEP[3].

To model a more realistic LEP, the simulation program has been upgraded to handle four interaction points (IP). The results show the important role of the errors. Hamiltonian analysis has been extended to interpret the simulation results.

II. HAMILTONIAN ANALYSIS WITH ERRORS

The Hamiltonian including the beam-beam interaction can be written as

$$H(x, p_x, y, p_y, s) = H_0 + V_{BB}(x, y, s) \qquad (1)$$

where H_0 is the unperturbed Hamiltonian of the storage ring, and V_{BB} is the beam-beam potential[4]. With B_{IP} interaction points, the beam-beam potential is

$$V_{BB}(x,y,s) = \frac{-Nr_e}{\gamma}\sqrt{\frac{2}{\pi\sigma_L^2}}\sum_{n=-\infty}^{\infty}\sum_{b=0}^{B_{IP}-1} V_F(x,y,s)\times \exp\left\{-\frac{2}{\sigma_L^2}\left[s-\left(nC+b\frac{C}{B_{IP}}+c\tau\right)\right]^2\right\} \qquad (2)$$

$$\tau = \frac{\hat{\tau}}{2}\cos(2\pi Q_s(n+\frac{b}{B_{IP}})), \qquad (3)$$

and V_F is defined in [4] with the additional feature that it depends on the parameters of interaction point b. By applying Fourier analysis, equation (1) becomes

$$H = H_0 - \frac{Nr_e}{C\gamma}\sum_{b=0}^{B_{IP}-1}\sum_{m,n,p,r=-\infty}^{\infty} T_{pr}^b \exp\{2\pi i(p\Delta Q_x^b + r\Delta Q_y^b) -(k_{pr}^b\sigma_L)^2/8\}\times i^m J_m\left(k_{pr}^b\hat{\tau}c/2\right)\exp\{i(p\psi_x + r\psi_y -2\pi(n-mQ_s)s/C) + i2\pi nb/B_{IP}\} \qquad (4)$$

where T_{pr}^b is a function of transverse actions and strong beam size at each IP , and k_{pr}^b is a wave number that also depends on IP parameters. ΔQ^b's are the phase advance errors from IP_{b-1} to IP_b relative to the standard phase advance Q/B_{IP}.

First, examine the phase in the second exponential function in equation (4). Requiring it to vary slowly gives the resonance condition

$$pQ_x + rQ_y + mQ_s = n, \qquad (5)$$

However, if there are no errors, *i.e.*, all the IP's are identical and there are no phase advance errors, one can drop the super script b in eq. (4) and the sum over b reduces to the factor $\sum_{b=0}^{B_{IP}-1}\exp\{i2\pi bn/B_{IP}\}$. This factor, which can be viewed as a sum of a group of vectors, equals zero unless n is a multiple of B_{IP}. Resonance with n not equal to a multiples of B_{IP} are eliminated as the vectors cancel each other. The resonances left are

$$p\frac{Q_x}{B_{IP}} + r\frac{Q_y}{B_{IP}} + m\frac{Q_s}{B_{IP}} = \text{integer}, \qquad (6)$$

This is equivalent to a storage ring with one IP and $1/B_{IP}$ of the size. If the IP's are not identical, or there are phase advance errors, or both, the cancellation will not happen. In case the IP's are not identical, the resonances have unequal magnitudes, so that they will not cancel completely. In the other case the phase advances between IP's are different and the vectors are no longer evenly spaced, so that the cancellation is incomplete. As one can see from eq. (4), the phase advance errors between IP's combined with each resonance give phase factors $\exp\{i2\pi(p\Delta Q_x^b + r\Delta Q_y^b)\}$. A rough estimate of the sum over b is about equivalent to the sum of unit vectors with different phases. As the result, the phase advance errors may select certain resonances and discriminate against others, depending on the phase errors.

The above analysis gives two consequences for multiple IP colliders with errors: First, more resonances are introduced. The resonance condition with errors is $pQ_x + rQ_y + mQ_s = n$, or, in terms of tune per IP, the condition is

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$$p\frac{Q_x}{B_{IP}}+r\frac{Q_y}{B_{IP}}+m\frac{Q_s}{B_{IP}}=\frac{\text{integer}}{B_{IP}}. \quad (7)$$

Comparing eq. (7) with eq. (6), One can see that many more resonances are allowed in this case. Second, among those resonances, some are possibly of lower order. Their appearance can dramatically change the tail distribution, or the lifetime.

In LEP whered B_{IP}=4, the errors introduce 4 times as many transverse resonances as a perfect symmetric machine. Considering the many synchrotron sidebands involved, the total number of resonances allowed in an asymmetric machine is significantly higher.

III. SIMULATION RESULT

The beam distribution from simulations is plotted in transverse amplitude space. The amplitudes are normalized to beam sizes. The contour lines give equal number density, and are logarithmic. Figure 1 gives the beam distribution of LEP with a linear lattice and 4 symmetric IP's. Resonance lines allowed by symmetry up to 8th order are plotted over the distribution. One can identify the resonance $2Q_x$-$2Q_y$-$2Q_s$ as the one that dominates the tail formation.

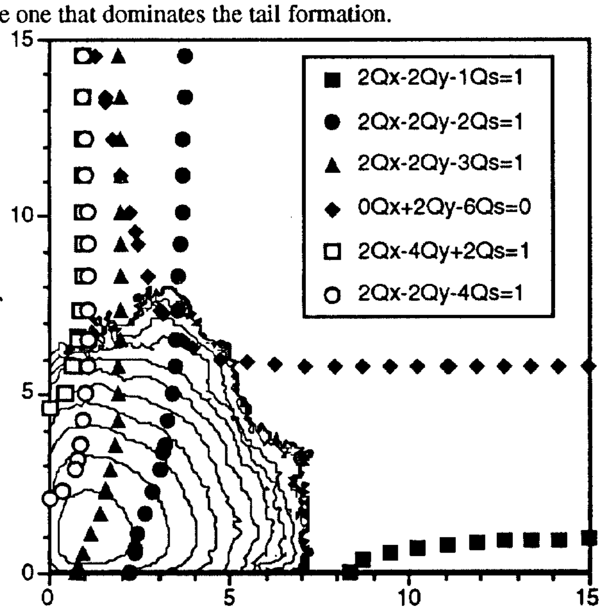


Fig 1. LEP beam-beam tail distribution and resonances. A linear, symmetric lattice is used. The tunes in the legend refer to 1/4 of the total tunes.

When random errors are included, the tail distribution changes dramatically, as we expected. The errors introduced include errors in the ß-functions and dispersion functions at each IP, and phase advance errors between IP's. The phase advance errors can be as large as 0.015 to 0.04 because only two arcs have RF cavities[5]. When the phase advance errors are introduced, the total tunes of the machine are held constant. Figure 2 shows the distribution.

With lattice errors, about 40 resonances appeared inside the footprint of the beam-beam interaction. Only four of those resonances, chosen because they appear related to the halo, are plotted in figure 2. The 6 resonances in figure 1 are still there. However, they apparently have little effect on the tail distribution. The resonances $2Q_x$+$2Q_y$+$0Q_s$=5/4 and $4Q_x$+$0Q_y$-$3Q_s$=9/4 seem responsible for the vertical tail, and we conjecture that the lower order resonance is more important, as discussed below. The resonance $2Q_x$+$2Q_y$-$1Q_s$=5/4 also has an effect at the up-left corner. Notice that all these resonances are forbidden in the symmetrical case.

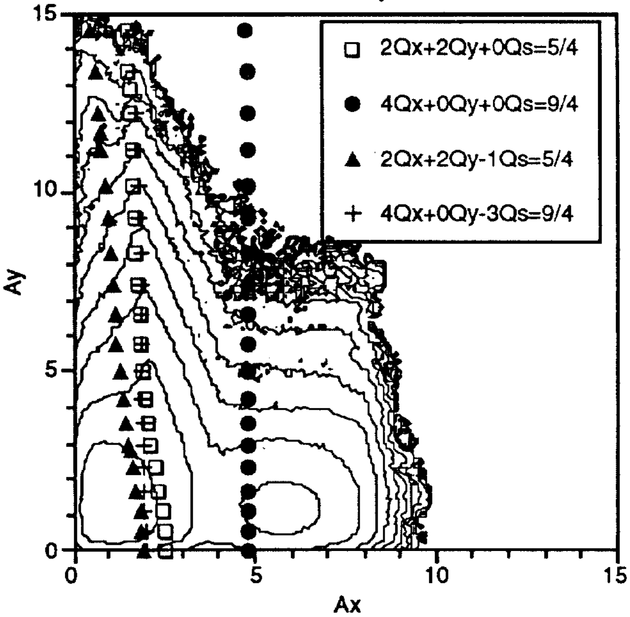


Figure 2. LEP beam-beam tail distribution with lattice including errors. The tunes in the legend refer to 1/4 of the total tunes.

The horizontal tail is believed to be related to the resonance $4Q_x$=9/4. This is a low order resonance forbidden by symmetry. As the result, a peak at A_x=5.5, A_y=1 is formed. Because the horizontal tune of LEP is very close to the 4th integer resonance, there is a good reason that this resonance is very strong. This could cause serious problems in lifetime limited by a horizontal aperture. Figure 3 plots the lifetime as a function of horizontal aperture for the perfect symmetric lattice (figure 1) and the accelerator with errors (figure 2) respectively.

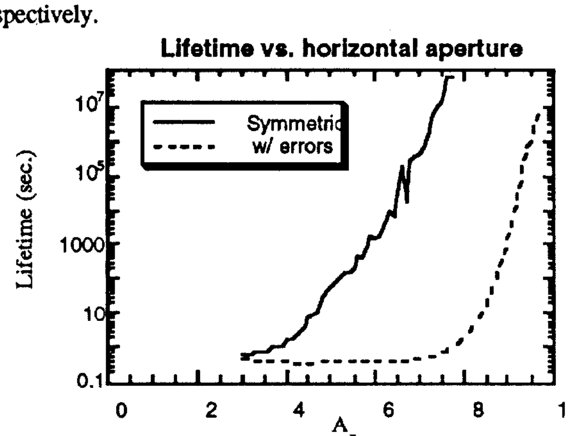


Figure 3. Lifetime versus horizontal aperture for LEP with and without errors.

Following the discussion in the previous section, we checked the sum of vectors $\exp\{i2\pi(p\Delta Q_x^b + r\Delta Q_y^b)\}$ for the particular errors in figure 2. We found the magnitude of the sum for resonances $2Q_x+2Q_y$ is 3.53, while for resonance $4Q_x+0Q_y$ is 2.44. (The maximum of this number is 4, which means the vectors are aligned). Considering $4Q_x+0Q_y-3Q_s$ =9/4 is a 7th order resonance, we tend to believe that resonance $2Q_x+2Q_y+0Q_s=5/4$ is the major contributor driving the tail.

IV. EFFECT OF COHERENT MOTION OF THE STRONG BEAM

Strong coherent quadrupole motion has been observed in LEP operation[6]. The depth of the beam size modulation can be as large as ±20%. Coherent motion has been found significant effect in the beam halos of proton linacs[7]. To answer the question whether the modulation contributes to the tail distribution, this effect is tested in the simulation. Since our program is based on strong weak picture, we cannot simulate the coherent core motion. However, we can include the motion in the strong beam and look the response of the weak beam tail. The horizontal beam size of the strong beam is modulated by ±20% at a frequency of twice of the horizontal tune. Figure 4 shows the results with and without coherent motion in the strong beam. To get this result, a symmetric lattice is used, but certain lattice nonlinearities, tuneshift with amplitudes and energy spread, are included[3]. Figure 5 gives the lifetime for these cases. These results suggest that the core modulation does not make dramatic change in the tail distribution.

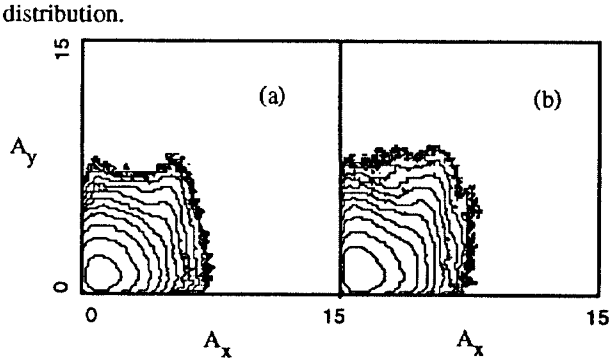
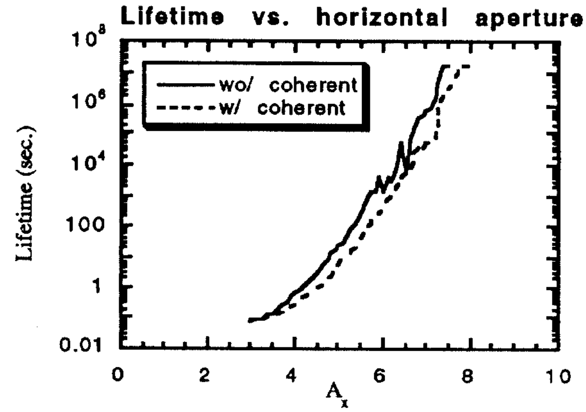


Figure 4. LEP beam-beam tail distributions. (a) without coherent motion; (b) with strong beam quadrupole mode coherent motion.

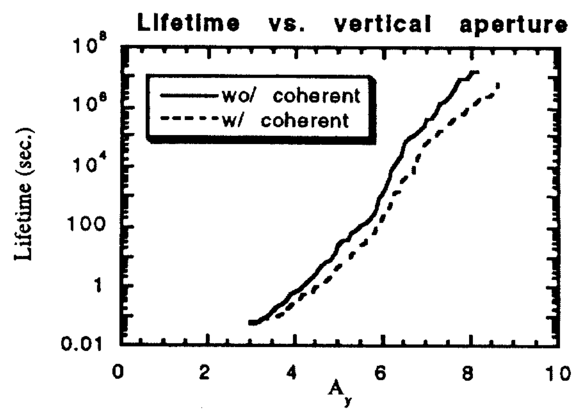
V. CONCLUSIONS

More simulation studies on beam-beam tail distribution and lifetime have been carried out. Resonances have the most important role in the tail formation. In a previous paper[3], we have discussed lattice nonlinearities and chromatic effects that make dramatic changes in tail distributions. Here, we discussed the errors in a multiple IP machine that introduce many low order resonances. All these effects result in modifying resonance structure and can be interpreted by analysis the structure.

From our study, we found that the tail distribution is sensitive to uncertain conditions, such as errors, lattice nonlinearities, chromaticities, etc. We believe that this is the reason that it is difficult to get good agreement between simulation and accelerator experiments. We are trying to understand more about this problem and pushing toward a reasonable comparison between simulation and experiment.



(a)



(b)

Figure 5. Lifetime as function of (a) horizontal and (b) vertical aperture, with and without strong beam coherent motion.

VI. REFERENCES

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