'Small and Large' Emittances in the CLIC Main Linac

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Abstract

In the context of the study presented in this paper, 'small' emittances are the direct measure of the transverse dilution of one single bunch travelling along the linac in static conditions of the parameters and misalignments, or at most in the presence of drifts that are slow with respect to the time interval between corrections. In contrast with this definition, 'large' emittances include, in addition, the effective pulse-to-pulse spread of the single-bunch emittance; this smear is due to the random walk of the ellipse in the phase space, which is a consequence of active element strength- and position-jitters, fast enough that any correction becomes impossible. Both kinds of emittances have been estimated numerically for the main linac of the CERN Linear Collider (CLIC), considering different sets of parameters. The results provide us with complementary information on the tolerances required according to the conditions retained and with the time dependence of the imperfections taken into account.

1. STARTING PARAMETERS

To compare the two kinds of emittances defined in the Abstract (see also Ref. [1]), let us start from a consistent set of parameters that include the most recent developments of the CLIC study. The first characteristic is the independent scalings of the cell length $2L_c$ and focal distance f [2], with α_a and α_q exponents different from 0.5. Since an optimum was found for $\alpha_a = 0.3$ and $\alpha_q = 0.6$ in static conditions, we begin with these values and the corresponding Twiss functions [2], though slightly higher values seem preferable in the presence of jitter (Section 3). The second characteristic, of importance to simultaneously minimize the energy spread and to preserve the transverse emittance at the linac exit, is the shape of particle distribution at injection. As suggested [3], longitudinal bunch-shaping aiming at a steeply rising charge density tends to reduce the energy spread $\langle \Delta p / p \rangle$, since the resulting longitudinal wakefield increase is closer to the sinusoidal shape of the RF voltage used in the compensation.

In CLIC, we looked therefore for a charge distribution, Gaussian-like but strongly truncated in the front and weakly truncated in the rear (Fig. 1). With such a distribution and a given charge q_b, the bunch length and the RF phase have to be adjusted for a minimization of $\langle \Delta p / p \rangle$ (Fig. 1, for $q_b = 10^{10}$).

But this process has to satisfy two conditions: (a) the particle density as function of the deviation in energy within the bunch must stay within the final-focus (FF) acceptance; (b) it should draw a 'petal-like' loop as symmetric and narrow as possible (Fig. 2). Since these conditions apply to the extracted bunch while the truncation is done before injection, an optimization requires iterations. They provided for $q_b = 6 \times 10^9$, a bunch length σ_z of 0.18 mm r.m.s and an RF phase of 11° (Fig. 2). Figure 3 shows the resulting energy distribution that fits the FF acceptance now equal to ~ 1% [4] and provides an energy spread of 0.15%. It was obtained by a truncation of the head at $+1.2\ \sigma_z$ and of the tail at $-2\ \sigma_z$. Studies of the bunch compressor in the injection scheme convinced us that such a bunch-shaping is possible by momentum collimation and subsequent rotation of the longitudinal emittance [5].

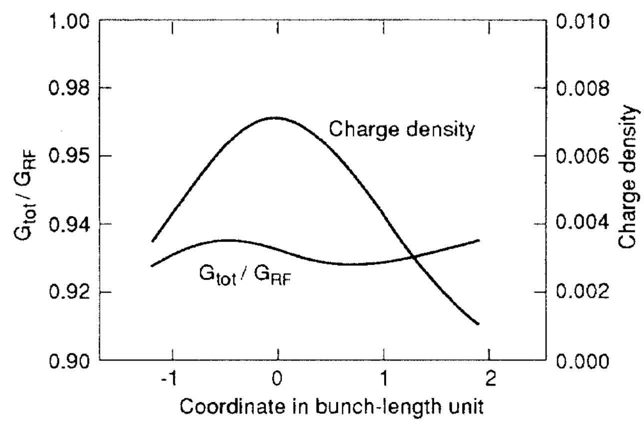

Figure 1. Relative accelerating voltage resulting from a truncated charge distribution.

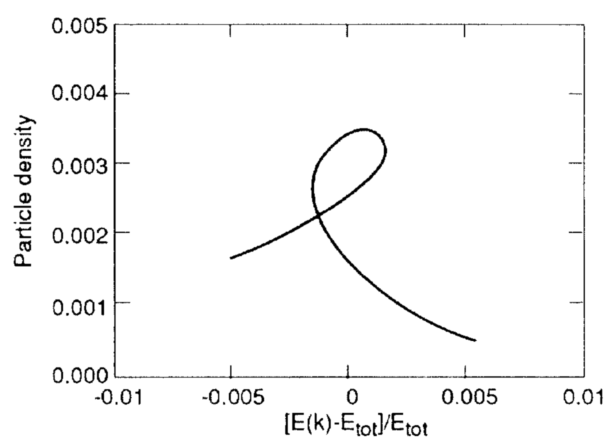

Figure 2. Energy deviation of slices of beam.

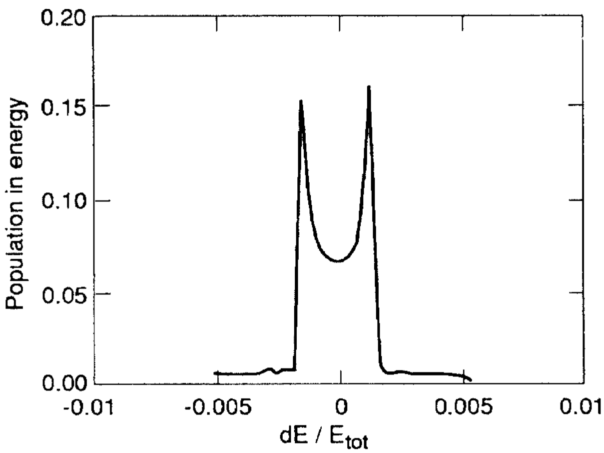

Figure 3. Energy distribution at linac end.

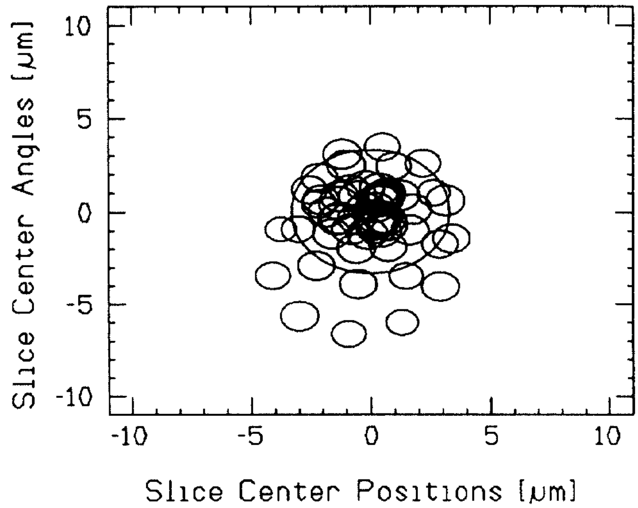

Figure 4. 'Small' emittance taken as reference.

2. 'SMALL' EMITTANCE

The 'small' emittance is a measure of the single-bunch dilution in static conditions or in presence of slow drifts. For CLIC with 500 GeV centre-of-mass energy and 9 GeV injection energy, the beam is fully relativistic and its 'small' emittance at linac exit can be represented by the superposition in the phase space of transverse 'slices of beam' related to different positions in the bunch. All previous simulations have been done on this basis and examples of transverse-emittance dilution with trajectory correction by iterations of a one-to-one (or one-to-few) algorithm are presented for instance in Ref. [2]. In the flat-beam assumption, the more critical result concerns the vertical emittance, the normalized value of which is equal to 0.5×10^{-7} rad·m at injection. Brief statistics show that the average value at extraction is then around 2×10^{-7} (by comparison the horizontal emittance blows up from 1.5×10^{-6} to ~ 1.8×10^{-6} on average). According to what precedes, the 'small' emittance (or its σ_s-matrix) can be depicted by the emittance ellipses of each slice, defined by their σ_i-matrices and their offset coordinates c_i. The 'small' emittance of a single bunch is then written as

$$\begin{aligned} \sigma_s(j,j) &= \sum_i w_i\, \sigma_i(j,j) + \sum_i w_i \left[\bar{c}(j) - c_i(j)\right]^2 \\ \sigma_s(j,k) &= \sum_i w_i\, \sigma_i(j,k) + \sum_i w_i \left[\bar{c}(j) - c_i(j)\right]\left[\bar{c}(k) - c_i(k)\right] \end{aligned} \tag{1}$$

where w_i is the fractional charge of slice i, and $\bar{c}$ the average coordinates of the whole bunch. Figure 4 gives the graphic representation of these slice ellipses in the normalized phase-space (angles multiplied by the β-function) as well as the 2σ contour of a typical vertical 'small' emittance of 2.1×10^{-7} rad·m, though it is obtained with scaling exponents $\alpha_a = 0.35$ and $\alpha_q = 0.65$ (instead of 0.3 and 0.6 as in Ref. [2]).

3. 'LARGE' EMITTANCE

If there is a jitter, i.e. a pulse-to-pulse variation in the strengths and positions of the active elements of the linac, two things happen: (a) the dilution of a single bunch is slightly different for each pulse, and (b) the centre of gravity of the 'small' emittance wanders in the phase space as its coordinates change with time. Since fast enough corrections are essentially impossible, this random walk of the bunch ellipse translates into a situation where the $e^+ e^-$ bunches do not collide exactly head-on anymore at the interaction point, reducing correspondingly the luminosity. This reduction can be worked out by considering the concept of a 'large' emittance, defined as the phase-space area including all the single-bunch emittances smeared by the jitter, and calculated around the average position taken over a large number M of pulses. Including the coordinated $\bar{\bar{c}}$ resulting from this averaging, the mathematical definition of the 'large' emittance (or of the associated σ_ℓ-matrix) becomes

$$\begin{aligned} \sigma_\ell(j,j) &= \frac{1}{M}\left[\sum_m \sigma_{s,m}(j,j) + \sum_m \left[\bar{\bar{c}}(j) - \bar{c}_m(j)\right]^2\right] \\ \sigma_\ell(j,k) &= \frac{1}{M}\left[\sum_m \sigma_{s,m}(j,k) + \sum_m \left[\bar{\bar{c}}(j) - \bar{c}_m(j)\right]\left[\bar{\bar{c}}(k) - \bar{c}_m(k)\right]\right] \end{aligned} \tag{2}$$

where $\sigma_{s,m}$ and $\bar{c}_m$ are the single-bunch quantities of Eqs. 1 (m being the bunch index).

It results from Eqs. (2) that the 'large' emittance is the quadratic sum of the single-bunch emittance and of the square of the offsets. Thus, an estimate of the second term will give an approximation of the jitter effect. Let us show how it is possible to do that for uncorrelated motions of the quadrupoles. Starting from the expression (3.8.4) of Ref. [6], a statistical addition including the scaling of strength $K\ell$ and function β gives for the contribution to emittance of a position-jitter $\langle\delta y\rangle$ (index n numbering the quadrupoles)

$$\frac{\gamma_f\langle y_c^2\rangle}{\beta_f} = (K\ell)_i^2\,\frac{\hat{\beta}_i+\check{\beta}_i}{4}\,\gamma_i^{2(1-\alpha_q)-\alpha_b}\langle\delta y^2\rangle\sum_n \gamma_n^{\alpha_r}. \quad (3)$$

The index i refers to the values at injection, the dependence on energy being rejected on the exponents of γ with $\alpha_r = 2\alpha_q + \alpha_b - 1$ (α_b is the scaling exponent of β average). Using the definitions

$$\gamma_n^{\alpha_r} = \gamma_i^{\alpha_r} + n\cdot d \quad \text{and} \quad d = \frac{\gamma_f^{\alpha_r} - \gamma_i^{\alpha_r}}{N_q}, \quad (4)$$

where N_q is the number of quadrupoles, together with the thin-lens approximations for $K\ell$, $\hat{\beta}$ and $\check{\beta}$ one gets

$$\frac{\gamma_f\langle y_c^2\rangle}{\beta_f} = \frac{2}{L_c}\,\mathrm{tg}\frac{\psi}{2}\,\gamma_i\langle\delta y^2\rangle\sum_n\left(1+n\frac{d}{\gamma_i^{\alpha_r}}\right), \quad (5)$$

and the last sum can be approximated by $N_q + N_q^2 d/2\gamma_i^{\alpha_r}$. Using the CLIC scaling exponents quoted above, a phase advance ψ of 90°, an initial period of 7 m, an injection energy of 9 GeV, and a number of quadrupoles close to 400, Eq. (5) gives an emittance contribution of $\sim 2\times10^{-8}$ rad·m (to a nominal of 2×10^{-7}) for a quadrupole jitter of 50 nm. Extending the treatment to accelerating cavities by replacing the focusing strength $K\ell$ with the wakefield intensity shows that the contribution is smaller by $\sim 10^4$. Hence, for the same blow-up of 10%, the cavity jitter amplitude can be $\sim$ 100 times larger (microns).

4. RESULTS AND DISCUSSION

Numerical simulations were based on the descriptions of Sections 2 and 3 and the parameters of Section 1. Three different jitters, uncorrelated since they are most dangerous when resonant situations are avoided, have been considered: jitter in the positions of the quadrupoles δy_q and of the cavities δy_c, and jitter in the quadrupole excitation. The dilution limitation was set arount 10% in the vertical plane (most critical). Section 3 tells us to expect then values of δy_q near 50 nm and of δy_c close to 5 µm. However, a 10% smear was already reached with δy_q = 10 nm when the 0.3/0.6 energy scaling was used. This explains the change to a 0.35/0.65 scaling (396 quadrupoles) which provided consistent results. Figures 5 and 6 give the 'large' emittances obtained with δy_q = 50 nm and 100 nm respectively. They correspond to dilutions of 30% and by a factor 2.5 approximately. Scaling down quadratically to get the 10% required gives δy_q = 30 nm. A jitter δy_c = 3 µm on cavities gave almost exactly the 10% growth allowed, while an excitation jitter on the quadrupoles of 0.5‰ generates a ~ 25% blow-up. The quasilinear dependence observed in the last effect implies a stability of 0.2‰ to satisfy the given condition.

Numerical results are in good agreement with statistical estimates and provide for CLIC the following jitter tolerances (r.m.s): 30 nm and 3 µm on quadrupole and cavity positions respectively, and 2×10^{-4} on quadrupole current stability. If the last two are probably achievable within design conditions, the first one is tight and might require active damping of the vibrations. It is, however, consistent with the 14 nm updated value for the Next Linear Collider [6] and remains above the amplitude of the uncorrelated seismic vibrations observed in the LEP tunnel [7].

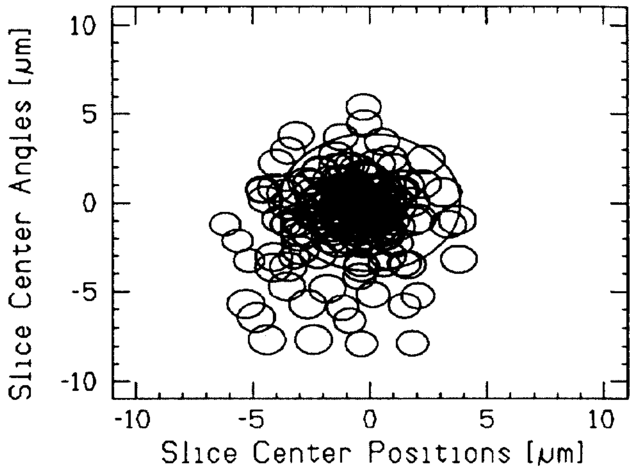

Figure 5. 'Large' emittance with 50 nm jitter on quadrupoles.

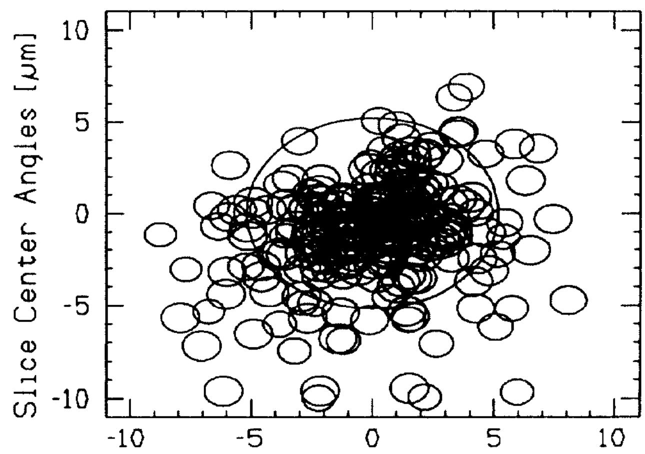

Figure 6. 'Large' emittance with 100 nm jitter on quadrupoles.

5. REFERENCES

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