Numerical Modeling of a Small Recirculating Induction Accelerator for Heavy-Ion Fusion*

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Abstract

A series of small-scale experiments has been proposed to study critical physics issues of a circular induction accelerator for heavy-ion fusion. Because the beam dynamics will be dominated by space charge, the experiments require careful design of the lattice and acceleration schedule. A hierarchy of codes has been developed for modeling the experiments at different levels of detail. The codes are discussed briefly, and examples of the output are presented.

1. Introduction

The idea of a circular induction accelerator or "recirculator" was introduced as a reduced-cost driver option for heavy-ion fusion (HIF). This configuration retains the best features of induction accelerators, namely high efficiency and peak current, and the ability to compress pulses during acceleration. In addition, the recirculator reduces the cost of a driver by using accelerator components repeatedly during acceleration, thereby reducing the size and cost of induction modules.

A small-scale experiment to validate the recirculator concept is being planned at the Lawrence Livermore National Laboratory in collaboration with Lawrence Berkeley Laboratory. Based on earlier theoretical work [1], it has been determined that a ring with a 4.4-m diameter is adequate to test most physics and technology issues facing a full-scale driver. As presently conceived, the ring would consist of 40 half lattice-periods, with permanent-magnet alternating-gradient quadrupole focusing and time-varying electric dipoles to bend the beam. Singly ionized potassium ions will be used, and during acceleration over 15 laps, the energy will increase from 80 to 320 keV while the current increases from 2 to 8 mA. Despite the small size of the proposed recirculator, it is designed to reproduce the dynamics of a full-scale driver. With an initial unnormalized emittance of 0.1 mm-mrad, the transverse dynamics is dominated by space charge, as in a driver. Furthermore, the 36-cm half lattice period is scaled from a driver to give the same dimensionless current or "perveance" as a driver. The circulation time of potassium ions at the chosen energy is similar to that of bismuth ions in a driver, so both machines have a peak repetition rate in the 50-100 kHz range. The need for high-precision control over acceleration voltages, bend fields, pulse shape, and centroid position is the same in the small experiment and a driver, and similar strategies for steering can be used in both.

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Because of the high space-charge density of the beam and the complicated field geometry, conventional accelerator codes are not adequate for modeling the small recirculator. Instead, a hierarchy of codes has been developed for analyzing the beam at different levels of detail. A pair of design codes employ simple analytic models to provide internally consistent beam and lattice parameters, as well as suitable waveforms for acceleration, bending, and longitudinal control. A macroscopic-dynamics code CIRCE [2] is used for jobs requiring iteration or a survey of parameter space, such as working out an acceleration schedule and determining tolerances to lattice or beam errors. Detailed beam dynamics is studied by the three-dimensional (3D) particle-in-cell (PIC) code WARP3d [3]. This code has proven invaluable in assessing the importance of the nonlinear field components of the electric dipoles and the permanent-magnet quadrupoles. Each of these codes is briefly described in this paper, and illustrative results are presented.

2. Design Codes

2.1 MATHEMATICA design code

Much of the initial scoping of the recirculator has been carried out using a systems-design code based on MATHEMATICA. The code is built on a collection of beam and accelerator formulas, and it exploits the graphics and numerical capabilities of MATHEMATICA to calculate hundreds of beam and accelerator quantities and to plot diagrams of the physical layout of the half-lattice period, the full ring, and the insertion/extraction section. Many "rules of thumb" are incorporated into the code, so that improper aspect ratios and other unsatisfactory regimes are avoided automatically as various design changes are explored.

2.2 REC design code

Using lattice and beam parameters from the MATHEMATICA design code, REC generates ideal acceleration waveforms required for self-similar current amplification. The method for finding waveforms, which ignores longitudinal space-charge effects, was first described by Kim and Smith [4]. The functional form of the current versus time at a fixed location is preserved throughout the accelerator, while the magnitude increases as the bunch shortens in time. Exact solutions for the current and the accelerating waveforms at every acceleration gap are determined completely by specifying the gap voltage as the beam head and tail enter each gap. REC also calculates the time-dependent dipole fields needed to bend the beam and the time-dependent E_z fields, commonly called "ear" fields, needed to balance the longitudinal space charge. Analytic

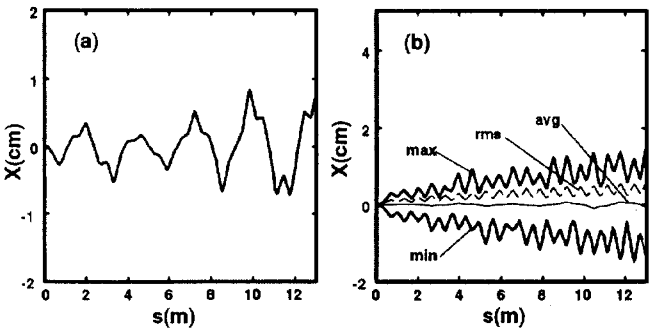

Fig. 1 CIRCE calculations of (a) centroid displacement of a beam slice for the first lap in a lattice with errors in quadrupole and dipole alignment and dipole strength, and (b) the maximum, minimum, and average displacement for 100 such runs using different errors.

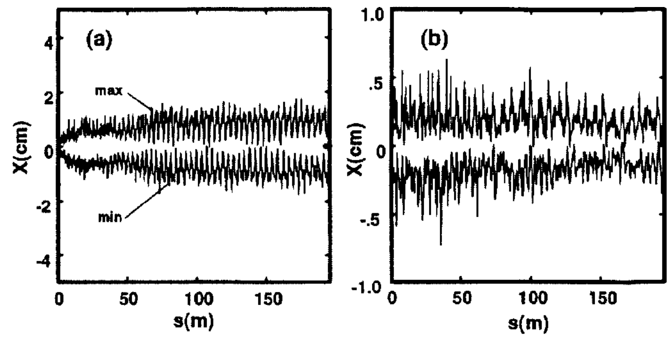

Fig. 2 CIRCE calculations of beam-centroid displacement over 15 laps in a lattice having random 1% rms errors in the bend-field strength (a) with no steering and (b) with time-independent steering at two steering stations per lap. Both plots show maximum and minimum values for 10 runs using different errors.

expressions, similar to those in the MATHEMATICA design code, are used to estimate such beam quantities as the transverse dimensions, current, energy, duration, and phase advance, and these quantities are plotted as functions of time along with the waveforms. This code permits the "trial-and-error" investigation of strategies for self-similar beam compression.

3. CIRCE

3.1 Code Description

CIRCE is a fast-running macroscopic beam-dynamics code developed to facilitate rapid design and analysis. The code uses a truncated moment or "envelope" description of transverse dynamics, with equations for the radii and the centroid co-ordinates of the elliptical beam. Equations for longitudinal dynamics are obtained by treating the beam as a Lagrangian fluid. Appropriate terms are included to account for the effects of image forces, beam emittance, and space charge in the limit of paraxial motion, and the beam is focused and accelerated by a user specified lattice of electric or magnetic dipoles and quadrupoles.

Extensive modification of CIRCE has been required to model features of the small recirculator that were not found in previously studied accelerators. Two types of electric dipoles have been added, along with the appropriate image forces The available types are "box" dipoles, which have flat plates, and "sector" dipoles, having plates that curve in concentric arcs. Different algorithms have been developed for automatically setting the bend fields in the two types, and centroid-dependent energy jumps are introduced when the beam enters or exits a dipole to account for the variable electric potential along the trajectory. In addition, high-frequency modulators using field-effect transistors will supply the acceleration voltage on the small recirculator, replacing conventional constant-format pulse-forming networks. To model the variable-format waveforms from these pulsers, the specification of acceleration fields in CIRCE has been changed to allow the field for each induction module to be specified separately on every lap.

3.2 Error Tolerances

Since CIRCE can follow a full-length beam for 15 laps around the small recirculator in about one minute of computer time on a CRAY2, it is well-suited to working out tolerances to lattice and beam errors. Such studies require a statistical treatment of many runs using different random errors. The code has been used so far to set tolerances for dipole and quadrupole errors. Based on a requirement that the beam centroid excursion remain less than 1.5 cm during the first lap in the absence of steering, we have specified root mean-square (rms) tolerances of ± 0.25 mm for quadrupole transverse alignment, $\pm 1\%$ for the dipole strength, and $\pm 0.1^\circ$ for the dipole roll angle. A typical slice trajectory with these errors is shown in Fig. 1a, and the envelope of maximum and minimum excursions for 100 such runs is presented in Fig. 1b, along with the average and rms values. These results underscore the need for steering during multiple-lap operation.

3.3 Steering

So far, only a very simple steering algorithm for the small recirculator has been tested. The procedure consists of estimating the beam trajectory from three beam-position monitors, and then returning the beam to the design trajectory by two suitable kicks at nearby dipoles. Although no provision is made in this scheme for error correction, it has been found to work acceptably in CIRCE runs of the small recirculator, even with realistic monitor errors. The calculated centroid displacement of a slice near the beam midpoint is shown in Fig. 2. In the absence of steering, the maximum excursion for an ensemble of 10 runs is found to be about 2 cm, as seen in Fig. 2a. With steering at two positions around the ring and monitor errors of ± 1 mm, this excursion is shown to be reduced to about 0.6 cm and to decrease from lap to laps.

4. WARP3d

4.1 Code Description

WARP is a family of codes developed specifically for modeling ion beams dominated by space-charge. There

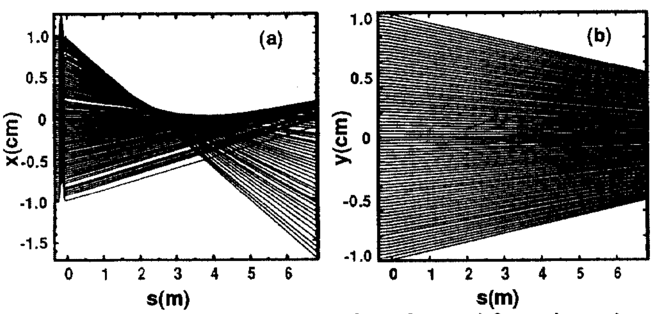

Fig. 3 WARP3d calculations of single particle trajectories through a single 9° bend, followed by a 7 m straight drift section.

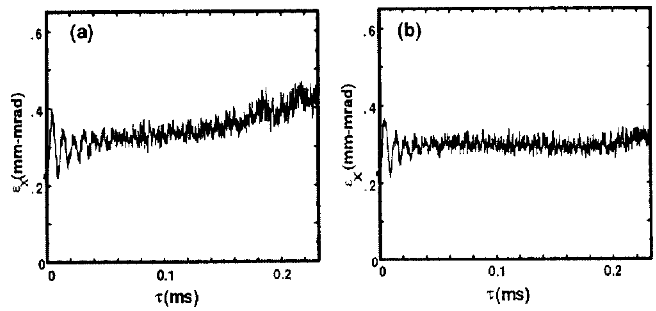

Fig. 4 WARP3d calculations of x-emittance growth over 15 laps in a lattice with (a) electric "box" dipoles and (b) electric sector dipoles.

are three principal WARP packages: a 3D PIC package WARP3d that uses "first-principles" models for beam dynamics as well as most lattice fields; an $r-z$ PIC modules WARPrz, used primarily to examine instability growth over long transport distances; and an envelope-equation solver, used by the other two packages for loading particles. The three packages are run through an interpreter that facilitates modular programming and provides a flexible interactive user-interface.

Simulations on WARP3d are done in the laboratory reference frame, and the computational mesh fills a moving window and is laid down at each time step. The beam self-field in taken to be electrostatic and is obtained by a Poisson solver. The accelerator lattice is a user-specified set of acceleration modules, and electric or magnetic focusing and bending elements. Dipole strengths can be set automatically using the bend radius and beam energy, but for most lattice elements the position, strength, and other parameters must be explicitly specified. For several of these elements, the user may choose among representations that have different levels of physicality. For example, electric dipoles may be represented by idealized "hard-edged" fields, varying linearly across the beam pipe, by hard-edged potentials, which account for the energy jumps entering and exiting the dipole, or by realistic fields, including fringe fields, calculated by solving Laplace's equation with the physically correct boundary conditions. Similar choices are available for electric and permanent-magnet quadrupoles. Although the more realistic models invariably entail longer computer runs, the added detail is necessary for credible modeling of the small recirculator.

4.2 Nonlinear Focusing

WARP3d has proven invaluable in understanding the focal properties of the electric dipoles in the small recirculator. For box bends, the focusing is quite different in the two transverse directions. The potential fluctuation along the beam trajectory leads to linear focusing in the plane of the accelerator (x-direction), and the fringe fields outside the plates contribute additional nonlinear focusing. In contrast, there is linear focusing in the y-direction resulting from the finite size of the dipole plates. The focusing in both directions is illustrated by the single-particle trajectories shown in Fig. 3. This focusing is also found to be sensitive to the transverse placement of the plates (or the location of the ground plane), the relative length of the two plates, and the proximity of the circular beam pipe, complicating the development of an optimum dipole design. Other WARP3d simulations have given preliminary indication that nonlinear components of the permanent-magnet quadrupoles will not be a problem.

4.3 Emittance Growth

The nonlinear fringe fields and energy jumps associated with the electric dipoles have been viewed as possible sources of emittance growth in the small recirculator. WARP3d simulations indicate that the energy jumps are the larger cause of emittance growth. This growth can be reduced by appropriately shaping the dipole plates. In the simulations shown in Fig. 4, the emittance jumps upon entry into the ring by about 50% due to an initial beam mismatch. With flat plates, the in-plane emittance then rises an additional 35% above this "matched" value over 15 laps, as seen in Fig. 4a, and the out-of plane emittance grows similarly. When sector dipoles are used, the in-plane emittance, shown in Fig. 4b, remains virtually constant after the initial jump, although the out-of-plane emittance behavior is similar to the flat-plate case. These levels of growth are well within the requirements for 15-lap operation, although the simulations do not yet incorporate possible misalignments or field-strength errors.

5. References

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