Hadron Production by Two Photons
Some General Comments on Form Factors

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Abstract

For describing the production of hadrons by two colliding photons, a general analysis of the vertex function is presented. As well, the quark/parton model is used. Thereby, the four-point-Greens function is considered as to be composed by two vertices and the complete Fermion propagator. Additional contributions are neglected.

1. INTRODUCTION

Because of interest to investigate the contribution of rates of the process $\gamma\gamma \to$ hadrons in linear e^+e^- colliders by virtue of beam beam radiation, a general analysis of the underlying structure of the vertex function independent of any argument of perturbation theory is presented. The corresponding 4 point Greens function of the underlying process of $\gamma\gamma \to$ hadrons is considered as to result from two vertex functions and the complete fermion propagator. Additional contributions of Feynman diagrams leading to one-particle irreducible 4 point Greens function are neglected. The general structure of the matrix element describing the Compton effect is determined by six functions (four form factors and two functions involved in the fermion propagator). In the vicinity of the parton mass shell, the two functions involved in the fermion propagator as usual exhibit the pole charcater. For establishing the matrix element of the underlying process of hadron production by two photons corresponding substitutions of 4 impulse variables are performed as motivated e.g. in quantum electrodynamics.

2. SOME GENERAL REMARKS

In order to describe the $\gamma\gamma \to$ hadrons production process, for the following, general considerations with respect to the properties of the underlying matrix element are presented. At first, the basic structure of a vertex with one photon line and one fermion line at mass shell and one virtual fermion line is investigated.

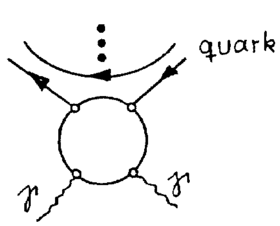


Figure 1. Four point Greens function.

Later on, the general 4 point Greens function [1] is considered as to be composed of two vertices and one complete fermion propagator, thereby from the outset neglecting all those contributions, which are 1 particle irreducible in the above mentioned Greens function. As well, the quark/parton [2], [3] model is used in the subsequent presentation.

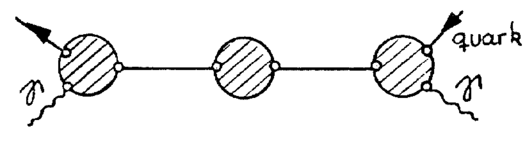


Figure 2. Four point Greens function without 1-particle irreducibele contributions.

3. GENERAL FERMION PROPAGATOR

The most general structure of spinor matrix valued propagator function $(q) as a linear combination of the 16 linear independent matrices

$$\left\{1, \gamma_\alpha, \sigma_{\alpha\beta} := \frac{i}{2}\left[\gamma_\alpha \gamma_\beta\right]_-, \gamma_5\gamma_\alpha, \gamma_5\right\} \quad \text{reads:}$$

$$\$(q^\mu) = a(q^\mu)1 + b^\alpha(q^\mu)\gamma_\alpha + c^{\alpha\beta}(q^\mu)\sigma_{\alpha\beta} \qquad (1)$$

from the outset negelecting parity-violating contributions (only strong and electromagnetic interaction). The scatar-valued, vector-valued, tensor-valued coefficient functions in (1) satisfy

$$\begin{aligned} a(q^\mu) &= \underline{a}(q^2) \quad ; \; c^{\alpha\beta}(q^\mu) = \underline{c}(q^2)q^{\alpha\beta} + \\ b(q^\mu) &= \underline{b}(q^2)q^\alpha; \qquad\qquad + \underline{\underline{c}}(q^2)q^\alpha q^\beta \end{aligned} \qquad (2)$$

where $\underline{a}, \underline{b}, \underline{c}, \underline{\underline{c}}$ depend on $q^2 = q^\nu q_\nu$ only, thus

$$\text{yielding } \$ = \frac{1}{\underline{B}(q^2)q + \underline{A}(q^2)} = \frac{Z(q^2)}{q - m(q^2)}; \qquad (3)$$

$$Z(q^2) = \frac{1}{\underline{B}(q^2)}, m(q^2) = -\frac{\underline{A}(q^2)}{\underline{B}(q^2)}$$

by in addition taking into account $c^{\alpha\beta}(q^\mu)\cdot\sigma_{\alpha\beta} = 0$ because of the symmetry $\sigma^{\alpha\beta}$ and because of antisymmetry of $\sigma_{\alpha\beta}$. The representation of $ in (3) is chosen such that immediately the pole structure can be read off. Obviously, the introduction of appropriate functions

$$\underset{\sim}{A}=\frac{\underline{a}(q^2)}{\underline{a}^2(q^2)-\underline{b}^2(q^2)\cdot q^2};\underset{\sim}{B}=\frac{\underline{b}(q^2)}{a^2(q^2)-\underline{b}^2(q^2)\cdot q^2} \quad (4)$$

permits to shift q into the denominator. In the vicinity of quark/parton mass shell, $ can be specified according to

$$\$=\underset{\sim}{a}+q\,\underset{\sim}{b}=\frac{1}{q-m_{part}};$$

$$\underset{\sim}{a}(q)=\frac{m_{part}}{q^2-m_{part}^2};\ \underset{\sim}{b}(q)=\frac{1}{q^2-m_{part}} \quad (5)$$

where the usual renormalization procedure

$$e=e_0\left[\hat{Z}\right]^{\frac{1}{2}};\ \hat{Z}(m_{part}^2):=\hat{Z}(m_{part}^2)/\left\{1-\frac{1}{16}m^{2^*}(m_{part}^2)\right\}$$

has been used. (6)

4. General Vertex Function

By neglecting parity violating contributions, the corresponding general representation of the vertex function ϑ^μ reads:

$$\vartheta^\mu(k,p)=\hat{a}^\mu(k,p)\mathbf{1}+\hat{b}^\mu(k^\alpha,p)\gamma_\alpha+ +\hat{C}^\mu(k^{\alpha\beta},p)\sigma_{\alpha\beta} \quad (7)$$

where the coefficient functions

$$\hat{a}^\mu(k,p)=\underline{a}k^\mu+\underline{b}p^\mu$$

$$\hat{b}^{\mu\alpha}(k,p)=\underline{c}q^{\mu\alpha}+\underline{d}k^\mu k^\alpha+\underline{e}k^\mu p^\alpha+\underline{f}p^\mu k^\alpha+gp^\mu p^\alpha$$

$$\hat{c}^{\mu\alpha\beta}(k,p)=\underline{h}(q^{\mu\alpha}k^\beta-q^{\mu\beta}k^\alpha)+\underline{n}(q^{\mu\alpha}p^\beta-q^{\mu\beta}p^\alpha)+$$

$$=\underline{r}(k^\mu\cdot p^\alpha k^\beta-k^\mu p^\beta k^\alpha)+\underline{s}(p^\mu p^\alpha k^\beta-p^\mu p^\alpha k^\beta) \quad (8)$$

are determined by only eleven scalar valued functions depending on q^2 (note, that $k^2=0;\ p^2=m^2;\ k\cdot p=1/2\{q^2-m^2\}$), thus yielding

$$\begin{aligned}\vartheta^\mu(k,p)&=\underline{a}k^\mu+\underline{b}p^\mu+\\&+\{\underline{c}\gamma^\mu+\underline{d}k^\mu \not{k}+\underline{e}k^\mu \not{p}+\underline{f}p^\mu \not{k}+gp^\mu \not{p}\}+\\&+\{\underline{h}\{k^\beta\sigma_\beta^\mu-k^\alpha\sigma_\alpha^\beta\}+n\{p^\beta\sigma_\beta^\mu-p^\alpha\sigma_\alpha^\mu\}+\\&+\underline{r}k^\mu\{p^\alpha k^\beta\sigma_{\alpha\beta}-p^\beta k^\alpha\sigma_{\alpha\beta}\}\}+\\&+\underline{s}p^\mu\{p^\alpha k^\beta\sigma_{\alpha\beta}-p^\beta k^\alpha\sigma_{\alpha\beta}\}.\end{aligned} \quad (9)$$

In electrodynamics, the relation $k^\mu\varepsilon_\mu(k)=0$ between photon wave number vector k^μ and polarisation vector $\varepsilon_\mu(k)$ as well as the relation $\not{p}u(p)m\cdot u(p)$

$$\begin{aligned}\vartheta^\mu(k,p)&=\underline{b}p^\mu+\{\underline{c}\gamma^\mu+\underline{f}p^\mu\not{k}+gp^\mu\not{p}\}+\\&+\{\underline{h}\{k^\beta\sigma_\beta^\mu-k^\alpha\sigma_\alpha^\mu\}+\\&+\underline{n}\{p^\beta\sigma_\beta^\mu-p^\alpha\sigma_\alpha^\mu\}+\\&+\underline{s}p^\mu\{p^\alpha k^\beta\sigma_{\alpha\beta}-p^\beta k^\alpha\sigma_{\alpha\beta}\}\}\end{aligned} \quad (10)$$

Finally,

$$(p^\alpha k^\beta\sigma_{\alpha\beta}-p^\beta k^\alpha\sigma_{\alpha\beta})u(p)=i(-2\not{k}m+2p\cdot k)u(p)$$

$$(p^\beta\sigma_\beta^\mu-p^\alpha\sigma_\alpha^\mu)u(p)=i(2\gamma^\mu m-2p^\mu)u(p)$$

permits to arrive at (11)

$$\begin{aligned}\vartheta^\mu(k,p)&=\underline{\underline{a}}(k\cdot p)p^\mu+\underline{\underline{b}}(k\cdot p)\gamma^\mu+\\&+\underline{\underline{c}}(k\cdot p)p^\mu\not{k}+\underline{\underline{h}}(k\cdot p)k^\beta\sigma_\beta^\mu\end{aligned} \quad (12)$$

An additional transcription involving the transverse momentum vector q^μ only, once more using $k^\mu\varepsilon_\mu=0$ as well, yields the result:

$$\begin{aligned}\vartheta^\mu(q^\nu)&=a^*(q^2)q^\mu+b^*(q^2)\gamma^\mu+\\&+c^*(q^2)q^\mu\not{q}+d^*(q^2)q^\beta\sigma_\beta^\mu\end{aligned} \quad (13)$$

being determined by at most four independent scalar valued functions.

5. Compton Scattering

The general matrix element of compton scattering of a photon γ at a fermion (quark/parton) is described by

$$\begin{aligned}\mathcal{M}_{fi}&=\left\langle\overline{u(-p_2)}\middle|\vartheta^\nu_{oben}\varepsilon^\nu_{II}(K_2)\$(q)\vartheta^\mu\varepsilon^\mu_I(K_1)\middle|u(p)\right\rangle\\&=\overline{u(-p_2)}\{a^*(q^2)\{-q^\nu\}+b^*(q^2)\gamma^\nu+\\&+c^*(q^2)q^\nu\not{q}+d^*(q^2)\{-q^\beta\}\sigma_\beta\}\cdot\\&\cdot\frac{Z(q^2)}{\not{q}-m(q^2)}\cdot\{a^*(q^2)q^\mu+b^*(q^2)\gamma^\mu+\\&+c^*(q^2)q^\mu\not{q}+d^*(q^2)q^\alpha\sigma_\alpha^\mu\}u(p_1)\varepsilon^\nu_{II}(K_2)\cdot\varepsilon^\mu_I(K_1)\end{aligned} \quad (14)$$

and is depicted in the following figure:

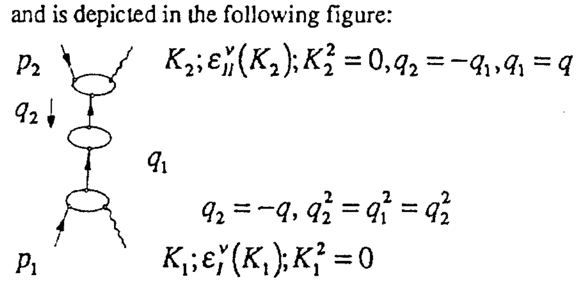


Figure 3. Copton Scattering

It is a tedious algebraic calculation which also comprises identities of Dirac γ matrices. In case of a known $\underline{a},\underline{b}$ the four newly introduced funtions can be transformed:

$$\left(a^{\bullet},b^{\bullet},c^{\bullet},d^{\bullet}\right)\xleftrightarrow{1:1}\left(A^{\bullet},B^{\bullet},C^{\bullet},D^{\bullet}\right)$$

$$d^{\bullet}=i\frac{\underline{a}C^{\bullet}+\underline{b}D^{\bullet}}{\underline{a}^2-q^2\underline{b}^2};\; b^{\bullet}=\frac{1}{\underline{b}}\frac{\underline{b}\left\{-q^2\underline{b}C^{\bullet}-\underline{a}D^{\bullet}\right\}}{\underline{a}^{\bullet}-q^2\underline{b}^2}$$

$$c^{\bullet}=\frac{B^{\bullet}}{2\underline{a}}-\frac{i\underline{b}}{2\underline{a}}i\frac{\underline{a}C^{\bullet}+\underline{b}D^{\bullet}}{\underline{a}^2-q^2\underline{b}^2}; \tag{15}$$

$$a^{\bullet}=\left\{-A^{\bullet}-q^2\left\{\frac{B^{\bullet}}{2\underline{a}}+\frac{\underline{b}}{2\underline{a}}\cdot\frac{\underline{a}C^{\bullet}+\underline{b}D^{\bullet}}{\underline{a}^2-q^2\underline{b}^2}\right\}^2\right\}^{\frac{1}{2}}$$

In the vicinity of the parton mass shell, evidently, the form factors can be expanded by using a Taylor series around $q^2=m_{part}^2$ thus yielding

$$\begin{aligned}\overline{u(-p_2)}=&\left\{q^{\nu}q^{\mu}\left\{m_{part}\cdot\hat{A}^{\bullet}+b_0^{\bullet}\hat{B}_0^{\bullet}+id_0^{\bullet}\hat{C}_0^{\bullet}\right\}\right.\\&+q^{\nu}\gamma^{\mu}\left\{a_0^{\bullet}\hat{D}_0^{\bullet}+b_0^{\bullet}\hat{C}_0^{\bullet}+m_{part}^2c_0^{\bullet}\hat{C}_0^{\bullet}\right\}\\&+q^{\nu}\gamma^{\mu}\left\{-a_0\hat{D}_0^{\bullet}+b_0^{\bullet}\hat{C}_0^{\bullet}+m_{part}^2c_0^{\bullet}\hat{C}_0^{\bullet}\right\}\\&+q^{\nu}q^{\mu}q\left\{\hat{A}_0^{\bullet}+id_0^{\bullet}\hat{B}_0^{\bullet}\right\}\\&+q^{\nu}q^{\alpha}\sigma_{\alpha}^{\mu}\left\{\left\{-ia_0^{\bullet}\right\}\hat{C}_0^{\bullet}-ic_0^{\bullet}\hat{D}_0^{\bullet}-d_0^{\bullet}\hat{C}_0^{\bullet}\right\}\\&+q^{\mu}q^{\alpha}\sigma_{\alpha}^{\nu}\left\{\left\{-ia_0^{\bullet}\right\}\hat{C}_0^{\bullet}+ic_0^{\bullet}\hat{D}_0^{\bullet}+d_0^{\bullet}\hat{C}_0^{\bullet}\right\}\\&+q^{\nu\mu}\left\{\left\{-b_0^{\bullet}\right\}\hat{D}_0^{\bullet}-m_{part}^2id_0^{\bullet}\hat{C}_0^{\bullet}\right\}\end{aligned}\tag{16}$$

(16)

$$\begin{aligned}&-i\sigma_{\mu}^{\nu}\left\{\left\{-b_0^{\bullet}\right\}\hat{D}_0^{\bullet}-m_{part}^2id_0^{\bullet}\hat{C}_0^{\bullet}\right\}\\&+q^{\alpha}\varepsilon_{\alpha}^{\nu\mu\chi}\gamma_5\gamma_{\chi}\left\{\left\{-b_0^{\bullet}\right\}\hat{C}_0^{\bullet}+d_0^{\bullet}\hat{D}_0^{\bullet}\right\}\\&-ig^{\nu\mu}q\left\{\left\{-ib_0^{\bullet}\right\}\hat{C}_0^{\bullet}+d_0^{\bullet}\hat{D}_0^{\bullet}\right\}\Big\}\varepsilon_{I\nu}\varepsilon_{II\mu}V(-p_1)\frac{1}{q^2-m_{part}^2}\end{aligned}$$

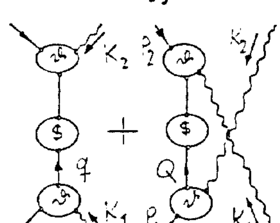


Figure 4. Two Photons and 'crossing'

In addition, taking into account the 'crossing effect', the total matrix element reads

$$\begin{aligned}\mathcal{M}_{total}=&\mathcal{M}_{fi}\left(q;K_1,p_1,K_2,p_2\right)+\\&+\mathcal{M}\left(Q;K_2,p_1,K_1,p_2\right)\end{aligned}\tag{17}$$

where $Q=p_1+K_2$ (see fig.)

6. PARTON MODEL AND SUBSTITUTION

By substitution (see fig.),

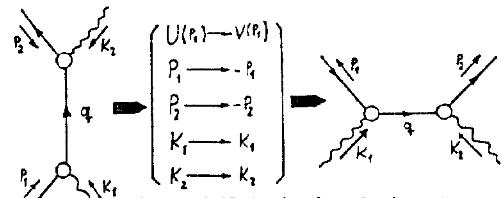
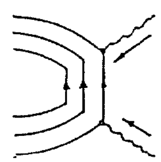


Figure 5. 'Substitution depicted'

which might be motivated by a corresponding mechanism in [3] and by using the parton model (see fig.)



$$K_2 \qquad \bar{p}_i=\zeta_i\bar{p}+p_i^T$$

$$q=K_1+\zeta_ip_1 \qquad \bar{p}\cdot\bar{p}_i^T=0,\;\zeta_i\geq 0$$

$$K_1 \qquad \sum_i\zeta_i=1$$

Figure 6. Compton Process in the Parton Model

the subsequent formula holds:

$$\overline{\left\|\mathcal{M}_{stoch,parton}\left(\gamma\gamma\rightarrow\overline{Had}\right)\right\|}^2=\sum_i r_j(\xi_i)\|\{\mathrm{M}\}\|^2,$$

$$\begin{aligned}\mathrm{M}=&\left\{\mathcal{M}_{parton}\left(K_1-\xi_jp_1;K_1;\xi_jp_1,K_2-\xi_jp_2\right)+\right.\\&\left.+\mathcal{M}_{parton}\left(K_2-\xi_jp_1;K_2;K_2;-\xi_ip_1,K_1-\xi_jp_2\right)\right\}\end{aligned}\tag{18}$$

7. SUMMARY

Taking into account the most general anaylsis of the propagator and vertex function and using the 'crossing symmetry' permit to arrive at a model for describing hadron production by two collding photons. The above-mentioned 'process rotation' (substitution) is motivated firstly by the corresponding method in Q.E.D. (invariance of trace) and secondly by the assumed additivity of the cross sections at the single partons, where usually the parton's transversal momentum is small but sufficiently great in order to form an entire hadron in connection with the remaining and nearly parallelly flying partons. It should be emphasized that the pole structure in the matrix element (16) only exists if p_1,k_1 or p_1,k_2 (incoming momenta) lead to the corresponding transfer momentum $q=k_1+p_1$, $Q=k_2+p_1$ being in the vicinity of a parton/quark mass shell. As the quark masses are not equal for all of them, obviously, in the above-mentioned case, only the summand containing the pole term yields the main contribution.

8. REFERENCES

[1] Bjorken, Drell, "Relativisitsche Quantenfeldtheorie", p. 23

[2] Jauch, Rohrlich, "The Theory of Photons and Electrons", p. 497

[3] Becher, Böhm, Joos, "Eichtheorien", p. 64